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PID TUNING FOR A MULTIVARIABLE PLANT USING TAGUCHI-BASED METHODS

E. F. Ryckebusch[α], I. K. Craig[β,*]

[α]*Ecole Centrale de Nantes, France*
[β]*Department of Electrical, Electronic and Computer Engineering, University of Pretoria, South Africa*

Abstract: This paper presents the use of Taguchi methods for tuning PID parameters in a multivariable plant. The process used is a single-tank system with two inflows. The level in the main tank and one of the input flows are the controlled variables. The variation of eight parameters was analysed and tests were performed with a L_{18} orthogonal array. After five stages of experimentation, a 65% improvement in signal-to-noise ratio was achieved. *Copyright © 2002 IFAC*

Keywords: PID, MIMO system, Taguchi method, Experiment design, Parameter optimisation.

1. INTRODUCTION

Since proportional-integral-derivative controllers are easy to use and to tune, they have been extensively used in industry for many years. Many tuning methods have been devised for SISO systems and one of the most famous is the Ziegler and Nichols closed-loop tuning method (Seborg, *et al.*, 1989).

PID tuning is still a very popular subject of research, especially for multivariable plants. A robust PID controller design based on a semidefinite programming approach was implemented by J. Bao, *et al.* (1999). Chien, *et al.* (2000), also proposed a method for Two-Input, Two-Output systems requiring two bias-relay feedback tests to generate information on the interaction measure of the two-by-two system.

This paper presents the use of general experiment design methods implemented by Dr. G. Taguchi since the late 1940s (Roy, 1990). This general quality method was first used for improving efficiency in industrial applications but it can be applied in many different cases where discrete changes in parameter values are made. For instance, Chen *et al.* (1996), used these methods for optimising laser micro-engraving of photomasks. The effects of five key parameters were analysed and the laser linewidth was optimised using a L_{16} orthogonal array. Griffin (2000) also used this design method to improve the quality of the Wheatstone bridge, which is an electrical device for precise measurement of values of resistors.

Lee, *et al.* (1999), published a paper on controller gain tuning of a simultaneous multi-axis PID controlled system. The parallel-mechanism machine tool used in this study (the Eclipse) provided a total of 32 controller gains to tune for robustness.

Anderson (2000) explains both the basic Taguchi procedure and how to use the analysis of variance for analysing data. In this paper, the Two-Input, Two-Output controller gains are tuned with this method.

The outline of this paper is as follows. Section 2 presents the process used for this experiment. The Taguchi method is introduced in Section 3 and the procedure of experimentation and results are presented in Section 4. Concluding discussions are given in Section 5.

2. THE PROCESS

The MIMO system used here is a single-tank system. One of the two inflows is the first controlled variable and is measured thanks to a basic flowmeter. The level of the tank, which is measured with an inductive device, is the second controlled variable.

Two '4-20mA' currents control the two valve openings. A PC-30 D/A and A/D card and Matlab Simulink © real time toolbox are used to perform the tests. There is only one pump for the two inflows.

***** Corresponding author. Tel.: +27-12-420-2172; Fax: +27-12-362-5000.
E-mail address: icraig@postino.up.ac.za (I. K. Craig).

A simplified representation of the process is provided in Fig. 1. The dynamic mass balance leads to the following equation:

$$\frac{dh}{dt} = \frac{1}{A}\left(Fi_1 + Fi_2 - a\sqrt{2gh}\right) \tag{1}$$

where A is the area of the tank, a is the cross sectional area of the outflow and h is the level in the tank. (Fi_1 and Fi_2 are the inflows).

A linearisation around the steady state level H_O was performed resulting in the following transfer function:

$$\begin{bmatrix}\Delta h(s)\\ \Delta f(s)\end{bmatrix} = \begin{bmatrix}\frac{\beta}{1+s\tau}H_1(s) & \frac{\beta}{1+s\tau}\\ H_1(s) & G_4(s)\end{bmatrix}\begin{bmatrix}\Delta I_1(s)\\ H_2(s)\Delta I_2(s)-\sigma\end{bmatrix} \tag{2}$$

with $\tau = \frac{2H_0 A}{\sigma}$, $\beta = \frac{\tau}{A}$ $\sigma = a\sqrt{2gH_0}$ and $G_4(s)$=0.

In equation 2, Δh is the variation of the level around the steady state H_O, Δf is the controlled flow, H_1 and H_2 are the two valve transfer functions and ΔI_1 and ΔI_2 are the two input currents.

The theoretical study and the simulation performed show that flowrate, Fi_1, depends only on the opening of the valve V_1. For the plant, it is not the case. This is because there is only one pump for the two inflows. The interaction $G_4(s)$ between the two flows can therefore not be neglected.

Moreover, the level device installed in the plant is an inductive level sensor and its resolution is only one centimetre. That implies that, for a given level, h', in the tank, the output signal will switch between two values (h'-0.5 and h'+0.5)

These two problems lead to the reconsideration of equation 1. System identification experiments were performed on the plant to obtain transfer functions for the two valves and for the interaction:

$$H_1(s) = \frac{4.085s - 3.817}{s^2 + 6.788s + 11.05} \tag{3}$$

$$H_2(s) = \frac{-0.4743s + 0.713}{s^2 + 1.452s + 0.8193} \tag{4}$$

$$G_4(s) = \frac{0.05737s - 0.08766}{s^2 + 1.289s + 0.7303} \tag{5}$$

The multivariable plant shown above is difficult to tune using traditional methods, and the Taguchi method was used instead.

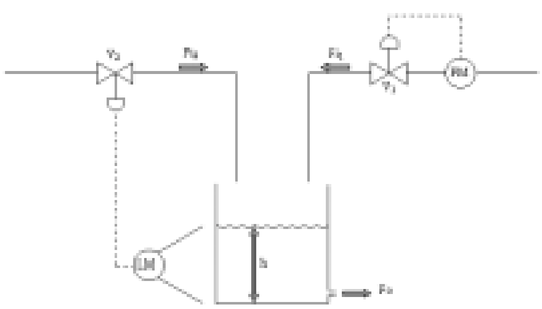

Fig. 1. Simple representation of the process.

3. THE TAGUCHI METHOD

3.1 The method

In the late 1940s, Dr G. Taguchi was given the responsibility of increasing productivity and improving quality at the Electrical Communications Laboratory in Japan. He developed methods that examine the sensitivity of a process to a discrete change of one or several given variables. R.A. Fisher (Fienberg and Hinkley, 1980) first introduced the classical approach for experiment design, which uses the full-factorial design where all combinations are tested. For a given number of parameters and a given number of levels for each parameter, Taguchi reduced the number of trials using special fractional factorial arrays called orthogonal arrays. These orthogonal arrays pit a given level of one factor against a given level of another factor only once in the entire design.

For this partial-factorial experiment design, the signal-to-noise ratio (*SNR*) parameter is used as a criterion of the performance of the system. The higher the *SNR* is the better the performance is. The *SNR* is given by:

$$SNR = -10Log_{10}\left[\frac{1}{n}\sum_n f(x_i)^2\right] \tag{6}$$

For instance, in equation 6, if results of the experiments, x_i, are intended to be large (productivity, efficiency), then $f(x_i) = (x_i)^{-1}$. In this paper, the criterion of performance is sum of the relative errors between the corresponding outputs and set-points. Thus, $f(x_i) = x_i$.

3.2 Analysis of results

Since the Taguchi method reduces the number of experiments over the full-factorial approach, it is useful to use the statistical analysis of experiments, called analysis of variance (ANOVA), to provide levels of confidence in the results. A computed F-value is compared to values in the Fisher criterion tables. Moreover, analysis of variance identifies and ranks variables that affect the variance of the output signal. The percentage contribution can also be computed which gives the effect of each level of each parameter.

This ANOVA is one of the main steps in using the Taguchi method. The procedure followed is explained in sections 4 and 5.

4. EXPERIMENTATION

Before performing the Taguchi method, the controllers were defined and the orthogonal array and the criterion of performance were chosen.

4.1 Choice of the controller

For the *level* and *flow* loops (1 and 2 respectively) the same controller is implemented (see Fig. 2). Given the level of noise in the output, derivative action was removed. For each controller, an anti-reset windup function was added to reduce the overshoots (especially for the level control). Moreover, a feedforward loop, called *set-point weighting,* was implemented in order to increase the frequency of the zero of the integral action, and thus speed up the transient set-point response and reduce overshoots.

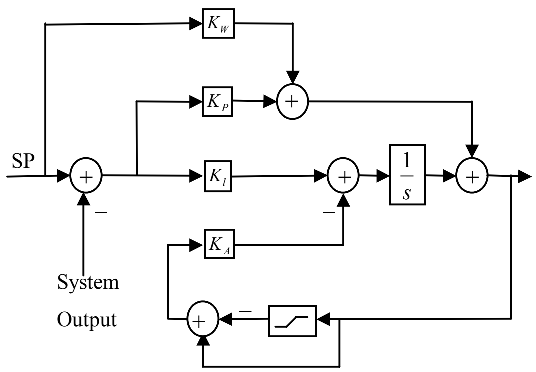

Fig. 2. Controller implemented for the two loops.

This configuration for the two controllers gives eight parameters to tune: K_{W1}, K_{A1}, K_{P1}, K_{I1} for controller 1 (level loop) and K_{W2}, K_{A2}, K_{P2}, K_{I2} controller 2 (flow loop). The next step is to choose the correct orthogonal array.

4.2 Choice of the orthogonal array

There are many examples of orthogonal arrays in the literature. Given that three levels for each factor will be defined, eight factors result in a L_{27} orthogonal array being chosen, which actually means 27 experiments and thirteen factors[1]. In this case, five factors would then be 'empty' in the array. Thus, in order to again reduce the number of experiments and consequently the cost of the method, a L_{18} orthogonal array was rather used. This array implies only eighteen experiments on condition that one of the eight factors would only have two levels (i.e. $3^7.2^1$). The experimental layout is shown in Fig. 3. The eighteen rows of this matrix represent the experiments to be conducted.

The eight columns correspond to the eight parameters or factors (respectively: K_{W1}, K_{A1}, K_{P1}, K_{I1}, K_{W2}, K_{A2}, K_{P2}, K_{I2}). The numbers '1', '2' and '3' correspond to the level number for each factor. For instance if 1,10 and 100 are the three values for parameter K_{P2}, then level '1' is 1, level '2' is 10 and level '3' is 100.

Note: The set point weighting gain for controller 1 has only 2 levels.

$$\begin{bmatrix} 1&1&1&1&1&1&1&1&1&2&2&2&2&2&2&2&2&2 \\ 1&1&1&2&2&2&3&3&3&1&1&1&2&2&2&3&3&3 \\ 1&2&3&1&2&3&1&2&3&1&2&3&1&2&3&1&2&3 \\ 1&2&3&1&2&3&2&3&1&3&1&2&2&3&1&3&1&2 \\ 1&2&3&2&3&1&1&2&3&3&1&2&3&1&2&2&3&1 \\ 1&2&3&3&1&2&2&3&1&2&3&1&1&2&3&3&1&2 \\ 1&2&3&3&1&2&2&3&1&2&3&1&3&1&2&1&2&3 \\ 1&2&3&3&1&2&3&1&2&1&2&3&2&3&1&2&3&1 \end{bmatrix}^T$$

Fig. 3. L_{18} orthogonal array.

4.3 Choice of the criterion

As explained in section 2, the choice of the criterion of performance is very important since the experimentation procedure is based on this result. For this experiment, the following error criterion was computed:

$$\varepsilon_i = \frac{1}{n}\cdot\left\{\sum_{k=1}^{n}\left|\frac{Level(k) - Level\_SP(k)}{Level\_SP(k)}\right|\right\} + \frac{1}{n}\cdot\left\{\sum_{k=1}^{n}\left|\frac{Flow(k) - Flow\_SP(k)}{Flow\_SP(k)}\right|\right\} \quad (7)$$

This criterion is intended to be small and a "smaller-the-better" configuration was chosen (Roy R., 1990). *SNR* Ratio is defined as follows:

$$SNR = -10 Log_{10}\left[\frac{1}{3}\sum_{i=1}^{3}\varepsilon_i^2\right] \quad (8)$$

Particular set-point profiles were chosen in order to achieve as much interaction between the two loops (see Fig. 7.) as possible. Each experiment in the L_{18} orthogonal array was repeated three times in order to increase the level of confidence. Such a repetition is possible since the testing time is relatively short.

4.4 Initial values and Stage A

The initial values for the experimentation were found with the traditional Ziegler-Nichols (Z-N) method. This closed-loop tuning method was applied separately to the level and flow loops.

For the flow loop, sustained oscillations are observed quite easily and the Z-N formulae give initial values for proportional and integrative parameters (K_{P2}=1.7 and K_{I2}=1.4s).

For the level loop, given that interactions and the windup phenomenon are significant, the Z-N method gives mediocre results. The initial values are K_{P1}=9 and K_{I1}=30s. Values for the set point weighting gain and anti-reset windup gains are logically initialised at zero. This corresponds to the basic PI controller configuration. First tests with these initial values give a *SNR*:

$$SNR_{init} = 14.25dB.$$

[1] An array for 8 factors with 3 levels does not exist

For stage A of the method, three levels are defined for each factor as given in Table 1.The experiments were then run according to the L_{18} array (Fig. 3) three times. For each experiment, the error criterion (equation 7) is computed to give the average *SNR*. Table 2 shows the results from this first stage. For instance, for parameter K_{P2}, the average of the *SNR* is calculated for all the experiments that use level 1 (experiments 1, 5, 9, 12, 14, 16). This results in:

SNR (K_{P2} - Level 1) = 14.87dB
SNR (K_{P2} - Level 2) = 16.13dB
SNR (K_{P2} - Level 3) = 12.82dB

Table 1. Stage A.

Factors	*Level 1*	*Level 2*	*Level 3*
K_{W1}	**0**	10	--
K_{A1}	**0**	10	20
K_{P1}	1	**9**	20
K_{I1}	1	**30**	50
K_{W2}	**0**	5	10
K_{A2}	**0**	10	20
K_{P2}	0.1	**1.7**	10
K_{I2}	0.1	**1.4**	10

These averages, shown graphically in Fig. 4, represent the *SNR* corresponding to the parameter. They also show the contribution of the levels of each parameter. The biggest *SNR* provides the best level and therefore for this stage, the best combination is given by highest values of each parameter (e.g. 16.13 dB for K_{P2}).

Three tests were performed with this best combination and the *SNR* increased to 49% from 14.25dB to 20.16B. Results are confirmed by the ANOVA analysis with the percentage contribution shown in Table 3. SNR values for the levels of K_{A1}, K_{P2} and K_{I2} differ significantly. It is important to note that the error contribution computed with ANOVA gives an idea of the confidence in the results.

Table 2. Errors and SNRs for stage A.

Exp.	ε_1	ε_2	ε_3	*SNR*
1	0.263	0.254	0.240	11.95
2	0.179	0.181	0.178	14.94
3	0.273	0.276	0.269	11.29
4	0.203	0.200	0.191	14.07
5	0.183	0.181	0.181	14.81
6	0.081	0.081	0.081	21.84
7	0.151	0.146	0.156	16.42
8	0.240	0.238	0.241	12.41
9	0.166	0.179	0.181	15.13
10	0.243	0.250	0.240	12.24
11	0.276	0.285	0.294	10.90
12	0.257	0.254	0.252	11.89
13	0.163	0.162	0.156	15.90
14	0.159	0.152	0.161	16.06
15	0.185	0.199	0.201	14.19
16	0.109	0.110	0.103	19.39
17	0.134	0.137	0.145	17.17
18	0.240	0.247	0.235	12.37

According to the ANOVA results, a finer tuning is implemented in the following stages. The levels of the factors are changed according to the tendency and the inclination of the parameters in Fig. 4. For instance, the K_{I1} values can be increased beyond 50 (see ■). The K_{P2} levels are set closer to 1.7 (see ▲). *SNRs* in stage B will be higher since the tuning is better.

The Taguchi method recommends first changing the levels of the factors that have the biggest contribution. This rule was used for stages A to E.

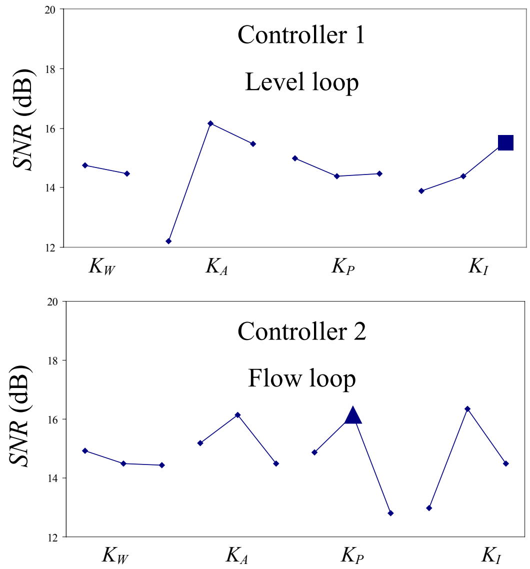

Fig. 4. Effect diagrams for stage A.

Table 3. Percentage contribution.

Controller parameter	*Percentage contribution*
K_{W1}	0.1%
K_{A1}	44.6%
K_{P1}	1.4%
K_{I1}	2.7%
K_{W2}	-0.3%
K_{A2}	0.6%
K_{P2}	24.7%
K_{I2}	19.3%
ANOVA Error	6.9%

4.5 Stages A to E

Appendix A gives the progression of level values for all stages and Fig. 5 and Fig. 6 summarise the SNR values.

Three reasons confirm then that the final tuning is completed after five stages. First, the improvement of the *SNR* between stage D and E is not significant. Secondly, the error contribution (given by ANOVA and presented in Table 4) is large for the last stage (39.5%).

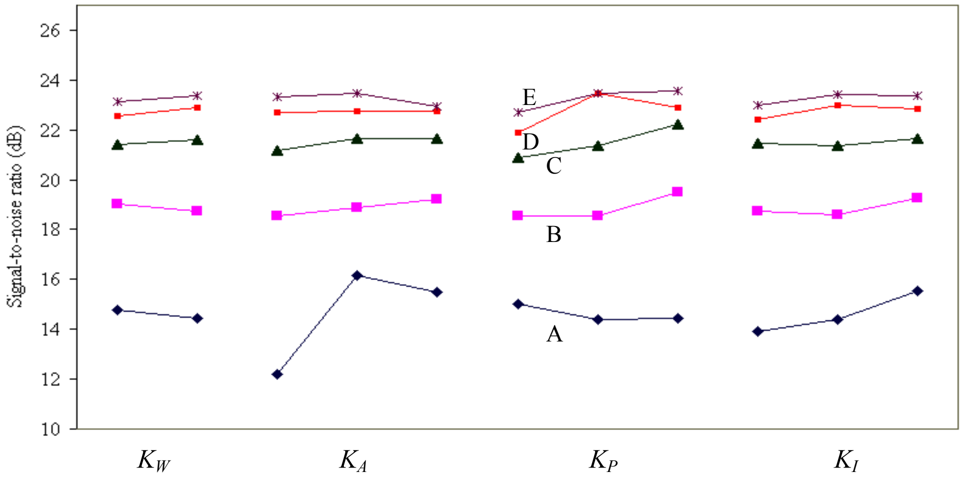

Fig. 5. Effect graph for controller 1.

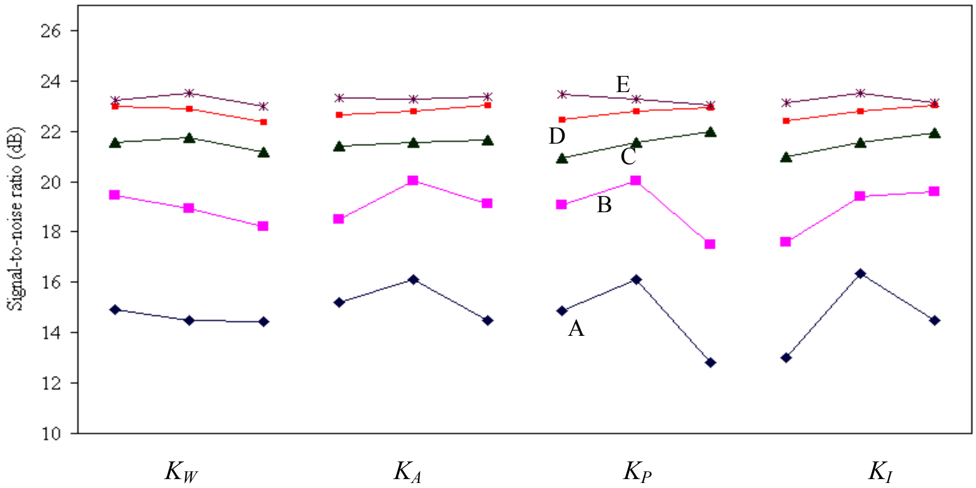

Fig. 6. Effect graph for controller 2.

Table 4. Error contribution.

Stage	*Error* (%)
A	6.9
B	4.9
C	13.7
D	28.6
E	39.5

Finally, Fig. 5 and Fig. 6 indicate that additional stages will not provide significant improvements. Highest points of stage E in Fig. 5 and Fig. 6 correspond to the final PI values used.

Table 5 gives improvement of the *SNR* from initial tuning to stage E. This improvement is quite significant for first, second and third stages. But for the two last stages, the improvement is small.

Table 5. Improvement of the criterion.

	SNR(dB)	I_i[2](%)	I_p[3] (%)
Initial Stage	14.25	--	--
Stage A	20.16	49.38	49.38
Stage B	21.57	56.96	14.98
Stage C	22.36	60.73	8.75
Stage D	22.98	63.41	6.83
Stage E	23.43	65.25	5.03

4.6 Final comparison

Fig. 7a and Fig 7b show the improvement of the time responses between the controller with initial Z-N tuning and the final tuning parameters obtained with the Taguchi method.

[2] Improvement regarding the initial tuning.
[3] Improvement regarding the previous stage.

5. CONCLUSIONS

In this paper, PI-tuning was performed using the Taguchi method with a L_{18} orthogonal array. The multivariable and non-linear behaviour of the plant, together with the poor resolution of the level sensor, made conventional tuning difficult.

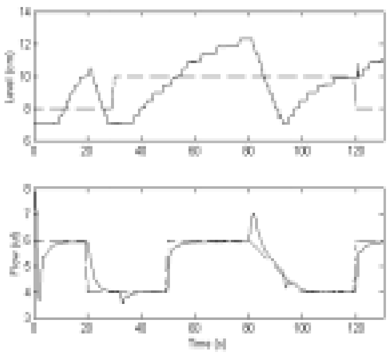

Fig. 7a. Initial PID-tuning.

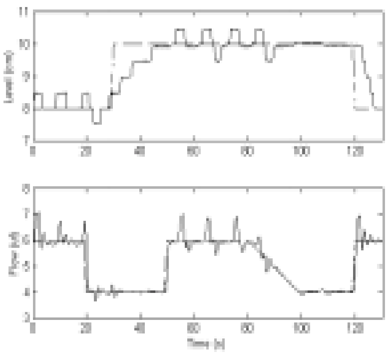

Fig. 7b. Best stage E values

Because the Taguchi experiments did not take long, three repetitions of the array of experiments were performed. This number of experiments can be reduced without affecting the results considerably.

Final values of the signal-to-noise ratio and error contributions reveal that the two last stages could have been left out. After five stages of experimentation, a 65% improvement in signal-to-noise ratio was achieved.

Appendix A. Level values for the five stages of the experimentation.

Controller 1		Stage A	Stage B	Stage C	Stage D	Stage E
K_W	Level 1	0	0	0	0	0
	Level 2	10	10	10	10	**10**
	Level 3	--	--	--	--	--
K_A	Level 1	0	10	10	10	10
	Level 2	**10**	20	20	20	**20**
	Level 3	**20**	30	30	30	30
K_P	Level 1	1	1	1	9	15
	Level 2	9	9	**9**	20	20
	Level 3	20	20	**20**	30	**25**
K_I	Level 1	1	50	50	50	50
	Level 2	30	70	70	70	**70**
	Level 3	**50**	90	90	90	90

Controller 2		Stage A	Stage B	Stage C	Stage D	Stage E
K_W	Level 1	0	0	0	0	0
	Level 2	5	**5**	2	2	**2**
	Level 3	10	10	5	5	5
K_A	Level 1	0	0	5	5	5
	Level 2	10	**10**	10	10	**10**
	Level 3	20	20	15	15	15
K_P	Level 1	0.1	0.5	1	2	**3**
	Level 2	**1.7**	**1.7**	1.7	2.5	3.5
	Level 3	10	5	**2.5**	**3**	4
K_I	Level 1	0.1	0.5	1	2.5	3.5
	Level 2	**1.4**	**1.4**	1.4	3	**4**
	Level 3	10	5	**2.5**	**3.5**	4.5

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