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SCHEDULED CONTROLLERS FOR LINEAR SYSTEMS WITH BOUNDED ACTUATORS

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Abstract: Output feedback scheduled controllers are designed for linear systems with saturating actuators. The scheduling is based on the closed-loop system response, thus resulting in quasi-linear parameter-varying structure for the compensator, as well as the performance measure. Linear splines are used to obtain solutions that can be obtained by standard LMI software.

Keywords: Saturation, scheduled controllers, disturbance attenuation.

1. INTRODUCTION

While actuator capacity limitation has been a major topic of research for decades, there has been a pronounced increase in research activity in this area recently (see, for example, Bernstein and Michel, 1995; Stoorvogel and Saberi, 1999). In many cases, linear control methods are used initially to obtain desirable nominal controllers, while in the second step anti-windup techniques are developed that reduce the controller gain so that saturation is avoided with only a graceful degradation of performance. Originally, much of this work was based on ad-hoc techniques, though recent progress has made this approach much more precise and rigorous (e.g., Campo and Morari, 1990; Gilbert and Tan, 1991; Kappor, *et al.*, 1998). An approach that has been receiving increasing attention more recently concerns the incorporation of actuator nonlinearity explicitly and exploiting advances in nonlinear and robust control techniques to develop guaranteed stability and performance bounds. When the operation of the system is faced with regular and persistent saturation regimes, this approach yields important stability and performance guarantees (Garcia *et al.*, 1999; Lin and Saberi, 1995). An example of this approach is that of Nguyen and Jabbari (1999, 2000), in which the issue of disturbance attenuation (in the sense of L_2 gain) was studied. There, the guaranteed performance levels are functions of actuator capacity (*i.e.*, larger actuators give better performance). To fully use the actuator capacity, a high gain controller similar Lin and Saberi (1995) was used.

Here, we focus on one of the main causes of conservatism faced in many of these techniques. When an LTI controllers is used for a reasonably large range of disturbances (or possible system response), the design often is based on a worst case description of the disturbance, leading to excessive conservatism. We rely on concepts of parameter dependent Lyapunov functions and parameter dependent performance measures (see Feron *et al.*, 1996; Lee and Spillman, 1997 for an example of each), as well as scheduling of the controller. Scheduled controllers have been used before (Gutman and Hagander, 1985; Henrion, *et al.*, 1998; Lin, 1998; Megretski, 1996; Teel 1995, to name a few). In most cases, these references concern the state feedback problem. In some others, the output feedback problem is solved for minimum-phase systems or systems with other restrictions.

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The basic approach here is to use ellipsoids –or rather, ellipsoid-like sets– that bound the location of the state in order to obtain a scheduling parameter that is used to adjust the controller. (Preliminary results for the state feedback problem based on this approach were presented in Srivastava and Jabbari (2000)). In the output feedback problem considered here, these sets depend on x_c, the state of the controller. The scheduling of the controller is aimed at finding the smallest ellipsoid-like set containing x_c. Smaller sets allow higher gain controllers, thus better performance. If, due to disturbance, x_c moves further from the origin, smaller gains are used, which lead to lower performance. As a result, the guaranteed performance bound is parameter-dependent as well. The choice of controller is thus a function of the system response and not any *a priori* estimate of the worst cases disturbance. Other features include dependence on the actuator capacity and ease of extension to general linear parameter-varying (LPV) models. For performance the L_2 gain is used, though other measures such as the peak-to-peak gain (Abedor, *et al.*, 1996) or the energy-to-peak gain (Rotea, 1993) can be used as well.

In gain-scheduling and LPV problems, when the main Lyapunov matrix is parameter-varying, solution of the main inequalities becomes difficult. Often, conservative sufficient conditions or gridding techniques are used. Here, we rely on spline functions (e.g., Masubuchi, *et al.*, 1998) to obtain a set of sufficient conditions which can be made less conservative by increasing the number of bases used.

Notation is standard: For $x \in \mathbb{R}^n$, $\|x\|$ denotes the common Euclidean norm. For $f \in \mathcal{L}_2[0,\infty]$, the $\mathcal{L}_2$-norm of f is defined by $\|f\|_2 \triangleq \int_0^\infty f(t)^T f(t)\, dt$. Given a matrix $M = M^T \in \mathbb{R}^{n\times n}$, $M > 0$ ($M < 0$) denotes positive (negative) definiteness of M. Positive and negative semi-definiteness are denoted similarly by $M \geq 0$ and $M \leq 0$.

2. PRELIMINARIES

Consider a system of the form

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1a}$$
$$z = C_1 x + D_{11} w + D_{12} u \tag{1b}$$
$$y = C_2 x + D_{21} w, \tag{1c}$$

where $w(t) \in \mathbb{R}^{m_1}$ is the external disturbance on the system, $u(t) \in \mathbb{R}^{m_2}$ is the control input to the system and $z(t) \in \mathbb{R}^{p_1}$ and $y(t) \in \mathbb{R}^{p_2}$ denote the controlled and measured outputs of the system, respectively. We assume a magnitude saturation bound is given for u,

$$\|u(t)\| \leq u_{sat} \qquad \forall t \geq 0. \tag{2}$$

An alternative saturation bound, namely

$$|u_i(t)| \leq u_{i_{sat}} \qquad \forall t \geq 0 \tag{3}$$

is also possible. For simplicity of exposition, we consider (2) here, although handling of (3) is also immediate. (Note that $u_{i_{sat}}$'s in (3) can be taken to be equal to each other without loss of generality.) Our goal is to design controllers that render the closed-loop system internally stable with a prescribed disturbance attenuation level so that the condition above is not violated. Throughout the paper, we assume $\|w(t)\| \leq w_{max}$ for all $t \geq 0$. While assuming a safe estimate for w_{max} might lead to conservatism, the results presented here can avoid the conservatism associated with such an overbound, as shown below.

For a given $P \geq 0$ in $\mathbb{R}^{n\times n}$, we define

$$\mathcal{E}(P,c) \triangleq \left\{x \in \mathbb{R}^n : x^T P x \leq c\right\}. \tag{4}$$

Whenever P is a constant matrix, $\mathcal{E}(P,c)$ defines an ellipsoid. However, in the next section, it will be shown that $\mathcal{E}\big(P(\rho), c\big)$ need not be an ellipsoid.

In the next section, we present the main results of the paper, namely scheduled controllers using dynamic output feedback. We first give a parameter-dependent condition for stabilization with disturbance attenuation. Due to space limitations, all proofs are omitted (see the full version of this paper for details).

3. MAIN RESULTS

In this section we consider system (1) and our goal is to design a dynamic output feedback controller of the form

$$\dot{x}_c = A_c\big(\rho(t)\big)x_c + B_c\big(\rho(t)\big)y \tag{5a}$$
$$u = C_c\big(\rho(t)\big)x_c \tag{5b}$$

such that the closed-loop system has a good performance while satisfying (2). The parameter $\rho(t)$ is related to the proximity of the state vector to the origin and will be explicitly defined below.

We start with a preliminary result that establishes the concept of a parameter varying controller as well as that of a parameter-varying performance measure. This result combines an inequality that establishes an estimate for the reachable set with a relatively standard bounded-real type inequality (for performance) and an additional inequality to accommodate the saturation bound. As mentioned before, the key to this approach is the use of the smallest ellipsoid that contains the state vector, which in turn allows the most aggressive control law. These ellipsoids are parameterized through a scheduling parameter ρ defined based on the "size" of the x_c. The resulting controller is thus linear parameter-varying.

Theorem 1. Let ρ denote a time-varying parameter such that $\rho(t) \in [\rho_{min}, \rho_{max}]$ ($\rho_{min} = \frac{1}{w_{max}^2}$). Suppose there exist a $\mathcal{C}^1$ function $X(\rho) \in I\!R^{n\times n}$ with $\frac{d}{d\rho}X(\rho) \leq 0$, Y, $F(\rho)$, $G(\rho)$ and $L(\rho)$ and a scalar $\alpha > 0$ such that for all $\rho \in [\rho_{min}, \rho_{max}]$,

$$\begin{bmatrix} M_X(\rho) & \star & \star & \star \\ A^T + L(\rho) & M_Y(\rho) & \star & \star \\ B_1^T & B_1^T Y + D_{21}^T G(\rho)^T & -\gamma(\rho) I & \star \\ C_1 X(\rho) + D_{12} F(\rho) & C_1 & D_{11} & -\gamma(\rho) I \end{bmatrix} < 0 \tag{6}$$

and

$$\begin{bmatrix} M_X(\rho) + \alpha X(\rho) & \star & \star \\ A^T + L(\rho) + \alpha I & M_Y(\rho) + \alpha Y & \star \\ B_1^T & B_1^T Y + D_{21}^T G(\rho)^T & -\alpha I \end{bmatrix} < 0 \tag{7}$$

where $M_X(\rho) \triangleq AX(\rho) + X(\rho)A^T + B_2 F(\rho) + F(\rho)^T B_2^T - \dot{X}(\rho)$, $M_Y(\rho) \triangleq A^T Y + YA + G(\rho)C_2 + C_2^T G(\rho)^T$ and

$$\begin{bmatrix} X(\rho) & I & F(\rho)^T \\ I & Y & 0 \\ F(\rho) & 0 & \rho\, u_{sat}^2 I \end{bmatrix} > 0. \tag{8}$$

Then, if $\rho(t)$ is chosen as

$$x_c(t)^T S\big(\rho(t)\big)^{-1} x_c(t) \leq \frac{1}{\rho(t)}, \tag{9}$$

then, the controller defined by

$$C_c(\rho) = F(\rho) S(\rho)^{-1} \tag{10a}$$

$$B_c(\rho) = -Y^{-1} G(\rho) \tag{10b}$$

$$\begin{aligned} A_c(\rho) = {} & (A - B_c(\rho) C_2) X(\rho) S(\rho)^{-1} + B_2 C_c(\rho) \\ & - Y^{-1} L(\rho) S(\rho)^{-1}, \end{aligned} \tag{10c}$$

where $S(\rho) \triangleq X(\rho) - Y^{-1}$, satisfies the following:

(i) For the closed-loop state vector, the set $\mathcal{E}\big(P(\rho_{min}), 1/\rho_{min}\big)$ is invariant, for $P(\rho_{min}) = \begin{bmatrix} Y & -Y \\ -Y & S(\rho)^{-1}+Y \end{bmatrix}$. That is, for a disturbance with $w(t)^T w(t) \leq w_{max}^2$ and any $x_{cl}(0) \in \mathcal{E}\big(P(\rho_{min}), 1/\rho_{min}\big)$, we have, for all $t \geq 0$, that $x_{cl}(t) \in \mathcal{E}\big(P(\rho_{min}), 1/\rho_{min}\big)$, where x_{cl} denotes the closed-loop state vector.

(ii) The closed-loop system is internally stable with

$$\begin{aligned} & \int_0^\infty \gamma\big(\rho(t)\big)^{-1} z(t)^T z(t)\, dt \\ & \qquad < \int_0^\infty \gamma\big(\rho(t)\big) w(t)^T w(t)\, dt. \end{aligned} \tag{11}$$

(iii) The control input satisfies (2). ■

Remarks on Theorem 1:

(a) Note that we have used a constant Y. While there may be benefits of using a parameter-varying Y, the cost associated with it are fairly substantial, since it results in $\dot{Y}(\rho)$ terms in the compensator. This, in turn, entails obtaining $\dot{\rho}$ on-line. For brevity, we only present the results for constant Y.

(b) The key parameter here is ρ which needs to satisfy (9) at all times. This parameter identifies the appropriate ellipsoid that contains x_c. By construction, smaller ellipsoids (or ellipsoidal sets) correspond to larger ρ. Note that ρ acts as the main constraint through inequality (8); larger values of ρ lead to better performance (lower γ). To have the most aggressive controller, the largest ρ satisfying (9) should be used This often leads to the ρ that results in equality in (9). Smaller values results in a more conservative controller, but still maintain the stability and performance measures discussed in the theorem. However, it is important that at no time the ρ used in the controller be larger than the maximum ρ satisfying (9), since that might lead to violation of the saturation limit which destroys the properties established in the theorem above.

(c) Typically, in LPV problems, the term $\dot{X}(\rho) = \dot{\rho}\, dX/d\rho$ requires that inequality (6) be satisfied at both $d_{min} < 0$ and $d_{max} > 0$, where $d_{min} \leq \dot{\rho}(t) \leq d_{max}$. Due to $dX/d\rho \leq 0$, however, it is sufficient to check the condition corresponding to d_{max} only. Since ρ represents the location of the state vector, it depends on the disturbance $w(t)$ and the system's response to it. Its limits, therefore, are not known. The result above allows for arbitrary fast reduction in ρ, which is the critical issue. If the state gets closer to the origin faster than the assumed d_{max}, ρ will be increased with a constant rate of d_{max}. The resulting ρ satisfies all of the conditions needed for the theorem above, but might not be the most aggressive control law possible at all times.

(d) Inequality (11) is somewhat unusual. It is easy to show that it implies $\|z\|_2 < \gamma_{max}^2 \|w\|_2$ where $\gamma_{max} = \max_\rho \gamma(\rho)$. Of course, we could have used a more familiar $\int_0^\infty z(t)^T z(t)\, dt < \int_0^\infty \gamma^2\big(\rho(t)\big) w(t)^T w(t)\, dt$ which would have resulted in a $-\gamma^2(\rho) I$ in (2,2) entry of (6) and $-I$ in (3,3) entry. This can lead to numerical problems , particularly for $\gamma^2 \ll 1$.

(e) A constant value of $\rho(t) = \rho_{min}$ yields an LTI controller that handles the worst case scenario, both in magnitude of the disturbance and the system response. As discussed earlier, this is too conservative. When $w(t)$ does not result in worst case response, the state of the system is confined in smaller ellipsoids, which makes a more aggressive controller; *i.e.*, $\rho > \rho_{min}$, feasible. This relaxes the constraint inequalities and allows larger gains and lower performance guarantees. Often, the most conservative controller is never is used, *i.e.*, the least aggressive controller implemented is the one associated with $\min_t \rho(t)$.

(f) Theorem 1 considers the constraint (2) only. Generalization to the case in (3) is straightforward. In this case, there will be one LMI of the form (8) for each input, where $F_i(\rho)$ –the i^{th} row of $F(\rho)$– replaces $F(\rho)$ for the i^{th} input, while everything remains the same. □

In the next lemma, we show how the conditions of the theorem above can be satisfied through a finite number of LMI's for an appropriately defined parameter $\rho(t)$. We use the notation below: Suppose for a discrete collection of points $\eta_1 < \eta_2 < \cdots < \eta_{n_\eta}$, matrix variables M_k are given for $k = 1 : n_\eta$. Then, a linear spline function based on η_k's and M_k's is defined by

$$M_S(\rho) \triangleq M_k + \frac{\rho - \eta_k}{\eta_{k+1} - \eta_k}(M_{k+1} - M_k) \qquad (12)$$

for $\rho \in [\eta_k, \eta_{k+1}]$. We will use this structure for $X(\rho)$, $F(\rho)$, etc.

Lemma 2. Let positive scalars η_k be defined as $\frac{1}{w^2_{max}} = \eta_1 < \cdots < \eta_k < \cdots < \eta_{n_\eta}$. Suppose there exist matrices $X_k = X_k^T$, $Y = Y^T$, F_k, G_k and L_k and a scalar $\alpha > 0$ such that for all $k = 1 : n_\eta$ and $m = k-1, k$

$$\begin{bmatrix} N_k - d_{max}\Delta X_m & \star & \star & \star \\ A^T + L_k & R_k & \star & \star \\ B_1^T & B_1^T Y + D_{21}^T G_k^T & -\gamma_k I & \star \\ C_1 X_k + D_{12} F_k & C_1 & D_{11} & -\gamma_k I \end{bmatrix} < 0, \qquad (13)$$

$$\begin{bmatrix} N_k + \alpha X_k - d_{max}\Delta X_m & \star & \star \\ A^T + L_k + \alpha I & R_k + \alpha Y & \star \\ B_1^T & B_1^T Y + D_{21}^T G_k^T & -\alpha I \end{bmatrix} < 0, \qquad (14)$$

where $N_k \triangleq AX_k + X_k A^T + B_2 F_k + F_k^T B_2^T$, $R_k \triangleq A^T Y + YA + G_k C_2 + C_2^T G_k^T$, $\Delta X_m \triangleq \frac{X_{m+1} - X_m}{\eta_{m+1} - \eta_m}$ and we set $X_0 = X_1$ and $X_{n_\eta + 1} = X_{n_\eta}$,

$$\begin{bmatrix} X_k & I & F_k^T \\ I & Y & 0 \\ F_k & 0 & \eta_k u_{sat}^2 I \end{bmatrix} > 0 \qquad \forall k = 1 : n_\eta \qquad (15)$$

$$X_{k+1} \leq X_k \qquad \forall k = 1 : n_\eta - 1 \qquad (16)$$

Then, the parameter $\rho(t)$ and functions $X\big(\rho(t)\big)$, $F\big(\rho(t)\big)$, $G\big(\rho(t)\big)$, $L\big(\rho(t)\big)$ and $\gamma\big(\rho(t)\big)$ defined as below satisfy the conditions (6)-(8) in Theorem 1:

$\underline{\rho(t)}$: Given $x(t)$, determine $k \triangleq \max j$ such that $x_c^T \left(X_j - Y^{-1}\right)^{-1} x_c \leq 1/\eta_j$, and let

$$\rho'(t) \triangleq \begin{cases} \dfrac{1}{x_c(t)^T \left[X_S(r) - Y^{-1}\right]^{-1} x_c(t)} & \text{if } k < n_\eta \\ \eta_{n_\eta} & \text{if } k = n_\eta \end{cases}$$

where X_S is in the form given in (12). Then, for a $T > 0$ small enough,

$$\rho(t) \triangleq \frac{1}{T}\int_{t-T}^{t} \rho'(s)\, ds. \qquad (17)$$

$\underline{X(\rho)}$: Given ρ, for a $l > 0$ small enough,

$$X(\rho) \triangleq \frac{1}{l}\int_{\rho - l/2}^{\rho + l/2} X'(s)\, ds \qquad (18)$$

where

$$X'(s) \triangleq \begin{cases} X_1 & \text{if } \eta_1 - l/2 \leq s \leq \eta_1 \\ X_S(s) & \text{if } \eta_k \leq s \leq \eta_{k+1},\ 1 \leq k < n_\eta \\ X_{n_\eta} & \text{if } \eta_{n_\eta} \leq s \leq \eta_{n_\eta} + l/2 \end{cases} \qquad (19)$$

and $F(\rho)$, $G(\rho)$, $L(\rho)$ and $\gamma(\rho)$ are defined similar to $X'(\rho)$. ■

Remarks on Lemma 2:

(a) For optimal performance, the inequalities (13)-(16) are solved while minimizing $\sum_{k=1}^{n_\eta} \gamma_k$. Also note that l and T are scalars that are needed for technical reasons, and do not affect the solvability of the problem (nor the resulting controllers). For example, while the formal definition of the scheduling parameter is ρ, in implementation, one uses ρ'. The results hold since we rely on strict inequalities and ρ and ρ' can be made arbitrarily close.

(b) The construction of $\rho(t)$ in the theorem assumes that $\dot{\rho}(t) \leq d_{max}$ for all $t \geq 0$. In implementation, if $\rho(t)$ ever increases with a rate higher than d_{max}, one can always limit its rate to d_{max} and keep $\dot{\rho}$ at that level until the $\rho(t)$ integrated on the basis of d_{max} reaches the largest $\rho(t)$ that satisfies (9). Note that this is consistent with Remark b on Theorem 1.

(c) The best choice of η_k's is essentially problem-dependent but the major role is played by the choice of η_{max}. The larger η_{max}, the more aggressive the controller is for small x_c (e.g., at the onset of the disturbance or other instances when the system response is small). A large value of n_η allow the spline functions to better approximate general functions of ρ, albeit at a significantly higher computational burden. As discussed earlier, we allow effectively infinitely large d_{min}, to avoid violating the saturation bounds. Large values of d_{max} indicate that the controller can be made more aggressive swiftly, though it might yield lower performance guarantees through higher γ (the full version of this paper contains comparisons regarding different values of n_η, d_{max}, etc - which cannot be repeated here due to space limitations).

(d) In certain applications, such as earthquake engineering, the critical period is the initial stages when the state of the system becomes large due to strong disturbance (e.g., ground motion). In such cases, to maximize the performance, one can set $d_{max} = 0$, which typically results in better performance when ρ is very large initially, but cannot increase ρ if - or when - states come closer to the origin. Such a control is then based on $\hat{\rho} = min_{t \geq \tau \geq 0}\rho(\tau)$ and the inequalities (6) and (7) can be simplified. This special case was discussed in Srivastava and Jabbari (2000). □

We finally give a simpler version of Lemma 2 where X is also taken as a constant, and only F, G and L are scheduled and depend on ρ. In this case, any information and any constraints about $\dot{\rho}(t)$ automatically disappear. While this introduces significant conservatism over Lemma 2, the implementation is greatly simplified.

Corollary 3. Let η_k's be defined as in Lemma 2. Suppose there exist matrices $X = X^T$, $Y = Y^T$, F_k, G_k and L_k and a scalar $\alpha > 0$ such that (13) and (8) are satisfied, and

$$\begin{bmatrix} \hat{N}_1 + \alpha X & \star & \star \\ A^T + L_1 + \alpha I & \hat{R}_1 + \alpha Y & \star \\ B_1^T & B_1^T Y + D_{21}^T G_1^T & -\alpha I \end{bmatrix} < 0,$$

where $\hat{N}_1 \triangleq AX + XA^T + B_2F_1 + F_1^T B_2^T$ and $\hat{R}_1 \triangleq A^TY + YA + G_1C_2 + C_2^T G_1^T$. Define $\rho(t)$ as $\rho'(t)$ in Theorem 1 and let $V_c(x_c) \triangleq x_c^T \left(X - Y^{-1}\right)^{-1} x_c$. Consider the controller given by (10), with $F(\rho)$ a linear spline on F_k along the notation in (12); i.e,

$$F(\rho) = \begin{cases} F_S(\rho) & \text{if} \quad \dfrac{1}{\eta_{n_\eta}} \le V_c \le \dfrac{1}{\eta_1} \\ F(\eta_{n_\eta}) & \text{if} \quad V_c < \dfrac{1}{\eta_{n_\eta}} \end{cases} \qquad (20)$$

while $G(\rho)$ and $L(\rho)$ are defined similarly. Then, the controller satisfies the following:

(i) The ellipsoid $\mathcal{E}(P, \rho_{min})$ is invariant for the closed-loop state vector x_{cl}.
(ii) The closed-loop system is internally stable and satisfies (11) for zero initial conditions.
(iii) Condition (2) holds. ■

4. NUMERICAL EXAMPLE

Due to space limitations, we consider the simple second order system below (a more reasonable example is presented in the journal version of this paper)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 5 \end{bmatrix} w + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u$$
$$z = y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

We assume the saturation bound on the control input is $u_{sat} = 10$. We choose $n_\eta = 30$ and set the η_k's to be 30 linearly spaced points between $\eta_1 = 1$ and $\eta_{20} = 350$. We solve the inequalities (13)-(15) for $d_{max} = 10000$, while minimizing $\sum_{k=1}^{n_\eta} \gamma_k$. The γ_k's we obtain are shown in figure Figure 1.

We obtaining $A_c(\rho)$, $B_c(\rho)$ and $C_c(\rho)$, using (10) and simulate the closed-loop system using the disturbance signal given in Figure 2. The resulting control input and the norm of the state vector are given in Figure 3. Clearly, the scheduled control law, while avoiding saturation, makes better use

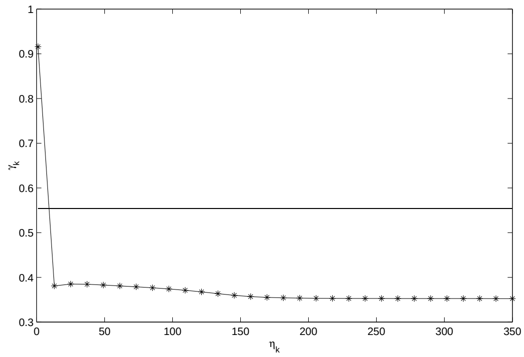

Fig. 1. γ_k vs. η_k.

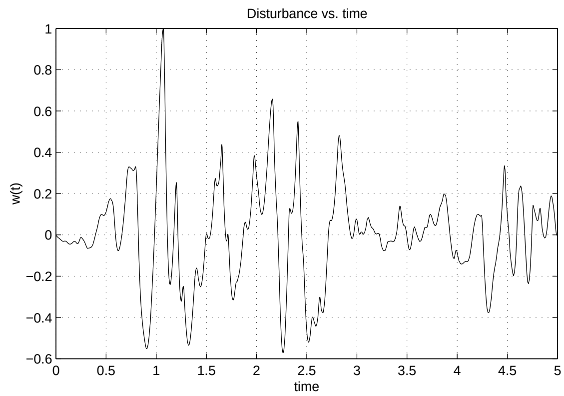

Fig. 2. Disturbance signal.

of actuator capacity. The time history of $\rho(t)$ is given in Figure 5.

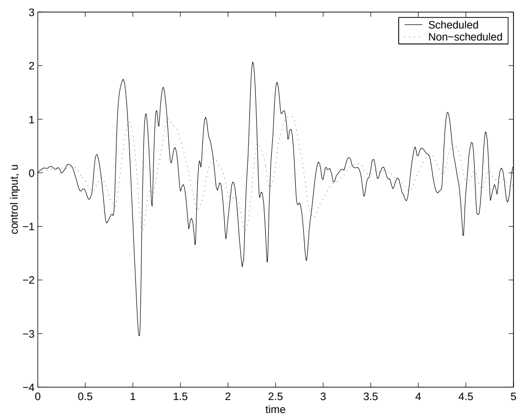

Fig. 3. Scheduled and nonscheduled control inputs.

REFERENCES

Abedor, J., K. Nagpal and K. Poolla (1996). "A Linear Matrix Inequality Approach to Peak-to-Peak Gain Minimization", *International Journal of Robust and Nonlinear Control*, Vol.6, pp.899-927.

Bernstein, D. S. and A. N. Michel, editors (1995). Special Issue on "Saturating Actuators", *Int. Jour. of Robust and Nonlinear Control*, Vol.5.

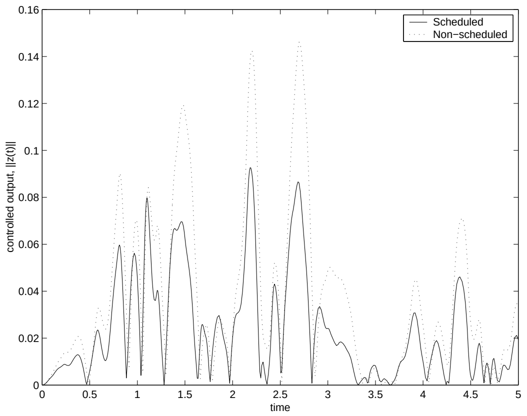

Fig. 4. Scheduled and nonscheduled controlled outputs.

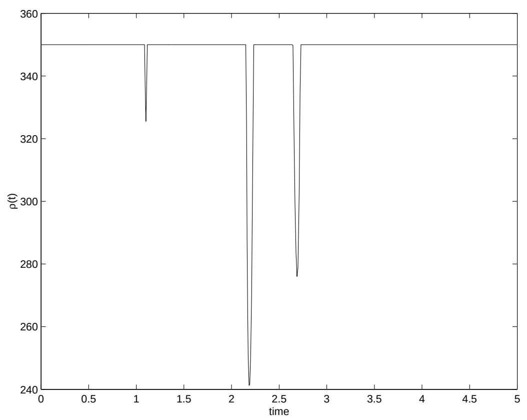

Fig. 5. Parameter $\rho(t)$.

Campo, P. J. and M. Morari (1990). “Robust Control of Processes Subject to Saturation Nonlinearities”, *Computers Chem. Engng*, Vol.14, No.4/5, pp.343-358.

Feron, E., P. Apkarian and P. Gahinet (1996). “Analysis and Synthesis of Robust Control Control Systems via Parameter-Dependent Lyapunov Functions”, *IEEE Transactions on Automatic Control*, Vol.41, No.7, pp.1041-46.

Garcia, G., S. Tarbouriech, R. Suarez and J. Alvarez-Ramirez (1999). “Nonlinear bounded control for norm-bounded uncertain systems,” *IEEE Transactions on Automatic Control*, Vol.44, No.6, pp.1254-1258.

Gilbert, E. G. and K.T. Tan (1991). “Linear Systems with State and Control Constraints: The Theory and Application of Maximal Output Admissible Sets”, *IEEE Transactions on Automatic Control*, Vol.36, pp. 1008-20.

Gutman, P.O. and P. Hagander (1985). “A new design of constrained controllers for linear systems,” *IEEE Transactions on Automatic Control*, Vol.30, No.1, pp.22-33.

Henrion, D., G. Garcia and S. Tarbouriech (1998). “ Piecewise-linear robust control of systems with input constraints,” *Proceedings of the ACC*, pp.3545-49.

Kappor, N., A.R. Teel and P. Daoutidis (1998). “An anti-windup design for linear systems with input saturation,” *Automatica*, Vol. 34, No. 5, pp. 559-574.

Kothare, M., P.J. Campo, M. Morari and C.N. Nett, “A unified framework for the study of anti-windup designs,” *Automatica* Vol. 30, No. 12, pp. 1869-1883, 1994.

Lee, H.L. and M. Spillman (1997). “A Parameter Dependent Performance Criteria for Linear Parameter-Varying Systems.”, *Proceedings of CDC*, pp.984-989.

Lin, Z. (1998). “Global Control of Linear Systems with Saturating Actuators,” *Automatica*, Vol.34, No.7, pp.897-905.

Lin, Z. and A. Saberi (1995). “A Semi-Global Low-and-High Gain Design Technique for Linear Systems with Input Saturation - Stabilization and Disturbance Rejection”, *Int. J. of Robust and Nonlinear Control*, Vol.5, pp.381-398.

Masubuchi, I., A. Kume and E. Shimemura (1998). “Spline-Type Solution to Parameter Dependent LMIs”, *Proceedings of CDC*, pp.1753-1758.

Megretski, A. (1996). “L_2 BIBO Output Feedback Stabilization with Saturated Control,” *Proceedings of 13th IFAC World Congress*, Vol.D, pp.435-440.

Nguyen, T. and F. Jabbari (1999). “Disturbance Attenuation for Systems with Input Saturation: an LMI Approach”, *IEEE Transactions on Automatic Control*, Vol. 44, No. 4, pp. 852-858.

Nguyen, T. and F. Jabbari (2000). “Output feedback controller for disturbance attenuation with actuator amplitude and rate saturation,” *Automatica*, Vol.36, No.9, pp.1339-46.

Rotea, A.R. (1993). “The generalized H_2 Control Problem,” *Automatica*, Vol.29, No.2, pp.373-385.

Srivastava, S. and F. Jabbari (2000). “Scheduled Controllers for Disturbance Attenuation of Systems with Bounded Inputs”, *Proceedings of ACC*, pp.735-739.

Stoorvogel, A.A. and A. Saberi, editors (1999). Special Issue on“Control Problems with Constraint”, *Int. Jour. of Robust and Nonlinear Control*, Vol.9.

Teel, A. R. (1995). “Linear Systems with Input Nonlinearities: Global Stabilization by Scheduling a Family of H_∞-Type Controllers”, *Int. Jour. of Robust and Nonlinear Control*, Vol.5, pp.399-411.