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COMPARISON AMONG SEQUENTIAL SUBOPTIMAL APPROACHES WITH FOCUS ON STOCHASTIC PRODUCTION PLANNING.

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Abstract: Within a decision-making hierarchy of production planning process, the majority of problems are dependent on the time component and strongly sensitive to endogenous and exogenous components. These problems can be related to an important class of stochastic optimal control. Difficulties in providing a closed-loop policy for them, lead to search for sub-optimal alternatives. In this paper, four different sub-optimal procedures are investigated in relation to their implementation by dynamic programming algorithm. Based on an aggregated production-planning problem, a case study is considered with the purpose of comparing the different procedures. The best one is used to provide scenarios relating to the future use of material resources of the company. *Copyright@2002IFAC*

Keywords: optimal control, dynamic systems, sub-optimal procedures, dynamic programming, production planning.

1. INTRODUCTION

The paper deals with a discrete time, linear-stochastic optimal control problem with chance constraints on state and control variables. Such a problem has been used to represent an extensive class of management problems like financing, marketing, advertising, production and inventory (Neck, 1984). The last class, that is, the production planning problem is of main interest here. The production-planning problem requires a set of decisions used to adapt the material resources of the company as a means to satisfy the demand. Such decisions are made over different planning horizons that depend on the levels of the planning hierarchy. For instance, in the strategic level, the decisions are made over a long-term horizon (i.e. from 1 to 2 years) and as a result, a production plan is developed. This plan usually enables the manager to make decisions about changes in the workforce, overtime scheduling, activating machines on stand-by, subcontracting production capacity, firing, hiring, etc. Therefore, it provides important insights concerning the rational use of the firm's industrial resources. This production plan is to be provided by a stochastic optimal control problem.

Due to factors like uncertainty, dimensionality, and physical constraints, this problem is not easily solved. A true optimal solution (i.e., closed-loop solution) is possible only in special cases, as with small dimension problems by using, for example, stochastic dynamic programming. Consequently, for larger problems, suboptimal approaches become very important. These procedures usually provide numerical solutions for various sequential optimization problems, which is calculated and implemented easier than the true optimal solution. There are a wide variety of sequential suboptimal procedures, and it is not easy to classify them in a unified manner. Some tentatives are found in Neck (1984) and Bertesekas (1995).

The basic objectives desired from this work are: a) to formulate a general linear stochastic control model that represents a production planning problem (section 2); b) to use the dynamic programming algorithm for implementing the optimal (section 4) and suboptimal approaches (section 5) with focus on the stochastic problem defined in section 2; c) from a simple example (section 6), to compare solutions provided by the different suboptimal approaches with the true optimal solution; and d) to show that the best performance approach can be used in a simulation scheme that helps the manager adjust the production plan and, as well, to gain insight concerning the future use of the company's material resources.

2. A LINEAR STOCHASTIC PROBLEM

Let's consider the following stochastic optimal control problem in which all states are assumed perfectly observed

$$\begin{aligned} &\underset{u_k}{\mathrm{Min}}\ \underset{d_k}{\mathrm{E}}\left\{ f_N(x_N) + \sum_{k=0}^{N-1} f_k(x_k, u_k) \right\} \\ &\text{s.t.} \\ &\quad x_{k+1} = x_k + Bu_k - d_k \\ &\quad \mathrm{Prob.}\{x_{k+1} \in X_{k+1}\} \geq \alpha \\ &\quad \mathrm{Prob.}\{u_k \in U_k\} \geq \gamma \\ &\quad k = 0, 1, \ldots, N-1 \end{aligned} \tag{1}$$

where $\{f_k,\ k=1,\ \ldots,\ N\}$ denotes functions that represent, for example, holding and production costs; $B \in \Re^{nxm}$ denotes a coupleable matrix, in which, each row can represent a sequence of machines which is responsible for producing a product; $x \in \Re^n$ denotes the vector of states of the system where information about inventory and workforce are available; $u \in \Re^m$ denotes the control variable of the system which contains information about the capacity of production of each machine; and $d \in \Re^n$ is a vector of non-negative random variables which represents, for example, the fluctuation of sales. Note that the state and control variables are constrained to the following spaces: $X_k=\{\underline{x}_k \leq x_k \leq \overline{x}_k\}$ and $U_k=\{\underline{u}_k \leq u_k \leq \overline{u}_k\}$. These constraints are considered in probability due to the stochastic nature of the system.

Since the production process is a dynamic system, the problem must be seen as a model for sequential decision making. Consequently, all information measured over the periods have a strong influence on the optimal solution. Besides, some features of the problem, such as: dimension, multi-period structure, probabilistic constraints, and stochastic nature, make it very difficult to be solved. A true optimal sequential solution can be provided by the stochastic dynamic programming algorithm (SDP), however, this is only valid for small dimension versions of (1). It is note worthy that an advantage of applying SDP is that it provides a global optimal solution for (1) that can be used as a benchmark for checking and comparing other alternative techniques.

For overcoming the above difficulty is to consider the suboptimal stochastic procedures available in the literature. Such approaches combine concepts of control theory (for example, feedback schemes) with mathematical programming techniques. Before starting to introduce the optimal and suboptimal procedures in the next section, a method for transforming the state and control probabilistic constraints into an equivalent deterministic constraint will be discussed.

3. CHANCE-CONSTRAINTS

Constraints on state and control variables strongly increase the complexity of solving an optimization problem. Particularly in stochastic cases, it is almost impossible to guarantee feasible solution face to these constraints. A possibility of overcoming such difficulty is to consider probabilistic constraints, as will be seen sequentially.

First, consider the linear stochastic system given in (1); and, second, assume that: (a) the random variable d_k has a Gaussian probability distribution function $\Phi_{d,k}$ with mean $\hat{d}_k$ and finite variance $\mathrm{Var}(d_k)=V_{dk}\geq 0$; and (b) a small variability of the control variance (i.e., $V_{uk}\approx 0$), so that, the risk of violation of the control constraint is low. This means that $u_k \in U_k$, $\forall \gamma$. Based on these characteristics follow the lemma:

Lemma: Let β_1 and β_2 be probabilistic measures, then $\mathrm{Prob.}\{x_{k+1} \in X_{k+1}\} \geq \alpha$ can be written:

$$\begin{cases} \mathrm{Prob.}\{x_{k+1} \geq \underline{x}_{k+1}\} \geq \beta_1 \\ \mathrm{Prob.}\{x_{k+1} \leq \overline{x}_{k+1}\} \geq \beta_2 \end{cases} \tag{2}$$

where $\alpha \leq \beta_1 + \beta_2 - 1$ ■ (see Silva Filho (2000)).

Assuming, for the sake of simplicity, that $\beta_1=\beta_2=\beta$ implies that $\alpha \leq 2\cdot\beta\text{-}1$ and, consequently the dynamic equation in (1) can be used to rewrite (2) as follows:

$$\begin{cases} \mathrm{Prob.}\left\{ \varepsilon_k \leq \left(x_k + Bu_k - \hat{d}_k - \underline{x}_{k+1} \right) / V_{dk} \right\} \geq \beta \\ \mathrm{Prob.}\left\{ \varepsilon_k \geq \left(x_k + Bu_k - \hat{d}_k - \overline{x}_{k+1} \right) / V_{dk} \right\} \geq \beta \end{cases} \tag{3}$$

where $\varepsilon_k=(d_k-\hat{d}_k)/V_{dk}$ denotes a normalized random variable with identical distribution to the d_k. Thus, assuming that the inverse distribution function $\Phi_{d,k}^{-1}$ exists, it is possible from (3) to determine statistics lower and upper boundaries for the control variable:

$$\begin{cases} \textbf{(a)}\ \Phi_{d,k}\left\{ \left(x_k + Bu_k - \hat{d}_k - \underline{x}_{k+1} \right) / V_{dk} \right\} \geq \beta \Leftrightarrow \\ u_k \geq (BB^T)^{-1} \cdot \left[\underline{x}_{k+1} + \hat{d}_k + V_{dk} \cdot \Phi_{d,k}^{-1}(\beta) - x_k \right] = \underline{u}(x_k, \beta) \\ \textbf{(b)}\ \Phi_{d,k}\left\{ \left(x_k + Bu_k - \hat{d}_k - \overline{x}_{k+1} \right) / V_{dk} \right\} \geq \beta - 1 \Leftrightarrow \\ u_k \leq (BB^T)^{-1} \cdot \left[\overline{x}_{k+1} + \hat{d}_k - V_{dk} \cdot \Phi_{d,k}^{-1}(\beta) - x_k \right] = \overline{u}(x_k, \beta) \end{cases} \tag{4}$$

Comparing the space defined in (4) with the original control space U_k, defined in section 2, it is possible to define a sub-space $U_\beta(x_k)$ that depends on the observed state at each period k, and on the probability measure β. The lower and upper limits of this space are respectively: $\underline{u}_{\hat{a}x} = \max[\underline{u}_k, \underline{u}(x_k, \hat{a})]$ and $\overline{u}_{\hat{a}x} = \min[\overline{u}_k, \overline{u}(x_k, \hat{a})]$. An important property of this subspace is that for two distinct probability measures β_1 and β_2, it is possible to show that $U_{\beta_2}(x_k) \subset U_{\beta_1}(x_k)\quad \forall k$, if $\beta_1 > \beta_2$.

4. TRUE OPTIMAL SOLUTION VIA SDP

Let's primarily change the probabilistic constraint of the problem (1) for the deterministic constraint $u_k \in U_\beta(x_k)$ described above. Due to the additive structure of the functional cost in (1), the principle of optimality can be applied and, as a result, a sequence of subproblems can be defined and solved interactively over the time. Thus, applying the stochastic dynamic programming SDP, the problem (1) becomes one of finding a sequence of control $\{u_k^* \in U_\beta(x_k),\ k=0,\ 1,\ ...,N-1\}$ that solves the following subproblem which proceeds backward from stage N-1 to stage 0 with x_0, x_N, and $J(x_N)=f(x_N)$, known *a prior*:

$$
\begin{aligned}
J_k(x_k) = &\min_{u_k \in U_\beta(x_k)} \mathop{E}_{d_k} \{f_k(x_k,u_k) + J_{k+1}(x_{k+1})\} \\
&\text{s.t.} \\
&\quad x_{k+1} = x_k + Bu_k - d_k \\
&\quad k = N-1,...,1,0
\end{aligned} \tag{5}
$$

where $J_{k+1}(x_{k+1}) = \min_{u_k \in U_\beta(x_k)} \mathop{E}_{d_k} \left\{ f_N(x_N) + \sum_{i=k}^{N-1} f_k(x_i,u_i) \right\}$.

Some comments: the optimal cost is $J^*=J_0(x_0)$ given by the last step of the algorithm (5). This true optimal solution is impossible for large dimension problems. However, for small dimension problems, it allows the user to compare alternative procedures for solving (1).

5. SUB-OPTIMAL PROCEDURES

The difficulties of providing a true optimal solution for the problem (1) has increased the interest for approximate (sub-optimal) procedures (Neck, 1984). These procedures usually depend on some simplifications of the original problem, which allows using mathematical programming and/or optimal feedback control techniques in order to provide a feasible solution. Next, three different alternative solution procedures will be presented.

5.1. Mean-Value Controller (MVC)

This procedure uses the certainty-equivalence principle (Bertesekas, 1995). Based on this principle, all available information about the decision variables is represented by their mean values. Therefore, the problem (1) can be approximated by a deterministic problem whose solution can be provided by any optimal control technique as, for instance, the maximum principle of Pontryagin (Parlar, 1985) or, by any applied mathematical programming technique. Using the deterministic dynamic programming algorithm, the MVC procedure can be implemented as follows:

$$
\begin{aligned}
&\text{Let be } J_N(\hat{x}_N) = f_N(\hat{x}_N) \text{ then compute:} \\
&J_k(\hat{x}_k) = \min_{\hat{u}(k) \in U(\hat{x}(k))} f_k(\hat{x}_k, \hat{u}_k) + J_{k+1}(\hat{x}_{k+1}) \\
&\qquad \text{s.t.} \\
&\qquad\quad \hat{x}_{k+1} = \hat{x}_k + B\hat{u}_k - \hat{d}_k \\
&\qquad\quad k = N-1,\ ...,\ 1,\ 0
\end{aligned} \tag{6}
$$

where $\hat{d}_k = E\{d_k\} \Rightarrow \hat{x}_k = E\{x_k\}$ e $\hat{u}_k = u_k$. Note that $\hat{u}_k \in U(\hat{x}_k)$ whose lower and upper bounds are respectively:

$$
\begin{cases}
\underline{u}(\hat{x}_k) = \max\left[\underline{u}_k,\ (BB^T)^{-1}(\underline{x}_{k+1} - \hat{x}_k + \hat{d}_k)\right] \\
\overline{u}(\hat{x}_k) = \min\left[\overline{u}_k,\ (BB^T)^{-1}(\overline{x}_{k+1} - \hat{x}_k + \hat{d}_k)\right]
\end{cases}
$$

Note that: since the he current state of the system are completely ignored at each period k, the solution provided by MVC is known as an open-loop solution.

5.2. Naive Feedback Controller (NFC)

The basic idea of this procedure is to consider the random variable of the system centered in any value know exactly, for example, the mean values of the decision variables. Different from MVC, the NFC takes into account the current state of the system to compute the control policy. This procedure can be implemented from the dynamic programming (DP) algorithm as follows:

(*Step 1*) At the beginning of period k, the current state of the system x_k is observed.

(Step *2*) This information is used as an initial condition (i.e., $\hat{x}_i = x_k$) to provide a deterministic optimal control policy from the algorithm:

$$
\begin{aligned}
&\text{Let be } J_N(\hat{x}_N) = f_N(\hat{x}_N) \text{ then compute:} \\
&J_k(\hat{x}_i) = \min_{\hat{u}_i \in U(\hat{x}_i)} \{f_k(\hat{x}_i, \hat{u}_i) + J_{k+1}(\hat{x}_{i+1})\} \\
&\qquad \text{s.t.} \\
&\qquad\quad \hat{x}_{i+1} = \hat{x}_i + B\hat{u}_i - \hat{d}_i \\
&\qquad\quad i = N-1,\ ...,\ k,\ k-1
\end{aligned} \tag{7}
$$

(*Step 3*) Since new measures of the state of the system are not allowed, the optimal policy provided by (7) is a deterministic sequence $\{\hat{u}_i, \hat{u}_{i+1}, ..., \hat{u}_N\}$. However, the NFC uses only the first element of this control sequence (i.e., $u_k^* = \hat{u}_i$) to apply to the input of the system, ignoring the others elements. For the next period k+1, as soon as a new measure is observed, the steps from *2* to *3* are repeated again.

As a result, in the implementation of the NFC procedure, problem (7) must be solved N times in an “on-line” fashion (Bertesekas, 1995).

5.3. Open-Loop Feedback Controller (OLFC)

Contrasting MVC and NFC, the OLFC procedure preserves the stochastic nature of the system. It takes into account the uncertainties about x_k and d_k whenever it calculates the optimal control policy. Consequently, its implementation is more complex than the previous ones. In fact, to apply the OLFC procedure it is essential to know beforehand the probability distribution of the state x_k. Using the dynamic programming algorithm, the procedure can be described as follows:

(*Step 1*) At each period k=0, 1, ..., N-1, as soon as a new measure of the state of the system is taken, the initial state is updated, that is: $\hat{x}_i = x_k$ and the probability distribution $\Phi_{x,k}$ is computed.

(*Step* 2) From this information, the optimal control sequence $\{u_i, u_{i+1}, ..., u_{N-1}\}$ is computed by the algorithm:

$$\begin{gathered}\text{Let be } J_N(\hat{x}_N) = F_N(\hat{x}_N) \text{ then compute:}\\ J_k(\hat{x}_i) = \underset{\hat{u}_i \in U_\beta(\hat{x}_i)}{\mathrm{Min}} \{F_k(\hat{x}_i, \hat{u}_i) + J_{k+1}(\hat{x}_{i+1})\}\\ \text{s.t.}\\ \hat{x}_{i+1} = \hat{x}_i + B\hat{u}_i - \hat{d}_i\\ i = N-1, ..., k, k-1\end{gathered} \tag{8}$$

where $F_i(\hat{x}_i, \hat{u}_i) = E\{f(x_i, u_i)\} = \int_{\Re} f_i(x_i, u_i)\partial\Phi_{xk}$ is the expected values of the functional cost $f_i(.)$.

(Step 3) Since new measures connected with the state of the system are not allowed, the solution of (8) is a deterministic sequence $\{\hat{u}_i, \hat{u}_{i+1}, ..., \hat{u}_N\}$. Similar to NFC, the OLFC procedure selects only the first element of the sequence (i.e., $u_k^* = \hat{u}_i$) to apply to the input of the system, ignoring the other elements. Thus, as soon as, a new measure of state is observed at the beginning of period k+1, the procedure returns to *step 2* and the problem (8) is solved again for stages i=k+1, ..., N.

5.4. Partial Closed-loop Controller (PCC)

The basic idea of this procedure is to combine an open-loop solution provided from the equivalent deterministic problem with a feedback control structure, see Silva Filho (1999) for details. A brief description of this approach is given as follows: First of all, a linear decision rule for the problem (1) is given by:

$$u_k = \hat{u}_k + G_k \cdot (x_k - \hat{x}_k) \tag{9}$$

where G_k is a linear feedback gain; and $\hat{u}_k$ and $\hat{x}_k$ are the mean optimal solution provided by the equivalent deterministic problem arisen from (1), i.e.:

$$\begin{gathered}\underset{u_k}{\mathrm{Min}}\, F_N(\hat{x}_N) + \sum_{k=0}^{N-1} F_k(\hat{x}_k, \hat{u}_k)\\ \text{s.t.}\\ \hat{x}_{k+1} = \hat{x}_k + B\hat{u}_k - \hat{d}_k\\ \underline{x}_{\alpha k} \le \hat{x}_k \le \overline{x}_{\alpha k}\\ \underline{u}_{\gamma k} \le \hat{u}_k \le \overline{u}_{\gamma k}\end{gathered} \tag{10}$$

where F(.) is calculated analogously to (8); and the lower and upper bounds of the state and control are given by (Silva Filho and Ventura, 1999):

$$\begin{cases}\underline{x}_{\beta k} = \underline{x}_k + \sqrt{V_{xk}} \cdot \Phi_x^{-1}(\beta)\\ \overline{x}_{\beta k} = \overline{x}_k - \sqrt{V_{xk}} \cdot \Phi_x^{-1}(\beta)\end{cases} \tag{11}$$

$$\begin{cases}\underline{u}_{\beta k} = \underline{u}_k + \sqrt{V_{uk}} \cdot \Phi_u^{-1}(\beta)\\ \overline{u}_{\beta k} = \overline{u}_k - \sqrt{V_{uk}} \cdot \Phi_u^{-1}(\beta)\end{cases} \tag{12}$$

where V_x and V_u denote the state and control variances and Φ_x^{-1} and Φ_u^{-1} the inverse distribution functions. Note that to guarantee some similarity with the results discussed in section 3, it has been assumed that $\alpha=\gamma=\beta$.

A particular characteristic of the system operating in open-loop (i.e. $G_k=0$) is that the evolution of variances of state and control grow over the periods, reaching their maximum values at periods N and N-1, respectively. The difficulty is that this growth can lead to the risk of infeasibility of problem (10) due to the degeneration of the inequality constraints given by (11)-(12). In practice, this degeneration means that $\underline{x}_{\beta k} \ge \overline{x}_{\beta k}$ and/or $\underline{u}_{\beta k} \ge \overline{u}_{\beta k}$. To overcome such difficulty, a feedback scheme (see linear rule in (9)) can be used to smooth the growth of the variances over the periods. For this purpose, an optimal linear gain G_k^* must be computed by a minimum variance problem, that is, (Silva Filho and Ventura, 1999):

$$\underset{G_k}{\mathrm{Min}} \{V_{x,k+1} + \lambda_k \cdot V_{uk}\} \tag{13}$$

As a result from (13), an optimal gain G_k^*=-$B^T \cdot [\lambda_k I + BB^T]^{-1}$ is provided. Note that the parameter λ_k denotes the tradeoff between the control and state variances whose evolutions are given by $V_{x,k+1} = (A + BG_k)V_{xk}(A + BG_k)^T + V_{dk}$ and $V_{uk} = G_k^2 \cdot V_{xk}$, respectively. This parameter can be computed by a search procedure that finds the minimum value of the function $\varphi(\lambda_k)=V_{x,k+1}+V_{uk}$, see Silva Filho and Ventura (1999) for details.

6. CASE STUDY

The idea here is to develop an optimal production plan for a single family of products. Thus, following the stochastic production problem described in (1), this production planning problem can be formulated by (14).

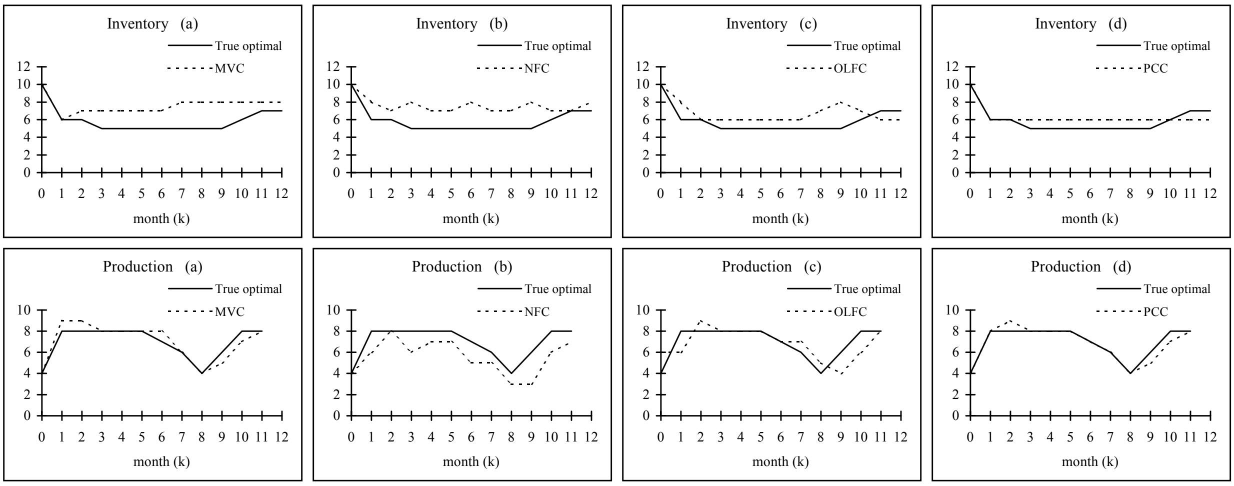

Figure 2. Comparing different trajectories

$$\begin{aligned} &\underset{u(k)}{\text{Min}}\ E\left\{h\cdot\sum_{k=0}^{12}{x_k}^2+c\cdot\sum_{k=0}^{11}{u_k}^2\right\} \\ &\text{s.t.}\quad x_{k+1}=x_k+u_k-d_k,\ x_0\ \text{given} \qquad (14) \\ &\qquad \Pr ob.(\underline{x}\le x_k\le\overline{x})\ge\alpha \\ &\qquad \Pr ob.(\underline{u}\le u_k\le\overline{u})\ge\gamma \end{aligned}$$

In practice, this plan usually allows managers to gain insights into the use of the company's material resources, therefore, helping him to anticipate managerial decisions that will improve the planning process as a whole. The main data of the problem are: (i) lower and upper boundaries of production capacities: $\underline{u}$ =2 and $\overline{u}=10$; (ii) lower and upper boundaries of storage: $\underline{x}$ = 5 and $\overline{x}=15$.

The production and inventory costs h=2 and c=1, respectively; and (iii) initial and final inventory levels are x_0=10 and x_N is free. The demand d_k, which is extracted from a historical sales report, is assumed Gaussian with: a) monthly mean $\hat{d}_k$ given by the sequence {8 8 9 8 8 8 7 6 4 5 7 8} from January to December, respectively; and b) the standard deviation given by $\sqrt{V_d}=1.42$. Lastly, it is assumed here that α=γ=½. This means that the manager assumes the risk that inventory and production constraints can be violated at any period k of the planning horizon N, see Silva Filho (1999).

The objective of this study is to perform a numerical analysis of the procedures discussed in the section 5, in order to consider them as alternatives of obtaining a feasible solution for (14). As a result, these sub-optimal solutions will be compared with the true optimal solution provided by stochastic dynamic programming (SDP) described in section 4.

6.1. True optimal solution

For the application of SDP algorithm, the first step is to define a space for the fluctuation of demand. The lower and upper boundaries of this space defined by: $\underline{d}_k=\hat{d}_k-2{,}58\sqrt{V_d}$ and $\overline{d}_k=\hat{d}_k+2{,}58\sqrt{V_d}$. Thus, the chances of the demand d_k occuring in this interval are close to 99% (Chou, 1972). Then, applying the algorithm given in (5) to the problem (14), it follows that the optimal cost of operation was \$ 1,560, and the true optimal trajectories (policies) of state and control are given in figure 2.

6.2. Sub-optimal procedures

Using the results of section 5, the suboptimal procedures were implemented via dynamic programming algorithm. With the exception of the MVC procedure, the others take into account the available information about the state of the system, that is, they use the level of inventory, observed at the beginning of each month, to improve the solution. In particular the PCC procedure deserves a little more attention due to require the computation of a linear optimal gain G^*. In such case, the first step was to compute the tradeoff parameter λ_k.Thus, using the results discussed in session 5.4, a search for $\lambda_k \in (0, +\infty)$ was employed in order to minimize: $\varphi(\lambda_k)=V_{x,k}+V_{u,k}$ subject to evolution of the variance equations given by: $V_{uk}=\left(1/(1+\lambda_k\right)^2V_{x,k}$ $V_{x,k+1}=\left[(2+\lambda_k)/(1+\lambda_k)\right]^2V_{xk}+V_{dk}$. Without loss of generality, it was assumed $\lambda_k^*=\lambda_N$ ∀k. The reason is that the maximum growth of variances is observed in the period k=N (this means a high risk of find a unfeasible solution to the problem (14)). As a result, the optimal value is λ^*=0.85, implying in $G^*(\lambda)$=0.46.

Analysis of Results: Table 1 allows comparison among these different approaches with relation to costs for problem solving. As was expected, the solution provided by SDP has the minimum cost while the open-loop solution provided by MVC has the maximum cost. It is worth mentioning that the values of the costs provided by the other procedures are higher and lower than the costs provided by SPD and MVC procedures. In fact, from table 1, it is possible to verify that: $J_{SDP}\le J_{PCC}\le J_{OLFC}\le J_{NFC}\le J_{MVC.}$ Thus, as a result, the best suboptimal solution is the one provided by the PCC procedure. The reason for this is that the PCC explicitly uses a feedback gain and, consequently its suboptimal solution is improved at each period k.

Table 1. Comparing the optimal costs

Procedure	SDP	PCC	OLFC	NFC	MVC
Costs	1560	1785	1812	1995	2830

This kind of solution can substitute the true optimal solution, providing a production plan and managerial insights about the use of material resources. The optimal inventory and production policies, for each one of these procedures are illustrated in figures 2(a)-(d). Note that they are compared individually with true optimal trajectories.

An interesting observation of these results is that the trajectories provided by the NFC and OLFC procedures (figure 2(b) e 2(c)) are more responsive to the fluctuation of demand than the PCC procedure (figure 2(d)) which uses the optimal gain G_k to smooth the production policy. Next, the use of the PCC procedure together with a simulation mechanism is investigated. The objective is to develop production scenarios that help managers gain insights into the use of the company resources.

6.3. Scenarios Analyses via PCC approach

The diagram in figure 3 describes the simulation scheme. The variables $\hat{x}(k)$ and $\hat{u}(k)$ denote respectively the mean optimal trajectories of inventory and production provided by the PCC procedure and illustrated in figure 2(d). These trajectories are used as goals during the simulation performance. The variables x(k) and u(k) will contain the current inventory and production levels which are provided by the simulation process per period k. The simulation was performed for a planning horizon of 24 months. Thus, the original trajectories for 12 months, see figure 2(d)), were duplicated for the other consecutive months, that is, k=13, 14, ..., 24. The objective was to provide a long-term vision for the production planning process. It was also considered that the fluctuation of demand was generated from a non-stationary seasonal forecasting model, described as follows: $d(k) = (6.0+0.25\cdot k)\cdot(1+\text{sine}(3.14\cdot k/6))+z(k)$, where z(k) is generated from a normal random process. Figure 4 illustrates the result of the simulation. It is worth mentioning that during the simulation process, the linear gain G_k is responsible for adjusting the production levels in order to guarantee that the inventory levels follow their optimal targets as closely as possible, i.e., x(k) close to $\hat{x}(k)$, k=1, ...,24.

7. CONCLUSION

In this paper, the solution of a general class of stochastic production planning problems via sub-optimal procedures was investigated.

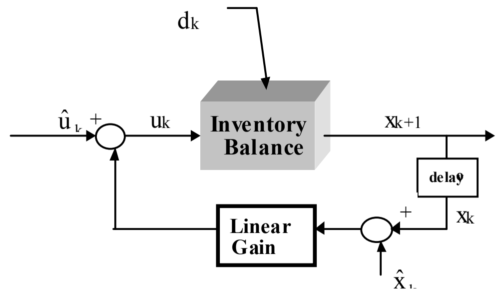

Fig. 3. Simulation scheme via PCC procedure

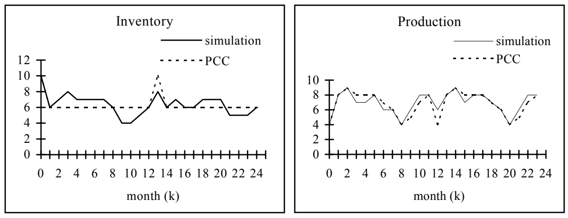

Fig. 4. Production scenarios with forecasting

Four different sub-optimal procedures were analyzed and compared with the true optimal solution provided by the application of stochastic dynamic programming. The PCC procedure had the best performance among them. The reason is that, in contrasting the other procedures, PCC uses a linear feedback gain to introduce (see (9)) some information about the current state of the system into the control policy. Another advantage of the PCC is that it can be used to simulate the production process, allowing managers to gain insights into the use of company's material resources.

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