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A MODELING STRATEGY FOR RECTANGULAR THERMAL CONVECTION LOOPS

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Abstract: In this paper a new modelling strategy based on the approximation of the Fourier series expansion of the analytic model of a rectangular thermal convection loop is proposed. This strategy has been derived in order to improve the accuracy of previous existing models. The computation of accurate models is an important step towards the design of suitable control strategies able to prevent the occurence of oscillations in the loop. The obtained model has been validated through extensive comparison between simulated and experimental data. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Thermal convection loops, often named closed loop thermosyphon, are thermo-fluid-dynamical systems mainly devoted to the refrigeration of a heat source by means of natural circulation of a fluid in a closed loop, without mechanical pumping. The absence of moving components drastically reduces the probability of a failure in the heat removal from the heat source. In fact, this is the main reason for which natural is preferred to force convection in those energy plants in which safety is a primary requirement. Therefore, refrigeration of reactors in nuclear power plants and electrical machine rotor cooling (Vijayan *et al*, 1992; Greif *et al*, 1979), represent the main applications of closed loop thermosyphons. Other important applications in which closed loop thermosyphons are preferred to forced circulation loops are those in which the absence of pumping elements allows a considerable costs reduction, e.g. geothermal plants or solar heaters that have low temperature thermal sources but relatively high circulating flow rate (Kreitlow *et al*, 1978; Zvirin *et al*, 1978), or, finally, where the pumping system cannot be conveniently positioned, such as cooling systems for internal combustion engines, turbine blade cooling or computer cooling (Zvirin , 1981; Japikse, 1973).

In their basic scheme natural circulation loops lye on a vertical plane, are symmetrical with respect to the vertical axis and consist of a heat source (placed in the bottom and cooled by the circulating flow), a heat sink (placed on top of the loop) in which the circulating flow is cooled, and may have or not two thermally isolated vertical legs connecting the heat exchanging sections. The boundary conditions usually examined in experimental studies are those in which the bottom section is heated using an imposed heat power (Cammarata *et al*, 1999; Singer and Wang, 1991; Satoh *et al* 1998) (though interesting analysis have been done also for an imposed temperature at the wall of the heated section (Wang *et al,* 1992; Yorke *et al,* 1987)), whereas the topmost section is generally refrigerated imposing a constant wall temperature at its wall. Whatever the boundary conditions are, the buoyancy caused by the density gradients existing in the loop represents the driving force for the fluid motion. Moreover, the flow direction is determined by the temperature difference existing between different points of the loop, on which the buoyancy depends.

The stability of natural circulation loops mainly depends on the entity of the buoyancy, which is proportional to the vertical temperature difference and therefore is determined by the boundary conditions imposed at the heating section. In particular, for increasing values of the vertical temperature difference, ΔT, the flow will be accelerated due to the growth of the forcing term. On the other hand, the growth both of ΔT and of velocity causes a destabilizing effect, which can be schematized as follows:

1. for low ΔT the buoyancy is too weak to cause the fluid motion: heat removal is ensured by conductive heat transfer;
2. when ΔT increases the fluid starts moving either in the clockwise or in the counterclockwise direction (ideally with the same probability); once the motion has started in one direction it keeps moving in this direction;
3. for higher ΔT the velocity of the flow becomes too high and the flow cannot be sufficiently cooled in its passage through the cooling section. The temperature at the outlet of the cooling section increases and its velocity decreases until the mass of fluid coming out from the cooling section has grown so much to exert a sort of sudden impulse, which makes the fluid move rapidly again.
4. when the temperature at the outlet of the cooling section is higher than that at the inlet, the buoyancy inverts its direction, causing therefore the flow inversion. The velocity in the opposite direction, which is initially very low, gradually increases so that steps 2) and 3) are repeated for the current flow direction.

In the last two cases, the dynamics is often characterized by non periodical oscillations which have been shown to be chaotic. Hence, the process leads to temperature oscillations and is associated to inversions of the flow direction. The flow inversion compromises the heat removal from the thermal source and should therefore be avoided.

Stabilizing the dynamics of the process therefore represents the main task in the field of natural circulation loops (Singer and Wang, 1991; Wang *et al*, 1992, Rubio, 1995). In particular suitable control action may consist in varying the refrigerant flow rate to the “cold” sink or the heat power to the “hot” source.

In order to design suitable control laws a reliable model of the system is needed. However the analytic model is non linear and with distributed parameters (Cammarata *et al*, 1999), therefore two different approaches have been proposed in order to obtain a simplified lumped parameter model. In the first approach a set of space-discrete time-continuos equations are obtained through the discretization of the partial differential equations describing the system. This method was firstly proposed in Vijayan (1992) and it was adopted to model the experimental natural circulation loop herein considered in Cammarata (1999). The obtained model was in good agreement with the experimental circuit as regard the stability regions, but was not able to suitably represent the system dynamic.

The other approach is based on an approximation of the equations by using a truncated Fourier series expansion of the variables. The geometry of the system plays an important role in the possibility of applying such method. In fact several results have been reported for toroidal circuits. In this configuration the torus is divided by a horizontal plane just in a heating and a cooling section without adiabatic legs. In this case the simplicity of the geometry has allowed to obtain a system of three first order non linear ordinary differential equations, mainly resembling the Lorenz system (Lorenz, 1963; Singer and Wang, 1991; Wang *et al* 1992; Rubio, 1995). This model does not apply to the case of rectangular circulation loops with adiabatic legs which conversely represent the most common configuration in real applications.

In Fichera *et al* (2000;2001) a neural based NARMAX model was obtained directly from experimental data. This model was useful to perform few step ahead prediction of the outputs in order to early detect the birth of oscillations, but is not suitable to apply classical control design strategies.

2. THE EXPERIMENTAL SYSTEM

The experimental natural circulation loop is depicted in Fig. 1 and its main dimensions are reported in Tab. 1. The circuit consists of two copper horizontal tubes (heat transfer sections), two vertical phirex tubes, four horizontal phirex tubes and four 90° phirex bends. The lower heating section consists of twelve independent electrical heating wires, able to provide 0.5 kW each, winding on the outside of the copper tube, so that the system is able to provide up to 6kW. The upper heat extraction system is a coaxial heat exchanger with tap water flowing in the annulus created by an external iron case (diameter 0.2 m). In this way it is possible to impose desired values both of the heat flux in the lower heating section and of the temperature of the coolant. The latter condition can be obtained by adopting high values of the water flow rate so to minimize the temperature difference between the inlet and the outlet of the cooling water. An expansion tank open to the atmosphere is installed on the topmost elevation of the loop allowing the fluid volumetric expansion.

The whole system is equipped with eight calibrated (± 0.1 K) *T*-thermocouples (diameter 1.6 mm) located (see Fig. 1): T2 and T4 on the left vertical tube; T5 and T6 on the right vertical tube; T1 and T3 on the lower horizontal tubes; T7 and T8 on the input and output of the cooling water.

An inductive flowmeter is inserted in the main loop while another inductive flowmeter and a electromechanical servo-valve are inserted to measure and regulate cooling flow rate, by means of an external computer.

During the tests the heat power supplied is controlled by a power board connected to the external computer via serial port. This system allows then to impose the desired values for both the heat flux in the lower heating section and the flow rate of the coolant in the upper section.

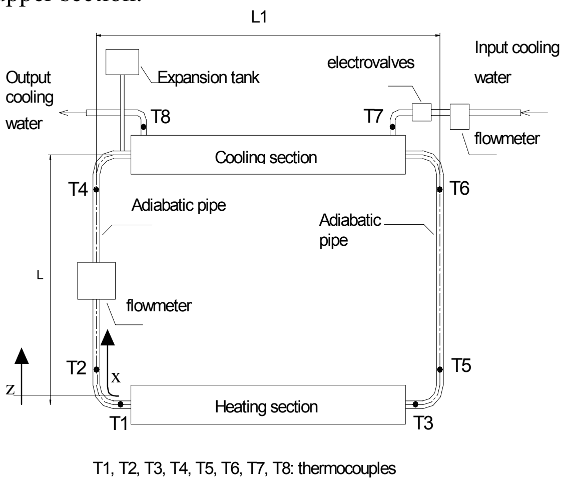

Fig.1 The Experimental system.

Tab. 1 Loop main dimensions

Loop height	1270 mm
Loop width	1780 mm
Loop inner diameter	26 mm
Heating section length	800 mm
Cooling section length	1000 mm
Expansion tank height	910 mm
Expansion tank diameter	300 mm

All the sensors are connected to a data acquisition board in the external computer. A software tool has been implemented in LabView environment that allows to monitor and record all data of each experiment and to perform the control action. The sampling frequency adopted was 1 Hz.

3. THE MATHEMATICAL MODEL

The aim of this section is to give a mathematical description of the behaviour of the natural circulation represented in Fig. 1, having generic height and width, indicated respectively with L and L_1, and constant tubes inner diameter equal to r. In the following, x represents an abscissa parallel to the loop pipes, with positive direction corresponding to the clockwise path on the loop and with an arbitrarily chosen origin in the left down corner of the loop, as indicated in Fig. 1.

The analytic model, obtained applying the thermo-fluid-dynamic equations to the rectangular geometry of the loop is given as:

$$\frac{dv(t)}{dt}+\frac{1}{r}b\left(\frac{\nu}{2r}\right)^{d}v^{2-d}=\frac{g\beta}{2(L+L_1)}\oint(T-T_0)f(x)dx \qquad (1)$$

$$v(0)=v_0$$

$$\frac{\partial T}{\partial t}+v(t)\frac{\partial T}{\partial x}=h(x)+a\frac{\partial^2 T}{\partial x^2} \qquad (2)$$

$$T(x,0)=T_0(x)$$

where: v is the fluid velocity, d and b are parameters to be determined experimentally, ν is the cinematic viscosity, g is the gravitational acceleration, β is the volumetric expansion coefficient, T is the fluid temperature, T_0 is a reference temperature, a is the fluid thermal diffusivity, $f(x)=dz/dx$ is approximated by the piecewise function:

$$f(x)=\begin{cases}1 & 0<x<L\\ 0 & L<x<L+L_1\\ -1 & L+L_1<x<2L+L_1\\ 0 & 2L+L_1<x<2(L+L_1)\end{cases}$$

and $h(x)$ describes the boundary conditions :

$$h(x)=\begin{cases}0 & 0<x<L\\ -\frac{2}{\rho_0 cr}\cdot\frac{\dot m c_p\Delta T_c}{2\pi r L_1} & L<x<L+L_1\\ 0 & L+L_1<x<2L+L_1\\ \frac{2}{\rho_0 cr}\cdot q & 2L+L_1<x<2(L+L_1)\end{cases}$$

where q is the heat flux, ΔT_C is the inlet-outlet temperature difference of the cooling section. In this way the heat extracted in an elementary area is $\dot m c_p \Delta T_C / 2\pi r L_1$ with $\dot m$ and c_p mass flow rate and specific heat of the cooling fluid.

As outlined in the introduction the spatial discretization of the partial differential equations was not able to give a good description of system dynamic. Rodriguez-Bernal and Van Vleck (1998a;1998b) demonstrated the possibility to reduce a model formally identical to that expressed by (1) and (2), by means of the following Fourier series expansion of the known function $f(x)$ and $h(x)$ and of the variable $T(x,t)$:

$$T(x,t)-T_0=\sum_{k\in Z^*}a_k(t)\cdot e^{i\frac{2\pi}{2(L+L_1)}kx} \qquad (3)$$

$$h(x)=\sum_{k\in K}b_k\cdot e^{i\frac{2\pi}{2(L+L_1)}kx} \qquad (4)$$

$$f(x)=\sum_{k\in J}c_k\cdot e^{i\frac{2\pi}{2(L+L_1)}kx} \qquad (5)$$

where $K,J\subset Z^*=Z\setminus\{0\}$. The substitution of (3)-(5) in (1) and (2) leads to the following system of infinite ordinary differential equations:

$$\frac{dv(t)}{dt}+\frac{1}{r}b\left(\frac{\nu}{2r}\right)^{d}v^{2-d}=g\beta\sum_{k\in K\cap J}a_k(t)\cdot\bar c_k \qquad (6)$$

$$\frac{da_k(t)}{dt}+\left[\frac{\pi}{(L+L_1)}ikv(t)+a\frac{\pi^2}{(L+L_1)^2}k^2\right]\cdot a_k(t)=b_k$$

$$k\subset K\cap J \qquad (7)$$

in which: $\bar a_k(t)=a_{-k}(t)$; $\bar b_k=b_{-k}$; $\bar c_k=c_{-k}$.

In order to proceed to the integration of (6)-(7), it is necessary to evaluate the coefficient of the Fourier series expansion, function of the geometrical configuration and of the boundary conditions.

This approach was followed in Wang *et al* (1992) to analyze and control a toroidal loop. In this case a system of three first order differential equations represented a suitable approximation. For some values of the parameters such equations are the Lorenz equations.

In the case of rectangular loops herein considered, the coefficients c_k of $f(x)$ expansion result from the following equation:

$$c_k = \frac{1}{2(L+L_1)} \oint f(x) \cdot e^{-i\frac{2\pi}{2(L+L_1)}kx} dx \qquad (8)$$

Substituting for simplicity the following expressions and considering the expression of *f(x)*:

$$\vartheta = \frac{\pi x}{L+L_1} \qquad \gamma = \frac{\pi L}{L+L_1} \qquad (9)$$

the coefficients c_k for the rectangular geometry become:

$$c_k = \frac{L+L_1}{\pi k i}\left[(1-\cos k\gamma + i sink\gamma)(1-\cos k\pi)\right] \quad (10)$$

that vanish when k is even.

Acting analogously for the calculation of the coefficient b_k of the expansion of *h(x)*, it is possible to obtain:

$$b_k = \frac{1}{2(L+L_1)} \oint h(x) \cdot e^{-i\frac{2\pi}{2(L+L_1)}kx} dx \qquad (11)$$

Substituting again (9) and the following:

$$\Gamma = \frac{2}{\rho_0 cr}\left(\frac{\dot{m}c_p \Delta T}{2\pi r L_1} + q\right) \quad \Gamma_1 = \frac{2}{\rho_0 cr}\left(\frac{\dot{m}c_p \Delta T}{2\pi r L_1} - q\right) \qquad (12)$$

the coefficient b_k in the case studied are expressed by:

$$b_k = \frac{\Gamma}{2\pi k i}\left(-1 - \cos k\gamma + i \sin k\gamma\right)$$

for odd k, k=2n+1 $(n = \pm1, \pm2, ...)$ (13)

$$b_k = \frac{\Gamma_1}{2\pi k i}\left(1 - \cos k\gamma + i \sin k\gamma\right)$$

for even k, k=2n $(n = \pm1, \pm2, ...)$ (14)

The coefficients a_k of the expansion *T(x,t)* are complex and can be expressed as $a_k(t) = \alpha_k(t) + i\beta_k(t)$.

Arresting the Fourier series expansion to the third mode (*k=3*) and applying the method of residuals in order to separate the terms of the same order, it is possible to rewrite equations (6)-(7) as:

$$\frac{dv(t)}{dt} + \frac{1}{r} b\left(\frac{\nu}{2r}\right)^d v^{2-d} = g\beta(a_{-3}c_3 + a_{-1}c_1 + a_1 c_{-1} + a_3 c_{-3}) \qquad (15)$$

$$\frac{da_1(t)}{dt} + \left[\frac{\pi i}{(L+L_1)} v(t) + a\frac{\pi^2}{(L+L_1)^2}\right] a_1(t) = b_1 \qquad (16)$$

$$\frac{da_2(t)}{dt} + \left[\frac{2\pi i}{(L+L_1)} v(t) + a\frac{4\pi^2}{(L+L_1)^2}\right] a_2(t) = b_2 \qquad (17)$$

$$\frac{da_3(t)}{dt} + \left[\frac{3\pi i}{(L+L_1)} v(t) + a\frac{9\pi^2}{(L+L_1)^2}\right] a_3(t) = b_3 \qquad (18)$$

Substituting a_i, b_i, and c_i and separating real and imaginary parts, leads to:

$$\dot{v}(t) = -\frac{1}{r} b\left(\frac{\nu}{2r}\right)^d v^{2-d} + \frac{2g\beta}{\pi} \cdot \left[\alpha_1(t)\sin\gamma - \beta_1(t)(1-\cos\gamma) + \alpha_3(t)\frac{1}{3}\sin 3\gamma - \beta_3(t)\frac{1}{3}(1-\cos 3\gamma)\right] \qquad (19)$$

$$\dot{\alpha}_1(t) = -a\frac{\pi^2}{(L+L_1)^2}\alpha_1(t) + \frac{\pi}{L+L_1}v(t)\beta_1(t) + \frac{\Gamma}{2\pi} sin\gamma \qquad (20)$$

$$\dot{\beta}_1(t) = -a\frac{\pi^2}{(L+L_1)^2}\beta_1(t) - \frac{\pi}{L+L_1}v(t)\alpha_1(t) + \frac{\Gamma}{2\pi}(1+\cos\gamma) \qquad (21)$$

$$\dot{\alpha}_2(t) = -a\frac{4\pi^2}{(L+L_1)^2}\alpha_2(t) + \frac{2\pi}{L+L_1}v(t)\beta_2(t) + \frac{\Gamma_1}{4\pi}\sin 2\gamma \qquad (22)$$

$$\dot{\beta}_2(t) = -a\frac{4\pi^2}{(L+L_1)^2}\beta_2(t) - \frac{2\pi}{L+L_1}v(t)\alpha_2(t) - \frac{\Gamma_1}{4\pi}(1-\cos 2\gamma) \qquad (23)$$

$$\dot{\alpha}_3(t) = -a\frac{9\pi^2}{(L+L_1)^2}\alpha_3(t) + \frac{3\pi}{L+L_1}v(t)\beta_3(t) + \frac{\Gamma}{6\pi}\sin 3\gamma \qquad (24)$$

$$\dot{\beta}_3(t) = -a\frac{9\pi^2}{(L+L_1)^2}\beta_3(t) - \frac{3\pi}{L+L_1}v(t)\alpha_3(t) + \frac{\Gamma}{6\pi}(1+\cos 3\gamma) \qquad (25)$$

This system of seven ordinary differential equation represents the mathematical model, approximated to the third order, of the dynamics of rectangular natural circulation loops.

An important assumption is that the global heat power supplied to the fluid in its passage through the heat source is extracted in its passage through the heat sink. This consideration leads to :

$$\oint h(x) = 0 \qquad (26)$$

and consequently

$$-\cdot\frac{\dot{m}c_p \Delta T_c}{2\pi r L_1} = q$$

and from (12) $\Gamma_1 = 0$.

In order to validate the model it is necessary to compare its simulation with measurements detected on an experimental loop. To this purpose, it is necessary to reconstruct the temperature function *T(x,t)* from the variables of the model $\alpha_k(t)$ and $\beta_k(t)$, i.e. real and imaginary part of its coefficients. This is easily performed according to (3), arresting the sum at the third mode:

$$\begin{aligned} T(x.t) - T_0 &= 2\alpha_1(t)\cos\frac{\pi}{L+L_1}x - 2\beta_1(t)sin\frac{\pi}{L+L_1}x + \\ &+ 2\alpha_2(t)\cos\frac{\pi}{L+L_1}2x - 2\beta_2(t)sin\frac{\pi}{L+L_1}2x + \\ &+ 2\alpha_3(t)\cos\frac{\pi}{L+L_1}3x - 2\beta_3(t)sin\frac{\pi}{L+L_1}3x \end{aligned} \qquad (27)$$

4. MODEL VALIDATION AND ANALYSIS

In order validate the model several simulations have been compared with experimental data.

The following parameters of the model have adopted:

$$\beta = 5.040 \cdot 10^{-4} °C^{-1},$$

$$\rho = 1000 \,\mathrm{kg/m^3},$$

$$c_p = 4186 \,\mathrm{kJ/kg \cdot K},$$

$$\nu = 1.002 \cdot 10^{-6} \,\mathrm{m^2/s},$$

corresponding to the fluid properties at a reference temperature of *T=55°C* and can be considered constant, while the parameters *a, b* and *d* are strongly function of the fluid motion condition and have been computed for four different working conditions, reported in Tab 2.

Tab. 2: Experimental parameters

Power	$a[m^2/s]$	B	d	
<900 W	0.004	2	0.5	Laminar flow
900W÷ 1600 W	0.0002	36	0.9	Turbulent flow with high eddy effect
>1600 W	0.0002	2	0.5	Turbulent flow with low eddy effect

A comparison between a simulation and experimental measurement for the temperature T4 with a constant heating power of 1800 W is reported in Fig. 2 and Fig. 3. Many other simulations have been compared with experimental data for a set of different power, confirming in all cases the good accuracy of the model. In Figures 4 and 5 the attractors of the difference T2-T5 for the simulated and the measured data respectively, are reported for an heating power of 2100W.
A set of comparison has been performed also for a reduced model with the Fourier series stopped to the first mode, in accordance to the model developed in Wang *et al* (1992) for a toroidal circulation loop. In this case a very simple third order model is obtained. From these simulations it resulted that for the rectangular circuit the presence of the third mode is fundamental and therefore the model cannot be further reduced. It can be observed that if the series expansion is stopped to the first mode in equations (4) and (5) there is no difference between a toroidal and a rectangular circuit. The second mode when the assumption (26) is considered becomes negligible. This mode is not controllable but at the same time is stable and rapidly decades.
In Fig. 6 the simulated attractor for the velocity of the first order model is reported. As it can be observed it is remarkably different from the experimental attractor reported in Fig. 8 for the same working conditions. The corresponding simulated attractor of the three mode model is reported in Fig. 7, thus confirming the importance of the introduction of the third mode in modelling the rectangular loop.
The next step towards the design of a controller consisted in the analysis of the linearized model at equilibrium points corresponding to a set of heating power values. It should be observed that for each input power three different equilibrium points have been obtained. Two are perfectly symmetrical and correspond to opposite fluid velocity, while the third correspond to null fluid velocity and is evidently always an unstable point. The first two other equilibrium points are stable for low power values (around 1100 W) and unstable for greater values. In Fig 9 the maximum real part of the eigenvalues of the linearized model is reported with respect to the power.

5. CONCLUSIONS

Thermal convection loops are fluid-dynamical systems widely used in a variety of applications to extract heat without using active pumping elements. Modelling of these systems is a fundamental task in order to design suitable controllers to avoid fluid oscillations. Oscillations considerably reduces the efficiency of the loops.
In this paper a new modelling strategy has been introduced to improve the performance of existing models. Several comparison among simulated and experimental data have been reported showing the accuracy of the proposed model.

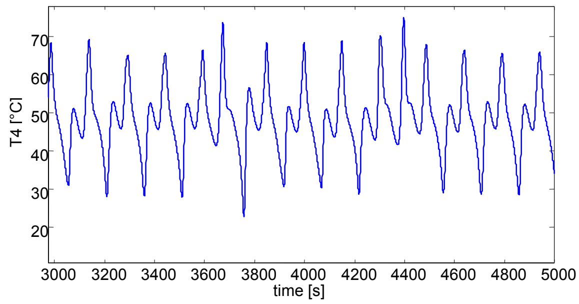

Fig. 2. Simulated temperature T4 for an heating power of 1800 W.

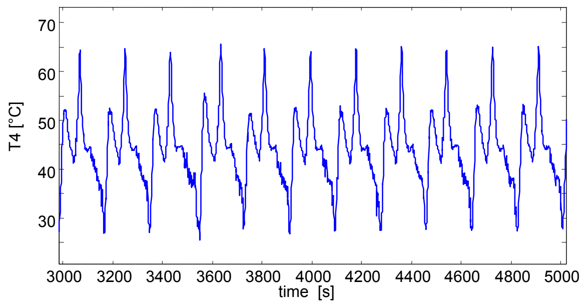

Fig. 3. Experimental temperature T4 for an heating power of 1800 W.

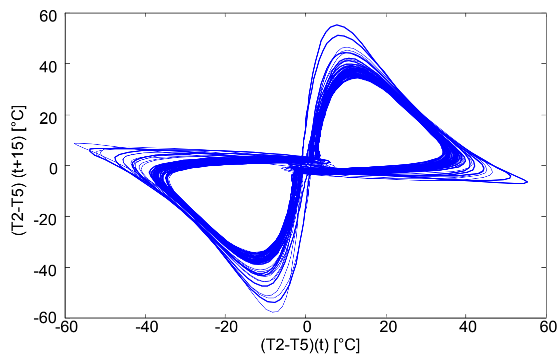

Fig. 4. Simulated attractor of T_2-T_5 for a power of 2100W.

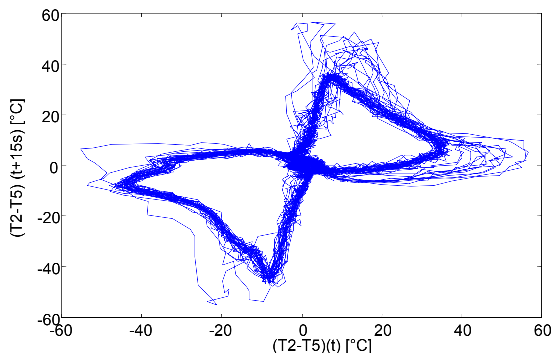

Fig. 5. Experimental attractor of T_2-T_5 for a power of 2100W.

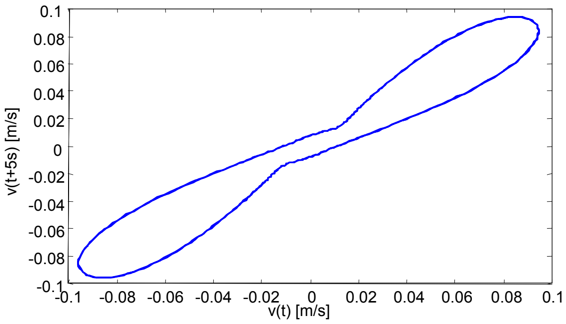

Fig. 6. Simulated attractor of the velocity for a single mode model for a power of 1800W.

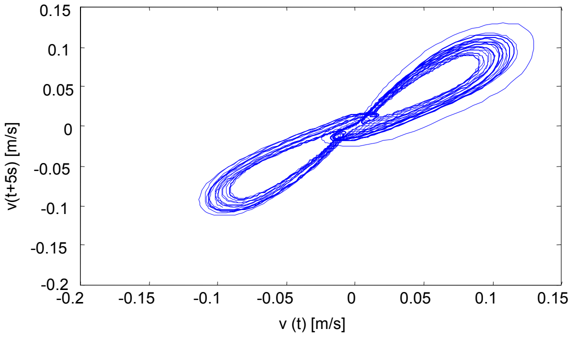

Fig. 7. Simulated attractor of the velocity for a three mode model for a power of 1800W.

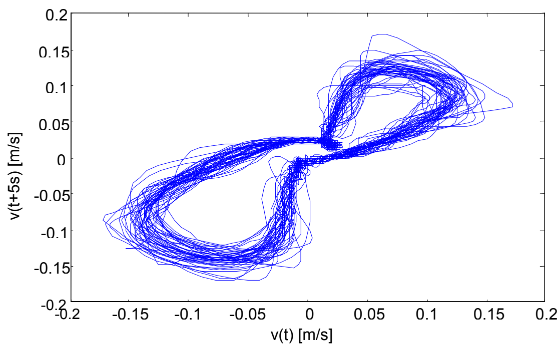

Fig. 8. Experimental attractor of the velocity for a three mode model for a power of 1800W.

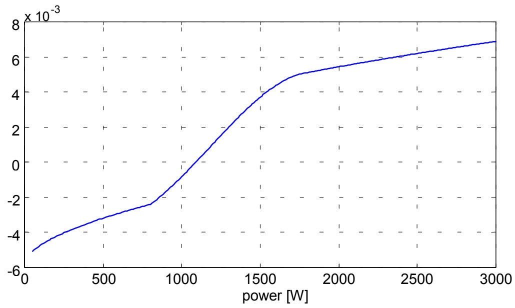

Fig. 9. Maximum real part of the eigenvalues of the linearized model.

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