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OPTIMAL FUZZY CONTROL TO OPTIMISE ENERGY CONSUMPTION IN DISTILLATION COLUMNS

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Abstract: Distillation columns constitute a significant fraction of the capital invested in the refineries around the world; their control requires a major part of the total operating cost of chemical processes, if the used strategy is not adequate. This article presents the application of optimal fuzzy control to reduce the energy consumption of a Benzene-Toluene distillation column. This method is based on the determination of the specific values of the fuzzy controller parameters such that certain performance criterion is minimised. Results of a simulation study are presented showing the potential improvement offered by this method. *Copyright©2002 IFAC*

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1. INTRODUCTION

The control of the overhead and bottom composition in distillation column has been the subject of research for many years. Luyben (1975) has shown that composition control minimizes the energy consumption of a distillation column under the influence of disturbances. However, implementing composition control is not easy due to the phenomenon of interaction or coupling that exists between the various control loops of distillation column (Khelassi, 1991). In addition, distillation column is usually non-linear, non-stationary, multivariable and is subject to constraints and disturbances.

These phenomenons pose a problem for the conception of a robust control system (Stogestad, 1992; Lundström, 1995; Christen, 1997).

Accordingly, much research and development has focused to determine a control that permitted to improve the performance of the distillation column and to optimize the energy consumed by this column. The goal of this work is to propose an optimal fuzzy controller developed by Wang (1998) to control a distillation column in view of optimization of the energy consumed by this column.

On the issue of optimal fuzzy control, Wang developed an optimal controller to stabilize a linear time invariant system via Pontryagin minimum principle (Wang, 1998). However, although fuzzy control of linear systems could be a good starting point for better understanding of some issues in fuzzy control synthesis, it does not have much practical

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implications since using the fuzzy controller designed for a linear system directly as the controller may not be a good choice (Wang, 1998).

Tanaka and co-workers (Tanaka, 1998a, b, 2000) tried to obtain a fuzzy controller to minimize the upper bound of the quadratic performance function by linear-matrix-inequality (LMI) approach based on the assumption of local-linear-feedback-gain control structure. Nevertheless, no theoretical analysis on this design scheme of optimal-fuzzy-control structure was proposed (Wu, 2000a). Wu and Lin (2000a, b) propose a global optimal fuzzy controller for a fuzzy system (i.e., the system described by a fuzzy model).

This paper is organized as follows; a dynamic model of this binary distillation column is presented in Section 2. The Pontryagin minimum principle for solving the optimal control problem is generalised in Section 3. In Section 4, the method developed by Wang to design an optimal fuzzy controller for linear systems is presented. Section 5 deals with the application of the Wang's method to control the study distillation column and gives the obtained simulations results.

2. MODEL OF THE COLUMN

Figure 1 shows a schematic representation of the binary distillation column studied in this work. The column separates a mixture of Benzene-Toluene. It is constituted of seven trays with feed F is entering at the feed tray f ($f = 4$).

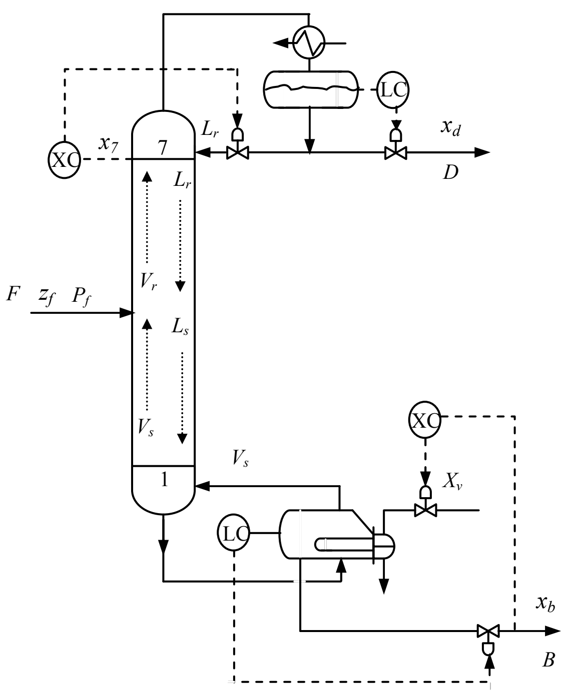

Fig. 1. Schematic of the distillation column.

In this process the top composition of the column (x_7) is controlled by the reflux (L_r), and the bottom composition of the column (x_b) is controlled by the vapour flow (X_v). The nominal data of the column are given by Khelassi (1991).

For the modelling of the distillation column, both the material balance and heat transfer equations are used (Cingara, 1990; Luyben, 1992), thus the obtained model will be constituted by a set of characteristic equations corresponding to the different stages of operating column. For the system of equations describing the operating column see Khelassi (1991).

The linear model of the distillation column is given by the state space representation (Khelassi, 1991):

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{u}\,, \qquad \boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{x}\,. \tag{1}$$

With:

$\boldsymbol{x}^T$, vector of state $\boldsymbol{x}^T = (x_d, x_7, \ldots, x_f, \ldots, x_1, x_b, P_c, V_s)$.
$\boldsymbol{u}^T$, vector of inputs $\boldsymbol{u}^T = (L_r, P_f, F, z_f, P_{ss}, X_v)$.
$\boldsymbol{y}$, vector of outputs $\boldsymbol{y}^T = (x_7, x_b)$.

The values of $\boldsymbol{A}$ and $\boldsymbol{B}$ are given by substitution of the linearised versions of equations around the nominal points.

The values of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are:

$$A=\begin{pmatrix}
-0.0135 & 0.0063 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.0290 & -0.0436 & 0.0168 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0490\\
0 & 0.0290 & -0.0457 & 0.0212 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0908\\
0 & 0 & 0.0290 & -0.0502 & 0.0290 & 0 & 0 & 0 & 0 & 0 & -0.1396\\
0 & 0 & 0 & 0.0270 & -0.0626 & 0.0346 & 0 & 0 & 0 & 0 & -0.1176\\
0 & 0 & 0 & 0 & 0.0356 & -0.0702 & 0.0446 & 0 & 0 & 0 & -0.1369\\
0 & 0 & 0 & 0 & 0 & 0.0356 & -0.0802 & 0.0548 & 0 & 0 & -0.1240\\
0 & 0 & 0 & 0 & 0 & 0 & 0.0356 & -0.0904 & 0.0628 & 0 & -0.0892\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0081 & -0.0157 & 0 & -0.0123\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -15.2240 & -5.0086 & 299.420\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0004 & 0.0283 & 0.0084 & -0.6868
\end{pmatrix}$$

$$B=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0\\
0.0533 & -0.0005 & 0 & 0 & 0 & 0\\
0.0988 & -0.0009 & 0 & 0 & 0 & 0\\
0.1520 & -0.0014 & 0 & 0 & 0 & 0\\
0.1653 & -0.0019 & -0.1169 & 0.0086 & 0 & 0\\
0.1129 & -0.0011 & 0.1129 & 0 & 0 & 0\\
0.1023 & -0.0010 & 0.1023 & 0 & 0 & 0\\
0.0736 & -0.0007 & 0.0736 & 0 & 0 & 0\\
0.0102 & -0.0001 & 0.0102 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0.6229 & 1.4409\\
0.0005 & 0 & 0.0005 & 0 & 0 & 0
\end{pmatrix},$$

$$C=\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}.$$

3. THE PONTRYAGIN MINIMUM PRINCIPLE

In this section, we state the Pontryagin minimum principle for solving the optimal control problem. Consider the system:

$$\dot{\boldsymbol{x}}(t) = g\left[\, \boldsymbol{x}(t), \boldsymbol{u}(t) \,\right], \tag{2}$$

with initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_0$ where $\boldsymbol{x} \in \Re^n$ is the state, $\boldsymbol{u} \in \Re^m$ is the control input, and g is a linear or non-linear function.

The optimal control problem for the system (2) is as follows (Macki, 1982): determine the control $\boldsymbol{u}(t)$ such that the following performance criterion

$$J = S\left[\, \boldsymbol{x}(T) \,\right] + \int_0^{T_f} L\left[\, \boldsymbol{x}(t), \boldsymbol{u}(t) \,\right] dt \,, \tag{3}$$

is minimized, where S and L are given functions and the final time T_f may be given.

The Pontryagin minimum principle for solving this optimal problem proceeds as follows. First, define the Hamilton function:

$$H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) = L\left[\, \boldsymbol{x}(t), \boldsymbol{u}(t) \,\right] + \boldsymbol{\lambda}^T(t)\, g\left[\, \boldsymbol{x}(t), \boldsymbol{u}(t) \,\right], \tag{4}$$

and find $\boldsymbol{u} = h(\boldsymbol{x}, \boldsymbol{\lambda})$ such that $H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda})$ is minimised with this $\boldsymbol{u}$. substituting $\boldsymbol{u} = h(\boldsymbol{x}, \boldsymbol{\lambda})$ into (4) and define

$$H^*(\boldsymbol{x}, \boldsymbol{\lambda}) = H\left[\, \boldsymbol{x}, h(\boldsymbol{x}, \boldsymbol{\lambda}), \boldsymbol{\lambda} \,\right]. \tag{5}$$

Then, solve the $2n$ differential equations (Anderson, 1990):

$$\dot{\boldsymbol{x}}(t) = \frac{\partial H^*}{\partial \boldsymbol{\lambda}}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \,, \tag{6}$$

$$\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H^*}{\partial \boldsymbol{x}}, \qquad \boldsymbol{\lambda}(T_f) = \left.\frac{\partial S}{\partial \boldsymbol{x}}\right|_{x(T_f)}. \tag{7}$$

and let $\boldsymbol{x}^*(t)$ and $\boldsymbol{\lambda}^*(t)$ denote the solution of (6) and (7). Finally, the optimal control is obtained as:

$$\boldsymbol{u}^*(t) = h\left[\, \boldsymbol{x}^*(t), \boldsymbol{\lambda}^*(t) \,\right]. \tag{8}$$

4. OPTIMAL FUZZY CONTROLLER

In this section a review of the method proposed by Wang to design an optimal fuzzy controller of linear systems is presented. This method is based to determine the specific values of the fuzzy controller parameters such that certain performance criterion is minimised.

Consider the time-invariant linear system:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\, \boldsymbol{x}(t) + \boldsymbol{B}\, \boldsymbol{u}(t), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0, \\ \boldsymbol{y}(t) &= \boldsymbol{C}\, \boldsymbol{x}(t). \end{aligned} \tag{9}$$

where $\boldsymbol{x}(t) = \left[\, x_1, ..., x_n \,\right]^T \in \Re^n$ is the state, $\boldsymbol{u}(t) \in \Re^m$ is the control input, $\boldsymbol{y}(t) \in \Re^{n'}$ is the output vector and $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are, respectively, $n \times n$, $n \times m$ and $n' \times n$ matrices. The performance criterion is given by the following quadratic function (Anderson, 1990):

$$J = \boldsymbol{x}^T(T_f)\, \boldsymbol{M}\, \boldsymbol{x}(T_f) + \int_0^{T_f} \left[\, \boldsymbol{x}^T(t)\, \boldsymbol{Q}\, \boldsymbol{x}(t) + \boldsymbol{u}^T(t)\, \boldsymbol{R}\, \boldsymbol{u}(t) \,\right] dt \,. \tag{10}$$

Where the matrices $\boldsymbol{M} \in \Re^{n \times n}$, $\boldsymbol{Q} \in \Re^{n \times n}$, $\boldsymbol{R} \in \Re^{m \times m}$ are symmetric and positive definite.

It is assumed that the desired controller is constructed from the fuzzy systems. If the rules using singleton fuzzifier, center-average defuzzifier and product inference engine (Wang, 1994; Mendel, 1995); the actuating signal from the controller $\boldsymbol{u}(t)$ is $\boldsymbol{u}(t) = (u_1, ..., u_m)^T$ with

$$u_j = f_j(\boldsymbol{x}) = \frac{\sum_{l_1=1}^{2N_1+1} \cdots \sum_{l_n=1}^{2N_n+1} \bar{y}_j^{l_1 \ldots l_n} \left(\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i) \right)}{\sum_{l_1=1}^{2N_1+1} \cdots \sum_{l_n=1}^{2N_n+1} \left(\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i) \right)}. \tag{11}$$

Here the membership functions $\mu_{A_i^{l_i}}(x_i)$ are fixed. Our task is to determine the parameters $\bar{y}_j^{l_1 \ldots l_n}$ such that J of equation (10) is minimised.

Define the fuzzy basis function:

$$\boldsymbol{b}(t) = \left(b_1(\boldsymbol{x}), ..., b_N(\boldsymbol{x})\right)^T,$$

as:

$$b_l(\boldsymbol{x}) = \frac{\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{2N_1+1} \cdots \sum_{l_n=1}^{2N_n+1} \left(\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i) \right)}, \tag{12}$$

where

$l_i = 1, \ldots, 2N+1$, $l_i = 1, \ldots, 2N$ and $N = \prod_{i=1}^{n} (2N_i + 1)$.

Define the parameter matrix $\boldsymbol{\theta} \in \Re^{m \times N}$ as:

$$\boldsymbol{\theta} = \left[\, \boldsymbol{\theta}_1^T \quad \cdots \quad \boldsymbol{\theta}_m^T \,\right]^T. \tag{13}$$

where $\boldsymbol{\theta}_j^T \in R^{1\times N}$ consists of the N parameters $\bar{y}_j^{l_1\ldots l_n}$ for $l_i = 1, 2, \ldots, 2N_i + 1$ in the same ordering as $b_l(\boldsymbol{x})$ for $l = 1, 2, \ldots, N$.

Using these notations, we can rewrite the fuzzy controller $\boldsymbol{u}(t) = (u_1, \ldots, u_m)^T = (f_1(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x}))^T$ of (11) as:

$$\boldsymbol{u} = \boldsymbol{\theta}\, \boldsymbol{b}(\boldsymbol{x}). \tag{14}$$

To achieve maximum optimality, we assume that the parameter matrix $\boldsymbol{\theta}$ is varying; that is, $\boldsymbol{\theta} = \boldsymbol{\theta}(t)$.

Substituting (14) into (9) and (10), we obtain the closed loop system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\, \boldsymbol{x}(t) + \boldsymbol{B}\, \boldsymbol{\theta}(t) \boldsymbol{b}(\boldsymbol{x}(t)), \tag{15}$$

and the performance criterion

$$\begin{aligned} J = \boldsymbol{x}^T(T_f) \boldsymbol{M}\, \boldsymbol{x}(T_f) + \int_0^{T_f} [\boldsymbol{x}^T(t) \boldsymbol{Q}\, \boldsymbol{x}(t) \\ + \boldsymbol{b}^T(\boldsymbol{x}(t)) \boldsymbol{\theta}^T(t) \boldsymbol{R}\, \boldsymbol{\theta}(t) \boldsymbol{b}(\boldsymbol{x}(t))\ ]dt. \end{aligned} \tag{16}$$

Hence, the problem of designing the optimal fuzzy controller becomes the problem of determining the optimal $\boldsymbol{\theta}(t)$ such that J of (16) is minimised (Wang, 1998). Viewing the $\boldsymbol{\theta}(t)$ as the control $\boldsymbol{u}(t)$ in the Pontryagin minimum principle, we can determine the optimal $\boldsymbol{\theta}(t)$ from (4)-(8). Specifically, define the Hamilton function:

$$\begin{aligned} H(\boldsymbol{x}, \boldsymbol{\theta}, \boldsymbol{\lambda}) = \boldsymbol{x}^T \boldsymbol{Q}\, \boldsymbol{x} + \boldsymbol{b}^T(\boldsymbol{x}) \boldsymbol{\theta}^T \boldsymbol{R}\, \boldsymbol{\theta}\, \boldsymbol{b}(\boldsymbol{x}) \\ + \boldsymbol{\lambda}^T [\boldsymbol{A}\, \boldsymbol{x} + \boldsymbol{B}\, \boldsymbol{\theta}\, \boldsymbol{b}(\boldsymbol{x})]. \end{aligned} \tag{17}$$

From $\dfrac{\partial H}{\partial \boldsymbol{\theta}} = 0$; that is:

$$\frac{\partial H}{\partial \boldsymbol{\theta}} = 2\boldsymbol{R}\, \boldsymbol{\theta}\, \boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \boldsymbol{B}^T \boldsymbol{\lambda}\, \boldsymbol{b}^T(\boldsymbol{x}) = 0. \tag{18}$$

We obtain, approximately, that (Wang, 1998):

$$\boldsymbol{\theta} \approx -\frac{1}{2} \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}\, \boldsymbol{b}^T(\boldsymbol{x}) \left[\boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \Delta \right]^{-1}. \tag{19}$$

Where Δ is a full-rank matrix with very small norm; we introduce Δ to make $\boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \Delta$ invertible (Δ may be generated by a small random number generator). Substituting (19) into (17), we can get:

$$\begin{aligned} H^*(\boldsymbol{x}, \boldsymbol{\lambda}) &= \boldsymbol{x}^T \boldsymbol{Q}\, \boldsymbol{x} + \boldsymbol{\lambda}^T \boldsymbol{A}\, \boldsymbol{x} + \frac{1}{4} \boldsymbol{b}^T(\boldsymbol{x}) \left[\boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \Delta \right]^{-1} \\ &\quad \times \boldsymbol{b}(\boldsymbol{x}) \boldsymbol{\lambda}^T \boldsymbol{B}\, \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}\, \boldsymbol{b}^T(\boldsymbol{x}) \left[\boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \Delta \right]^{-1} \boldsymbol{b}(\boldsymbol{x}) \\ &\quad - \frac{1}{2} \boldsymbol{\lambda}^T \boldsymbol{B}\, \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}\, \boldsymbol{b}^T(\boldsymbol{x}) \left[\boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \Delta \right]^{-1} \boldsymbol{b}(\boldsymbol{x}) \\ &= \boldsymbol{x}^T \boldsymbol{Q}\, \boldsymbol{x} + \boldsymbol{\lambda}^T \boldsymbol{A}\, \boldsymbol{x} + \left[\alpha^2(\boldsymbol{x}) - \alpha(\boldsymbol{x}) \right] \boldsymbol{\lambda}^T \boldsymbol{B}\, \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}, \end{aligned} \tag{20}$$

where $\alpha(\boldsymbol{x})$ is defined as:

$$\alpha(\boldsymbol{x}) = \frac{1}{2} \boldsymbol{b}^T(\boldsymbol{x}) \left[\boldsymbol{b}(\boldsymbol{x}) \boldsymbol{b}^T(\boldsymbol{x}) + \Delta \right]^{-1} \boldsymbol{b}(\boldsymbol{x}). \tag{21}$$

Using this H^* in (6) and (7), we obtain:

$$\dot{\boldsymbol{x}}(t) = \frac{\partial H^*}{\partial \boldsymbol{\lambda}} = \boldsymbol{A}\, \boldsymbol{x} + 2\left[\alpha^2(\boldsymbol{x}) - \alpha(\boldsymbol{x}) \right] \boldsymbol{B}\, \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}, \tag{22}$$

$$\begin{aligned} \dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H^*}{\partial \boldsymbol{x}} = -2\, \boldsymbol{Q}\, \boldsymbol{x} - \boldsymbol{A}^T \boldsymbol{\lambda} - [2\, \alpha(\boldsymbol{x}) - 1] \\ \times \frac{\partial \alpha(\boldsymbol{x})}{\partial \boldsymbol{x}} \boldsymbol{\lambda}^T \boldsymbol{B}\, \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}. \end{aligned} \tag{23}$$

with boundary condition $\boldsymbol{x}(0) = \boldsymbol{x}_0$ and $\boldsymbol{\lambda}(T_f) = 2\, \boldsymbol{M}\, \boldsymbol{x}(T_f)$. Let $\boldsymbol{x}^*(t)$ and $\boldsymbol{\lambda}^*(t)$ $(t \in [0, T_f])$ be the solution of (22) and (23), then the optimal fuzzy controller parameters are (Wang, 1998):

$$\boldsymbol{\theta}^*(t) = -\frac{1}{2} \boldsymbol{R}^{-1} \boldsymbol{B}^T \boldsymbol{\lambda}^*(t) \boldsymbol{b}^T(\boldsymbol{x}^*(t)) [\boldsymbol{b}(\boldsymbol{x}^*(t)) \boldsymbol{b}^T(\boldsymbol{x}^*(t)) + \Delta]^{-1}, \tag{24}$$

and the optimal fuzzy controller is:

$$\boldsymbol{u}^* = \boldsymbol{\theta}^*(t) \boldsymbol{b}(\boldsymbol{x}). \tag{25}$$

Note that the optimal fuzzy controller is a state feedback controller with time varying coefficients.

The optimal fuzzy controller algorithm (Wang, 1998)

Step 1. Specify the membership functions $\mu_{A_i^{l_i}}(x_i)$ to cover the state space where $l_i = 1, 2, \ldots, 2N_i + 1$ and $i = 1, 2, \ldots, n$. The membership functions may not be chosen as triangular because the function $\alpha(\boldsymbol{x})$ with these membership functions is not differentiable [we need $\dfrac{\partial \alpha(\boldsymbol{x})}{\partial \boldsymbol{x}}$ in (23)]. We choose $\mu_{A_i^{l_i}}(x_i)$ to be Gaussian functions.

Step 2. Compute the fuzzy basis functions $b_l(\boldsymbol{x})$ from (21) and the function $\alpha(\boldsymbol{x})$ from (21). Compute the derivative $\dfrac{\partial\alpha(\boldsymbol{x})}{\partial\boldsymbol{x}}$.

Step 3. Solve the two point boundary differential equations (22) and (23) and let the solution be $\boldsymbol{x}^*(t)$ and $\boldsymbol{\lambda}^*(t)$, $t \in [0, T_f]$. Compute $\boldsymbol{\theta}^*(t)$ from (24).

Step 4. The optimal fuzzy controller $\boldsymbol{u}^*$ is obtained as given by relation (25).

5. SIMULATION RESULTS

To demonstrate the contribution of the optimal fuzzy control depicted above, a comparison with classical optimal control is performed on the basis of a simulation study. In order to apply the two listed techniques, the **RGA** (MacAvoy, 1983) of the considered distillation column is generated to select the best control configuration. According to the values of the **RGA** given bellow

$$\mathbf{RGA} = \begin{bmatrix} 23.7 & -22.7 \\ -22.7 & 23.7 \end{bmatrix}, \tag{26}$$

The best control configuration is defined as follows:

$$[\, L_r - x_7 \,] \,;\, [\, X_v - x_b \,] \,.$$

The stability condition is verified since the corresponding relative gains of the control configuration pairs of this system are positive.
The quadratic function is chosen as:

$$J = \int_0^{T_f} \left[\boldsymbol{e}^T(t) \boldsymbol{Q}\, \boldsymbol{e}(t) + \boldsymbol{u}^T(t) \boldsymbol{R}\, \boldsymbol{u}(t) \right] dt \,, \tag{27}$$

where e is error vector $\boldsymbol{e} = [\, e_1 \,, e_2 \,]^T$ with:

$$e_1 = x_{7set} - x_7 \,,\; e_2 = x_{bset} - x_b \,,$$

and

x_{7set} is the set point for the top composition.
x_{bset} is the set point for the bottom composition.

The matrix $\boldsymbol{Q}$ and $\boldsymbol{R}$ are chosen as follow:

$$\boldsymbol{Q} = \begin{bmatrix} 0.008 & 0 \\ 0 & 0.01 \end{bmatrix} \text{ and } \boldsymbol{R} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.05 \end{bmatrix}.$$

The membership functions $\mu_{A_i^{l_i}}(e_i)$ are chosen as Gaussian form and are given by:

$$\mu_{A_i^l}(e_i) = \exp\left[-2\left(e_i - e_i^{l_i} \right)^2 \right], \tag{28}$$

l_i $(i = 1, 2)$ makes reference to the considered fuzzy set NB (Negative Big), NS (Negative Small), ZE (Zero), PS (Positive small) or PB (Positive Big), and $e_i^{NB} = -2$, $e_i^{NS} = -1$, $e_i^{ZE} = 0$, $e_i^{PS} = 1$ and $e_i^{PB} = 2$.

The linguistic rule table is given by the table 1 and the fuzzy basis function is given by:

$$b_l(\boldsymbol{e}) = \frac{\mu_{A_1^{l_1}}(e_1)\, \mu_{A_2^{l_2}}(e_2)}{\sum_{l_1=1}^{2N_1+1} \sum_{l_2=1}^{2N_2+1} \mu_{A_1^{l_1}}(e_1)\, \mu_{A_2^{l_2}}(e_2)}, \tag{29}$$

where

$$l = (2N_1 + 1)(2N_2 + 1) \text{ with } N_1 = N_2 = 2 \,.$$

The dynamic responses of set point change of the top composition ($x_7 = 0.8983 \to 0.91$) and the bottom composition ($x_b = 0.04878 \to 0.0537$) are presented in figure 2. It is shows that every controller in the two considered control techniques assures the tracking of the assigned reference input.

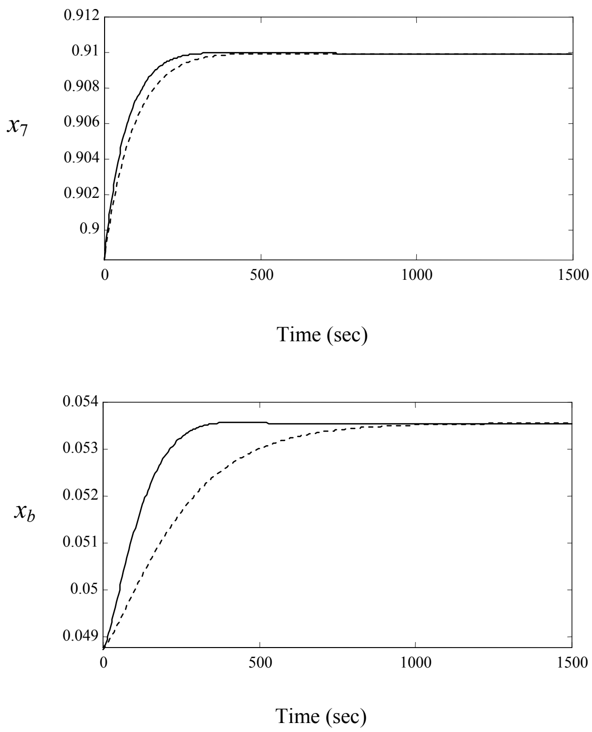

Fig. 2. Compositions of top x_7 and bottom x_b of the column.
------- Classical optimal control
——— Optimal fuzzy control

Table 1. The linguistic rule table

e_2 \ e_1	NB	NS	ZE	PS	PB
NB			PB		
NS			PS		
ZE	PB	PS	ZE	NS	NB
PS			NS		
PB			NB		

Table 2. Calculation of the IAE of the two loops for the optimal fuzzy control and classical optimal control of the column.

Optimal control	Top composition (x_7)	Bottom composition (x_b)
Classic	0.1298	0.1004
fuzzy	0.0942	0.0889

To express the energy consumed by the distillation column we calculate the integral absolute error IAE of the two loops for the optimal fuzzy control and classical optimal control. The obtained IAE values are given in table 2. The total IAE values (0.2302 for the classical optimal control and 0.1831 of the optimal fuzzy control), shows that the dynamic error is reduced in the optimal fuzzy control, what implies that the energy consumption is reduced in relation to the classical optimal control. Therefore, according to the obtained IAE values, the optimal fuzzy controller gives better results.

6. CONCLUSION

In this paper an optimal fuzzy controller for a binary distillation column was presented and compared to the classical optimal control. The obtained simulation results show the effectiveness of the optimal fuzzy control. The comparison of the obtained performance criterion IAE values, demonstrated that the consumed energy by the distillation column is optimised in the case of the optimal fuzzy control in relation to the classical optimal control, what constitutes a significant advantage in process industry capital investment, when we know that operating costs of these systems are often amongst the highest. According to these results, the optimal fuzzy control made a good forward in the optimisation of energy consumption in distillation columns.

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