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ROBUST FILTERING FOR BILINEAR STOCHASTIC SYSTEMS: THE DISCRETE-TIME CASE

Zidong Wang [*,1] **K. J. Burnham** [**]

[*] *School of Mathematical and Information Sciences, Coventry University, Coventry CV1 5FB, U.K.*
[**] *School of MIS, Coventry University, Coventry CV1 5FB, U.K.*

Abstract: This paper deals with the robust filtering problem for uncertain bilinear stochastic discrete-time systems with estimation error variance constraints. The uncertainties are allowed to be norm-bounded, and enter into both the state and measurement matrices. We focus on the design of linear filters, such that for all admissible parameter uncertainties, the error state of the bilinear stochastic system is mean square bounded, and the the steady-state variance of the estimation error of each state is not more than the individual prespecified value. It is shown that the design of the robust filters can be carried out by solving some algebraic quadratic matrix inequalities. In particular, we establish both the existence conditions and the explicit expression of desired robust filters. A numerical example is included to show the applicability of the present method. *Copyright © 2002 IFAC*

Keywords: Bilinear stochastic systems, Discrete-time systems, Quadratic matrix inequalities, Robust filtering, Uncertain systems.

1. INTRODUCTION

The well-known Kalman filtering is one of the celebrated H_2 filtering approaches widely used in various fields of signal processing and control, see (Anderson and Moore, 1979). This filtering approach assumes that the system under consideration has known dynamics described by certain well-posed model and its disturbances are Gaussian noises with known statistics. These assumptions limit the application scope of the Kalman filtering technique when there are uncertainties in either the exogenous input signals or the system model. It has been known that the standard Kalman filtering algorithms will generally not guarantee satisfactory performance when there exists uncertainty in the system model. This has led to the recent development of alternative design methods for H_∞ filters and robust filters.

For uncertain stochastic systems, it is reasonable to evaluate the filter performance in terms of mean square error and strive for a suitable robustification of the classical Kalman filter. Therefore, the study of the so-called cost guaranteed filters which *minimize* an easy-to-compute upper bound on the worst performance has recently gained growing interest, and many significant results have been obtained, see (Geromel, 1999; Zhu *et al.*, 2001) and the references therein.

On the other hand, for a large class of practical filtering problems such as the tracking of a maneuvering target, the filtering performance objectives are naturally formulated as the upper bounds on the estimation error variances, see *e.g.* (Skelton and Iwasaki, 1993; Yaz and Skelton, 1991). In this case, the steady-state error variance is not required to be minimal, but should not be more

[1] Partially supported by the University of Kaiserslautern of Germany and the Alexander von Humboldt Foundation of Germany. E-mail: Zidong.Wang@coventry.ac.uk.

than a prescribed upper bound. Note that it is usually difficult to utilize traditional methods to deal with this class of *variance-constrained* filtering problems. Fortunately, the error covariance assignment (ECA) theory developed in (Yaz and Skelton, 1991) provides an alternative and more straightforward methodology for designing filter gains which satisfy the above performance objectives. Subsequently, the ECA theory has been extended to the sampled-data systems in (Wang, *et al.*, 2001), and to the parameter uncertain systems in (Wang and Huang, 2000), respectively.

Among many practical systems, plants may be modeled by bilinear systems (BLSs), since some characteristics of nonlinear systems can be closely approximated by bilinear models rather than linearized models. Up to now, it has been known that BLSs could describe many real processes in the fields of socioeconomics, ecology, agriculture, biology, and industry, etc., see (Mohler and Kolodziej, 1980). In particular, the observer design problem has been intensively studied in (Yaz, 1992) for discrete-time stochastic bilinear systems (also called "state-dependent noise systems" because the structured parameter *perturbations* on the system matrix are modeled as zero mean white noises (Bernstein and Haddad, 1987; Skelton *et al.*, 1991; Yaz, 1992). However, so far, the issue of variance-constrained filtering for *bilinear uncertain* stochastic systems has not been fully investigated and remains to be important.

This paper is concerned with the design of robust filters for bilinear uncertain stochastic discrete-time systems subjected to the upper bound constraints on the estimation error variance. The purpose of the problem addressed is to design the filters for the bilinear uncertain stochastic discrete-time systems such that the steady-state estimation error variances are less than the prespecified upper bounds. A simple, effective matrix inequality approach is developed to solve this problem. Specifically, a set of the upper bounds on estimation error covariances that certain bilinear error dynamic processes may obey are first presented, all filters that assign these upper bounds to the estimation error variances are then explicitly characterized, and finally the solvability of the assignability conditions is discussed. An illustrative example is used to demonstrate the effectiveness of the proposed design procedure.

2. PROBLEM FORMULATION AND PRELIMINARY RESULTS

Consider the following bilinear uncertain discrete-time stochastic system

$$x(k+1) = (A+\Delta A)x(k) + \sum_{i=1}^{m} H_i x(k) v_i(k) + w(k) \qquad (1)$$

and the measurement equation

$$y(k) = (C+\Delta C)x(k) + z(k) \qquad (2)$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^p$ is the measurement output, $w(k) \in \mathbb{R}^n$ and $z(k) \in \mathbb{R}^p$ are uncorrelated stationary zero mean white noise sequences with respective covariances $W \geq 0$ and $Z > 0$. $v(k) := [v_1(k)\ v_2(k)\ \cdots\ v_m(k)]^T \in \mathbb{R}^m$ is a vector stochastic sequence satisfying $\mathcal{E}v(k) = 0$, $\mathcal{E}v(l)v^T(j) = I\delta_{ij}$, $l, j = 1, 2, \cdots, m$, where $\mathcal{E}\{\cdot\}$ means the mathematical expectation operator. The initial state $x(0) = x_0$ is a random vector that is independent of $w(k)$ and $z(k)$. A, H_i $(i = 1, 2, \cdots, m)$ and C are known constant matrices with appropriate dimensions. The matrix A is assumed to be Schur stable and nonsingular. The matrices ΔA and ΔC, which may be time-varying, represent the norm-bounded parameter uncertainties and satisfy

$$\begin{bmatrix} \Delta A \\ \Delta C \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} FN \qquad (3)$$

where $F \in \mathbb{R}^{i\times j}$ is a real uncertain matrix with Lebesgue measurable elements and meets

$$F^T F \leq I \qquad (4)$$

and M_1, M_2, N are known real constant matrices of appropriate dimensions which specify how the uncertain parameters in F enter the nominal matrices A and C. The uncertainties ΔA, ΔC are said to be admissible if both (3) and (4) are satisfied.

Remark 1. The kind of bilinear stochastic discrete-time systems formulated by (1)-(2) *without* uncertainties has been extensively studied in many papers, see (Bernstein and Haddad, 1987; Skelton *et al.*, 1991; Yaz, 1992), and is sometimes called the "state-dependent noise system". The parameter uncertainty structure as in (3)-(4) has been widely used in the problems of robust control and robust filtering of uncertain systems (Wang and Huang 2000; Wang and Burnham 2001).

In this paper, we adopt the following linear full-order filter

$$\hat{x}(k+1) = G\hat{x}(k) + Ky(k) \qquad (5)$$

where $\hat{x}(k)$ stands for the state estimate, G and K are filter parameters to be designed.

Define the estimation error and the estimation error covariance, respectively, as follows

$$e(k) = x(k) - \hat{x}(k),\ P(k) := \mathcal{E}[e(k)e^T(k)]. \qquad (6)$$

Then, it follows from (1)-(2) and (5)-(6) that

$$
\begin{aligned}
e(k+1) = Ge(k) + [(A - G - KC) \\
+ \sum_{i=1}^{m} H_i(v_i(k)I) + \Delta A - K(\Delta C)] \\
\cdot x(k) + w(k) - Kz(k).
\end{aligned} \tag{7}
$$

Define

$$
x_f(k) := \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \; A_f := \begin{bmatrix} A & 0 \\ A - G - KC & G \end{bmatrix},
$$

$$
M_f := \begin{bmatrix} M_1 \\ M_1 - KM_2 \end{bmatrix}, \; N_f := \begin{bmatrix} N & 0 \end{bmatrix}, \tag{8}
$$

$$
\Delta A_f = M_f F N_f, \; H_f := \begin{bmatrix} \sum_{i=1}^{m} H_i(v_i(k)I) & 0 \\ \sum_{i=1}^{m} H_i(v_i(k)I) & 0 \end{bmatrix},
$$

$$
J_i := \begin{bmatrix} H_i & 0 \\ H_i & 0 \end{bmatrix}, W_f := \begin{bmatrix} W & W \\ W & W + KZK^T \end{bmatrix}, \tag{9}
$$

$$
X(k) := \mathcal{E}[x_f(k)x_f^T(k)] := \begin{bmatrix} X_{xx}(k) & X_{xe}(k) \\ X_{xe}^T(k) & X_{ee}(k) \end{bmatrix}. \tag{10}
$$

Considering (1) and (7), we obtain the following augmented system

$$
\begin{aligned}
x_f(k+1) = (A_f + \Delta A_f + H_f)x_f(k) + W_f^{\frac{1}{2}} w_f(k), \\
x_f(0) = [x_0 \;\; x_0 - \hat{x}_0]^T.
\end{aligned} \tag{11}
$$

where $w_f(k)$ stands for a zero mean Gaussian white noise sequence with covariance I.

Now, by taking the expectation of both sides of (11), we have

$$
\begin{aligned}
X(k+1) = (A_f + \Delta A_f)X(k)(A_f + \Delta A_f)^T \\
+ \sum_{i=1}^{m} J_i X(k) J_i^T + W_f
\end{aligned} \tag{12}
$$

where $X(k)$, J_i and W_f are defined in (9)-(10).

We know from (Agniel and Jury, 1971) that, if the state of the system (11) is mean square bounded, the steady-state covariance X of the system (11) defined by $X := \lim_{k \to \infty} X(k)$ exists and satisfies the following discrete-time modified Lyapunov equation

$$
X = (A_f + \Delta A_f)X(A_f + \Delta A_f)^T + \sum_{i=1}^{m} J_i X J_i^T + W_f. \tag{13}
$$

Remark 2. It is necessary to discuss the conditions for the existence of the solution to (13). It follows from (Agniel and Jury, 1971) that, there exists a unique symmetric positive semi-definite solution to (13) if and only if

$$
\rho\big\{(A_f + \Delta A_f) \otimes (A_f + \Delta A_f) + \sum_{i=1}^{m} J_i \otimes J_i\big\} < 1 \tag{14}
$$

where ρ is the spectral radius and $\otimes$ is the Kronecker product. Furthermore, we know from (Agniel and Jury, 1971) that the condition (14) is equivalent to the mean square boundedness of the state of the system (11), and hence (14) will guarantee the convergence of $X(k)$ in (12) to a constant value X.

The purpose of this paper is to design the filter parameters, G and K, such that for all admissible perturbations ΔA and ΔC, the following requirements are met simultaneously: 1) the state of the augmented system (11) is mean square bounded; 2) the steady-state error covariance X_{ee} ($X_{ee} := \lim_{k \to \infty} \mathcal{E}[e(k)e^T(k)]$ meets

$$
[X_{ee}]_{ii} \leq \sigma_i^2, \quad i = 1, 2, \cdots, n. \tag{15}
$$

where $[X_{ee}]_{ii}$ means the steady-state variance of the ith error state and σ_i^2 $(i = 1, 2, \cdots, n)$ denotes the prespecified steady-state error estimation variance constraint on the ith state.

3. MAIN RESULTS AND PROOFS

Let us first recall the some intermediate results which are introduced in the sequel as lemmas.

Lemma 1. (Wang *et al.* 1992) Let a positive scalar $\varepsilon > 0$ and a positive definite matrix $Q_f > 0$ be such that $N_f Q_f N_f^T < \varepsilon I$. Then

$$
\begin{aligned}
(A_f + \Delta A_f)Q_f(A_f + \Delta A_f)^T \\
\leq A_f(Q_f^{-1} - \varepsilon^{-1}N_f^T N_f)^{-1}A_f^T + \varepsilon M_f M_f^T
\end{aligned} \tag{16}
$$

holds for all perturbations ΔA and ΔC.

Lemma 2. For a given negative definite matrix $\Pi < 0$ $(\Pi \in \mathbb{R}^{n \times n})$, there always exists a matrix $L \in \mathbb{R}^{n \times p}$ $(p \leq n)$ such that $\Pi + LL^T < 0$.

For technical convenience, we define the following additional notation:

$$
\Phi := (P_1^{-1} - \varepsilon^{-1}N^T N)^{-1}A^T,
$$

$$
\begin{aligned}
\widehat{A} := A + (W + \sum_{i=1}^{m} H_i P_1 H_i^T \\
+ \varepsilon M_1 M_1^T)\Phi^{-1},
\end{aligned} \tag{17}
$$

$$
\widehat{C} := C + \varepsilon M_2 M_1^T \Phi^{-1},
$$

$$
\Gamma := \Phi^{-1}(P_1^{-1} - \varepsilon^{-1}N^T N)^{-1}(\Phi^{-1})^T, \tag{18}
$$

$$
\begin{aligned}
R := Z + \varepsilon M_2 M_2^T \\
+ \varepsilon^2 M_2 M_1^T \Gamma M_1 M_2^T + \widehat{C} P_2 \widehat{C}^T,
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\Theta := \widehat{A} P_2 \widehat{C}^T + \varepsilon M_1 M_2^T + \varepsilon(W + \sum_{i=1}^{m} H_i P_1 H_i^T \\
+ \varepsilon M_1 M_1^T)\Gamma M_1 M_2^T.
\end{aligned} \tag{20}
$$

Now we are in a position to establish our main results in this paper.

Theorem 1. Assume that there exist a positive scalar $\varepsilon > 0$ such that the following two quadratic matrix inequalities

$$
\begin{aligned}
& AP_1A^T - P_1 + AP_1N^T(\varepsilon I - NP_1N^T)^{-1} \\
& \cdot NP_1A^T + \varepsilon M_1M_1^T \\
& + W + \sum_{i=1}^m H_iP_1H_i^T < 0 \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
\Pi := {} & \widehat{A}P_2\widehat{A}^T - P_2 - \Theta R^{-1}\Theta^T \\
& + (W + \sum_{i=1}^m H_iP_1H_i^T + \varepsilon M_1M_1^T)\Gamma \\
& \cdot (W + \sum_{i=1}^m H_iP_1H_i^T + \varepsilon M_1M_1^T) \\
& + W + \sum_{i=1}^m H_iP_1H_i^T + \varepsilon M_1M_1^T < 0 \qquad (22)
\end{aligned}
$$

respectively have positive definite solutions $P_1 > 0$ $(NP_1N^T \leq \varepsilon I)$ and $P_2 > 0$. Moreover, Let $L \in \mathbb{R}^{n\times p}$ $(p \leq n)$ be an arbitrary matrix satisfying $\Pi + LL^T < 0$ (see Lemma 2), and $U \in \mathbb{R}^{p\times p}$ be an arbitrary orthogonal matrix (i.e., $UU^T = I$). Then, the filter (5) with the parameters determined by

$$K = \Theta R^{-1} + LUR^{-1/2}, \quad G = \widehat{A} - K\widehat{C} \qquad (23)$$

will be such that, for all admissible perturbations ΔA and ΔC, 1) the state of the augmented system (11) is mean square bounded; 2) the steady-state error covariance X_{ee} meets $X_{ee} < P_2$.

Proof. Since A is assumed to be nonsingular, Φ^{-1} exists and the definitions (17)-(20) are meaningful. We set $P_f :=$ Block $-$ diag$(P_1,\ P_2)$. Then, by means of Lemma 1, the definitions (17)-(20), it is easily verified that

$$
\begin{aligned}
& (A_f + \Delta A_f)P_f(A_f + \Delta A_f)^T \\
& \quad - P_f + \sum_{i=1}^m J_iP_fJ_i^T + W_f \\
& \leq A_f(P_f^{-1} - \varepsilon^{-1}N_f^TN_f)^{-1}A_f^T \\
& \quad + \varepsilon M_fM_f^T - P_f + \sum_{i=1}^m J_iP_fJ_i^T + W_f \\
& := \Psi := \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22} \end{bmatrix} \qquad (24)
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi_{11} = {} & A(P_1^{-1} - \varepsilon^{-1}N^TN)^{-1}A^T - P_1 \\
& + \varepsilon M_1M_1^T + W + \sum_{i=1}^m H_iP_1H_i^T, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\Psi_{12} = {} & A(P_1^{-1} - \varepsilon^{-1}N^TN)^{-1}(A - G - KC)^T \\
& + \varepsilon M_1(M_1 - KM_2)^T \\
& + W + \sum_{i=1}^m H_iP_1H_i^T, \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\Psi_{22} = {} & GP_2G^T - P_2 + (A - G - KC) \\
& \cdot (P_1^{-1} - \varepsilon^{-1}N^TN)^{-1}(A - G - KC)^T \\
& + \varepsilon(M_1 - KM_2)(M_1 - KM_2)^T + W \\
& + \sum_{i=1}^m H_iP_1H_i^T + KZK^T. \qquad (27)
\end{aligned}
$$

It follows immediately from the matrix inverse lemma that

$$
\begin{aligned}
& (P_1^{-1} - \varepsilon^{-1}N^TN)^{-1} \\
& = P_1 + P_1N^T(\varepsilon I - NP_1N^T)^{-1}NP_1
\end{aligned}
$$

and then the inequality (21) implies that $\Psi_{11} < 0$.

To continue, by resorting to the definitions of $\widehat{A}$, $\widehat{C}$, Π, and the expression $G = \widehat{A} - K\widehat{C}$ in (23), we can rewrite (27) as follows

$$
\begin{aligned}
\Psi_{22} = {} & \widehat{A}P_2\widehat{A}^T - P_2 + (W + \sum_{i=1}^m H_iP_1H_i^T \\
& + \varepsilon M_1M_1^T)\Gamma(W + \sum_{i=1}^m H_iP_1H_i^T \\
& + \varepsilon M_1M_1^T) + W + \sum_{i=1}^m H_iP_1H_i^T \\
& + \varepsilon M_1M_1^T - K\Theta^T - \Theta K^T + KRK^T \\
= {} & \widehat{A}P_2\widehat{A}^T - P_2 + (W + \sum_{i=1}^m H_iP_1H_i^T \\
& + \varepsilon M_1M_1^T)\Gamma(W + \sum_{i=1}^m H_iP_1H_i^T \\
& + \varepsilon M_1M_1^T) + W + \sum_{i=1}^m H_iP_1H_i^T \\
& + \varepsilon M_1M_1^T - \Theta R^{-1}\Theta^T + (KR^{1/2} \\
& - \Theta R^{-1/2})(KR^{1/2} - \Theta R^{-1/2})^T \\
= {} & \Pi + (KR^{1/2} - \Theta R^{-1/2}) \\
& \cdot (KR^{1/2} - \Theta R^{-1/2})^T. \qquad (28)
\end{aligned}
$$

Noticing the expression of $K = \Theta R^{-1} + LUR^{-1/2}$ in (23) and the fact of $UU^T = I$, we obtain

$$(KR^{1/2} - \Theta R^{-1/2})(KR^{1/2} - \Theta R^{-1/2})^T = LL^T.$$

Thus, it follows from (28), the definition of the matrix L $(L \in \mathbb{R}^{n\times p})$ and the inequality (22) that $\Psi_{22} = \Pi + LL^T < 0$.

Moreover, substituting $G = \widehat{A} - K\widehat{C}$ into (26) immediately yields $\Psi_{12} = 0$. To this end, we arrive at the conclusion that $\Psi < 0$. Therefore, it follows from (24) that

$$
\begin{aligned}
&(A_f + \Delta A_f)P_f(A_f + \Delta A_f)^T \\
&\quad - P_f + \sum_{i=1}^{m} J_i P_f J_i^T = -W_f + \Psi < 0 \quad (29)
\end{aligned}
$$

which leads to (14). As discussed in Remark 2, we know that the state of the augmented system (11) is mean square bounded, and there exists a symmetric positive semi-definite solution to (13). This proves the first conclusion of this theorem.

Furthermore, subtract (13) from (29) to give

$$
\begin{aligned}
&(A_f + \Delta A_f)(P_f - X)(A_f + \Delta A_f)^T - (P_f - X) \\
&\quad + \sum_{i=1}^{m} J_i(P_f - X)J_i^T = \Psi < 0 \qquad (30)
\end{aligned}
$$

which indicates again from Remark 2 that $P_f - X > 0$ and therefore $X_{ee} = [X]_{22} < [P_f]_{22} = P_2$. This completes the proof of this theorem.

Remark 3. It is clear from Theorem 1 that, if the quadratic matrix inequalities (21)(22) respectively have positive definite solutions P_1, P_2 (P_2 meets $[P_2]_{ii} \leq \sigma_i^2,\ i = 1, 2, \cdots, n$), then the filter (5) determined by (23) will be such that: 1) the augmented system (11) is mean square bounded; and 2) $[X_{ee}]_{ii} < [P_2]_{ii} \leq \sigma_i^2,\ i = 1, 2, \cdots, n$. Hence, the design task of variance-constrained robust filtering for the uncertain bilinear systems will be accomplished. Note that the existence of a positive definite solution to (21) implies the asymptotical Schur stability of the system matrix A.

Remark 4. In practical applications, we can solve the quadratic matrix inequalities (QMIs) (21)(22) subjected to the constraints $[P_2]_{ii} \leq \sigma_i^2$ ($i = 1, 2, \cdots, n$), and then obtain the expected filter parameters immediately from (23). When we deal with the QMIs (21)(22), the local numerical searching algorithms suggested in (Beran and Grigoriadis, 1996; Geromel *et al.*, 1993) are very effective for a relatively low-order model. The detailed discussion on the solving algorithms for QMI's can be found in (Saberi *et al.*, 1995).

Remark 5. It should be pointed out that, in the present design procedure of robust filters for bilinear systems, there exists much *explicit* freedom, such as the choices of the the free parameters L (L satisfies $\Pi + LL^T < 0$), the orthogonal matrix U, etc. This remaining freedom provides the possibility for considering more performance constraints (*e.g.*, the transient requirement and reliability behavior on the filtering process) which requires further investigations. Note that in (Li *et al.*, 1999), a similar freedom on an arbitrary orthogonal matrix in the parameterization of the set of filters was successfully employed to minimize the H_2 norm of the filtering error transfer function by solving an unconstrained parametric optimization problem over the set of filters.

4. A NUMERICAL EXAMPLE

Consider a bilinear discrete-time uncertain stochastic system (1)-(2) with parameters as follows

$$
A = \begin{bmatrix} 0.8 & 0.05 \\ -0.08 & -0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},
$$

$$
H_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix},
$$

$$
M_1 = \begin{bmatrix} 0.08 \\ 0.06 \end{bmatrix}, \quad M_2 = 0.1, \quad N = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix},
$$

$$
W = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad Z = 0.0164.
$$

The goal of this example is to design the robust filter (5) such that 1) the augmented system (11) is mean square bounded; and 2) the steady-state error covariance X_{ee} meets $[X_{ee}]_{11} \leq \sigma_1^2 = 0.5,\quad [X_{ee}]_{22} \leq \sigma_2^2 = 1.2$.

Choosing $\varepsilon = 0.5$, we can obtain a positive definite solution to QMI (21), and subsequently $\Phi, \widehat{A}, \widehat{C}, \Gamma$, as follows

$$
P_1 = \begin{bmatrix} 0.0410 & -0.0006 \\ -0.0006 & 0.0174 \end{bmatrix},
$$

$$
\Phi = \begin{bmatrix} 0.0335 & -0.0032 \\ 0.0007 & -0.0088 \end{bmatrix},
$$

$$
\widehat{A} = \begin{bmatrix} 1.2029 & -0.3712 \\ 0.0194 & -1.8813 \end{bmatrix},
$$

$$
\widehat{C} = \begin{bmatrix} 1.4468 & -1.4175 \end{bmatrix},
$$

$$
\Gamma = \begin{bmatrix} 40.2445 & 26.0997 \\ 26.0997 & 232.2372 \end{bmatrix}.
$$

Then, solve the QMI (22) to give

$$
P_2 = \begin{bmatrix} 0.4271 & -0.1484 \\ -0.1484 & 1.1617 \end{bmatrix}.
$$

It is easily seen that $[P_2]_{ii} < \sigma_i^2$ ($i = 1, 2$) and the constraints (15) are satisfied. Next, select the parameter L which meets $\Pi + LL^T < 0$ (Π is defined in (22)) as $L = [0.0800 \ \ 0.1000]^T$. Then, for the two cases of $U = 1$ and $U = -1$, we obtain the corresponding desired filter parameters from (23), respectively, as follows

$$
\text{Case1}: \quad U = 1, \quad K = \begin{bmatrix} 0.4812 \\ 0.9643 \end{bmatrix},
$$

$$
G = \begin{bmatrix} 0.5068 & 0.3108 \\ -1.3758 & -0.5144 \end{bmatrix},
$$

$$
\text{Case2}: \quad U = -1, \quad K = \begin{bmatrix} 0.4002 \\ 0.8631 \end{bmatrix},
$$

$$
G = \begin{bmatrix} 0.6240 & 0.1960 \\ -1.2293 & -0.6579 \end{bmatrix}.
$$

It is not difficult to test that the prescribed performance objectives are all realized.

5. CONCLUSIONS

We have studied the robust filtering problem for uncertain bilinear stochastic discrete-time systems with estimation error variance constraints. Attention has focused on the design of a linear filter, such that for all admissible parameter uncertainties, the error state of the bilinear stochastic system is mean square bounded, and the the steady-state variance of the estimation error of each state is not more than the individual prespecified value. We have established both the existence conditions and the explicit expression of desired robust filters, in terms of the positive solutions to two quadratic matrix inequalities. A numerical example has been used to show the usefulness of the theory developed.

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