15th Triennial World Congress, Barcelona, Spain

FEEDBACK LINEARIZATION IMPROVING AN LMI-BASED DESIGN: APPLICATION TO POWER SYSTEMS

Rodrigo Andrade Ramos, Newton Geraldo Bretas and Luís Fernando Costa Alberto

Electrical Engineering Department, Engineering School of São Carlos - University of São Paulo, Av. Trabalhador Sãocarlense, 400, São Carlos - SP - Brazil, 13566-590

Abstract: This paper presents a new control scheme for the design of damping controllers for power systems. The scheme is based on the combination of two powerful control design techniques, namely the Linear Matrix Inequalities (LMI) techniques and the Direct Feedback Linearization (DFL). It is shown that the DFL can cancel nonlinearities in the power system model, thus reducing the uncertainty in the LMI design. The obtained results show the proposed controller provides smooth control actions and fast transient response, being well-suited for the low frequency oscillations problem.

Keywords: Robust Control, Feedback Linearization, Power System Control, Power System Stabilizers, Control Applications, Nonlinear Models

1. INTRODUCTION

Low frequency oscillations are a problem of major concern in power systems, as they place limits on power transfers, restricting the full utilization of transmission capacity. This problem has been investigated since the late 60's (see (DeMello and Concordia, 1969) and (Larsen and Swann, 1981)). To provide damping for these oscillations, decentralized lead-lag compensators, called Classical Power System Stabilizers (CPSS), have been the most used control structures. However, the design of these compensators has always been based upon the linearization of the power system model around a typical operating point.

For economical and environmental reasons, power systems are forced to operate near their limits nowadays. Therefore, they are exhibiting a stronger nonlinear behaviour and the classical compensators are failing to provide damping to the oscillations, as their designs were based on linearized techniques.

[1] This research is finantially supported by FAPESP, a brazilian research foundation.

Robust controllers have been recently proposed to deal with this nonlinear behaviour. (Boukarim, Wang, Chow, Taranto and Martins, 2000) and (Fischman, Bazanella, Silva, Dion and Dugard, 1997) are examples of such proposals. In particular, a powerful approach called Linear Matrix Inequalities (LMI) has received much attention by the designers over the past years. The LMI formulation allows the designer to represent a control problem in the form of an optimization, subjected to matrix restrictions, and then solve it numerically with fast algorithms called LMI solvers.

Various kinds of system models, such as Linear Time Invariant (LTI) systems or Linear Differential Inclusions (LDIs), can be employed in an LMI design. Among them, the Polytopic Linear Differential Inclusion (PLDI) model is particularly well-suited to robust control design problems in power systems. In this representation, different points of operation constitute the vertices of a convex region (called polytope) where the system is expected to operate.

One of the problems of the LMI-based approaches is the inherent overdesign associated to these kinds of optimization. The LMI design seeks for a controller to stabilize the entire operating region. This often leads to an overdesign, generating controllers with high feedback gains, which may not be feasible due to physical restrictions.

In this paper, a partial feedback linearization scheme is proposed to cancel strong nonlinearities in a power system model with network reduction. The system is not completely linearized, but the remaining nonlinearities are treated in an LMI framework to allow for a linear control design. The main advantage of this scheme (over the conventional LMI designs) is the reduction of the overdesign due to the elimination of nonlinearities in the model.

The partial linearization scheme is based on the measurement of the terminal bus voltage, so the linearizing feedback can be precisely implemented. Also, an H_2 performance criterion is included in the design, to generate a controller able to reject the cancellation error. No remote information is necessary for the linearization, which is another advantage of this control scheme.

This paper is organized as follows: section 2 presents the advantages and drawbacks of the polytopic model for power systems and section 3 explains how feedback linearization can be employed to reduce the uncertainties and improve the previously presented design; section 4 briefly presents the control design procedure employed in this research, while section 5 shows the obtained results, comparing the proposed approach to other conventional types of design.

2. THE POLYTOPIC MODEL FOR POWER SYSTEMS

A multimachine power system can be described, after network reduction, by the following set of equations:

$$\dot{\delta}_i = \omega_i \tag{1}$$

$$\dot{\omega}_i = \frac{1}{2H_i}[P_{mi} - (\sum_{k=1}^{n} E'_{qi}E'_{qk}(G_{ik}\cos(\delta_k-\delta_i) - B_{ik}\sin(\delta_k-\delta_i))) - D_i\omega_i] \tag{2}$$

$$\dot{E}'_{qi} = \frac{1}{\tau'_{doi}}[E_{fdi} - E'_{qi} + (\sum_{k=1}^{n} E'_{qk}(G_{ik}\sin(\delta_k-\delta_i) + B_{ik}\cos(\delta_k-\delta_i)))(x_{di}-x'_{di})] \tag{3}$$

$$\dot{E}_{fdi} = \frac{1}{T_{ei}}[K_{ei}V_{refi} + K_{ei}V_{ci} - K_{ei}V_{ti} - E_{fdi}] \tag{4}$$

where

$$\begin{aligned} V_{ti} = [&{E'}^2_{qi} + 2E'_{qi}x'_{di}\sum_{k=1}^{n} E'_{qk}(B_{ik}\cos(\delta_k-\delta_i) \\ &+G_{ik}\sin(\delta_k-\delta_i)) \\ &+{x'}^2_{di}\sum_{k=1}^{n}\sum_{l=1}^{n} E'_{qk}E'_{ql}(G_{ik}G_{il}\cos(\delta_k-\delta_l) \\ &+2G_{ik}B_{il}\sin(\delta_k-\delta_i) \\ &+B_{ik}B_{il}\cos(\delta_k-\delta_l))]^{1/2} \end{aligned} \tag{5}$$

In equations (1)-(5), δ_i is the rotor angle, ω_i is the angular rotor speed, E'_{qi} is the internal voltage, E_{fdi} is the field voltage, V_{ti} is the terminal bus voltage and V_{ci} is the control input to the voltage regulator, all referred to generator i. The reader should consult (Anderson and Fouad, 1994) for the description of the parameters in this model. It can be seen from (4) and (5) that the term V_{ti} introduces a very strong nonlinearity in the model.

After linearization around an operating point, the model can be put in the form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_p\mathbf{x}(t) + \mathbf{B}_p\mathbf{u}(t) \tag{6}$$

$$\mathbf{y}(t) = \mathbf{C}_p\mathbf{x}(t) \tag{7}$$

The classical techniques used in the design of damping controllers for power systems are based on this linearized model. These controllers are not robust to parameter variation and/or uncertainties in the operating conditions, because the model is valid only in a neighborhood of the operating point.

The polytopic model is an alternative to overcome this drawback of the classical techniques. In this model, the system is linearized at a number of different typical operating points, producing a series of matrices

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{pi} & \mathbf{B}_{pi} \\ \mathbf{C}_{pi} & \mathbf{0} \end{bmatrix} \tag{8}$$

for $i = 1, \ldots, L$. These matrices are called vertex systems. Then, a convex set $\Omega \in \mathbb{R}^{n\times n}$ is formed by taking the convex hull of the vertex systems. This can be expressed by

$$\Omega = \mathbf{Co}\left\{\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_L\right\} \tag{9}$$

where

$$\mathbf{Co}\,\mathbb{S} \triangleq \left\{\sum_{\mathbf{i}=1}^{\mathbf{n}+1}\lambda_{\mathbf{i}}\mathbf{v}_i \mid \mathbf{v}_i \in \mathbb{S},\ \sum_{\mathbf{i}=1}^{\mathbf{n}+1}\lambda_{\mathbf{i}} = \mathbf{1}\right\} \tag{10}$$

A model in the form

$$\dot{\mathbf{x}}(t) \in \Omega\mathbf{x}(t)\ ,\ \mathbf{x}(0) = \mathbf{x}_0 \tag{11}$$

with Ω described by (9) is a PLDI. Any $\mathbf{x} : \mathbb{R}_+ \to \mathbb{R}^{\mathbf{n}}$ satisfying (11) is called a trajectory of this PLDI.

Now, let

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \quad (12)$$

be a condensed form of equations (1)-(4) and suppose that, for every $\mathbf{x}(t)$, $\mathbf{u}(t)$ and t, there exists a matrix $\mathbf{G}(\mathbf{x}(t), \mathbf{u}(t), t) \in \Omega$ such that

$$\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) = \mathbf{G}(\mathbf{x}(t), \mathbf{u}(t), t)\mathbf{x}(t) \quad (13)$$

Then it follows immediately that every trajectory of the nonlinear system (12) is also a trajectory of the PLDI (11) defined by Ω. The conditions for the existence of such $\mathbf{G}(\mathbf{x}(t), \mathbf{u}(t), t)$ are presented in (Boyd, El Gahoui, Feron and Balakrishnan, 1994).

The convexity of the Ω set can be exploited in some interesting ways. For example, when certain properties (such as quadratic stability or decay rates) are proved for all vertices of Ω, they extend to all matrices within the polytope.

A number of control problems with PLDI models can be expressed in the form of LMI optimizations and numerically solved with high efficiency (Gahinet, Nemirovski, Laub and Chiali, 1995). Also, the PLDI model is very well-suited for the power system case, because the vertex systems can represent the various typical operating conditions throughout the day.

However, the fact that the design seeks for a controller to stabilize the entire polytope often leads to an overdesign, because some regions of the polytope might be modeling very unusual, or even impossible, operating conditions. This is a major disadvantage for the power system control problem, since controllers with high feedback gains are difficult to implement in power systems, due to physical reasons.

The next section presents an alternative to improve these LMI designs. The main idea is the reduction of the uncertainty in the model (consequently reducing the polytope) using partial feedback linearization, which leads to a less conservative design and yields controllers more suited for implementation.

3. FEEDBACK LINEARIZATION FOR UNCERTAINTY REDUCTION

Nonlinear control techniques have also been proposed to damp low frequency oscillations in power systems. The Direct Feedback Linearization (DFL) is one of these techniques. Roughly speaking, feedback linearization uses state feedback to cancel nonlinearities in the model, allowing the application of a linear control design to a nonlinear model. DFL is a kind of feedback linearization, which uses the original variables of the model, without a coordinate transformation (Wang, Guo and Hill, 1997).

The use of DFL to cancel nonlinearities in the power system model has been reported in (Zhu, Zhou and Wang, 1997), (Guo, Wang and Hill, 2000) and (Guo, Hill and Wang, 2000). For the implementation of these proposals, the Automatic Voltage Regulator (AVR) structure, described by equation (4) has to be changed. AVRs are integrating parts of most generating units in operation nowadays, and changing their structures in all units of a large power system might be a problem.

As mentioned previously, equation (4) introduces a strong nonlinearity in the power system nonlinear model, due to the presence of the term V_{ti}. However, if V_{ti}^* is a terminal bus voltage feedback and

$$V_{ci} = u_{ci} + V_{ti}^* \quad (14)$$

where u_{ci} is a control input, the term V_{ti} is canceled in (4), and the remaining equation is

$$\dot{E}_{fdi} = \frac{1}{T_{ei}}[K_{ei}(V_{refi} + u_{ci} + e_i) - E_{fdi}] \quad (15)$$

where

$$e_i = V_{ti}^* - V_{ti} \quad (16)$$

is the cancellation error. With this simple modification (which is done before the linearization process), a strong nonlinearity is eliminated from the power system model. The new equation (15) of the AVR is linear, with e_i viewed as a disturbance. Since the nonlinearities are converted to uncertainties in the polytopic model, this modification reduces the Ω subset (by eliminating the nonlinear term), thus reducing the overdesign in the controller.

Now, in addition to stabilizing the plant, the objective of the controller is also the rejection of the cancellation error e_i. This can be achieved by introducing an H_2 criterion to be accomplished in the design.

Implementation of this linearization scheme needs only a local measurement of the terminal voltage V_{ti}, which is already available to the AVR. The cancellation can be made very precisely, and the AVR structure is not changed.

4. CONTROLLER DESIGN

A benchmark two-area system from (Kundur, 1994) was chosen as the base system for the design and the tests of the controller proposed in this paper. This system has a strong inter-area mode of oscillation, which varies significantly with the change in operating conditions. Figure 1 presents this system, and the complete data related to it can be obtained from (Kundur, 1994).

In power systems, normal changes of the operating conditions are due to load variations throughout the day or the week. For this reason, the vertex systems were determined combining extreme variations of the load levels in both areas of the system from figure 1. The load levels of the base case are P_{L1} = 967 MW, Q_{L1} = 100 MVAr, P_{L2} = 1767 MW and Q_{L2} = 100

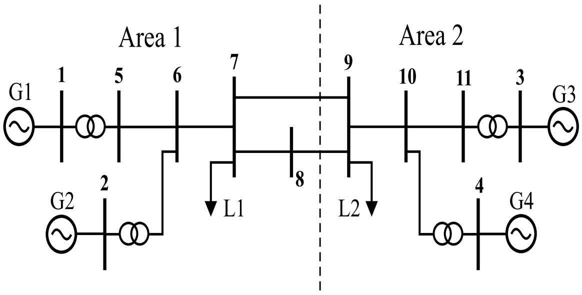

Fig. 1. Benchmark two-area system used for the design.

MVAr, and this condition is also taken as a vertex system.

Table 1 shows the eigenvalues related to the inter-area mode for the vertex systems. As shown in this table, the main factor affecting the stability of this system is the power flow in the tie lines (P_{tie}). Heavier power flows lead to a more unstable system.

Table 1. Vertex systems.

Load Level	P_{tie} [MW]	Mode	Freq. [Hz]	Damping Factor
Base Case	391	0.0051 ±j2.0405	0.3248	-0.0025
L1:+20% L2:-20%	205	-0.0026 ±j2.3228	0.3697	0.0011
L1:-20% L2:-20%	570	0.0302 ±j1.5579	0.2479	-0.0194
L1:-20% L2:+20%	564	0.0145 ±j1.2244	0.1949	-0.0118
L1:+20% L2:+20%	204	-0.0227 ±j2.2555	0.3590	0.0101

After the construction of the polytopic model, a mixed H_2/H_∞ design with regional pole placement constraints was carried out. Figure 2 summarizes this type of design, where $\mathbf{P}(s)$ is the polytopic transfer function matrix of the plant and $\mathbf{K}$ is the controller. In the present design, $\mathbf{x}$ is the state vector, $\mathbf{u}$ is a vector of inputs of the AVRs, $\mathbf{e}$ is a vector of cancellation errors and $\mathbf{z}_2$ is a vector with angular speeds of all generators. The output $\mathbf{z}_\infty$ was not considered in this design, since the main concern was the rejection of the cancellation error, which was modeled as a weighted white noise.

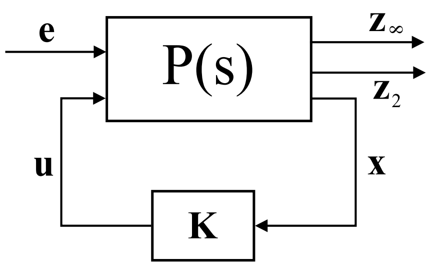

Fig. 2. Mixed H_2/H_∞ design.

The objective of the design was to maintain the H_2 norm of the transfer function from $\mathbf{e}$ to $\mathbf{z}_2$ below 0.5, while placing the poles of the polytopic closed loop system inside the region of damping factors higher than 25% ($\xi > 0.250$).

A similar design was done for a smaller polytope, considering variations of ±10% of the load levels in both areas. Also, conventional LMI designs, based on the same polytopic models (±10% and ±20%) without employing DFL, were carried out. The next section presents a comparison of results between these two types of control design.

5. TESTS AND RESULTS

As expected, controllers with smaller feedback gains were obtained with the DFL/LMI designs, comparing to the controllers provided by the conventional LMI designs (CLMI). A comparison between the DFL/LMI controller, designed for the ±20% polytope (DLMI20), and the conventional LMI controller, designed for the ±10% polytope (CLMI10), revealed a difference in the magnitude of the feedback gains ranging in the 10^3 order. The feedback gains obtained for the conventional LMI controller, designed for the ±20% polytope (CLMI20), were extremely high. For this reason, the comparisons presented in this section are made between the DLMI20 and CLMI10 controllers.

Nonlinear simulations were carried out to test the robustness and performance of the designed controllers. The exciter limiters were included in the simulation models. The initial operating conditions used in the various simulations correspond to system models lying inside the uncertainty polytope. For all the simulations shown in this section, a solid three-phase short circuit of 150 ms at bus 8 was used to stimulate the modes of oscillation.

Figure 3 shows the rotor speed responses of generators 1, 2 and 4 (without damping controllers) to the perturbation mentioned above, when the system is operating at the load levels of the base case. As expected, the base case of the open loop system is unstable.

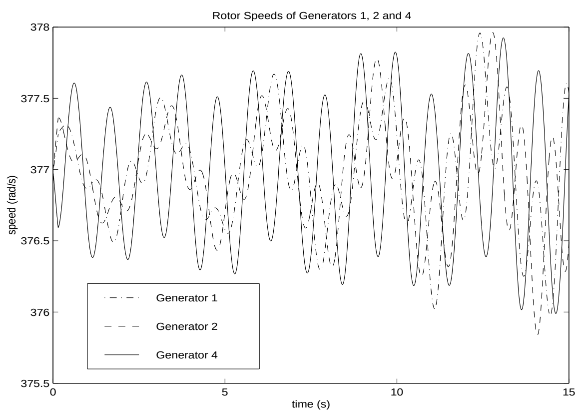

Fig. 3. Response of the open loop system.

Three different controllers are compared in figure 4: CPSS, CLMI10 and DLMI20. The rotor speed of generator 4 is the variable for comparison, and the system is operating at the base case load levels. It can be seen that the LMI designs achieve better damping than the CPSS, because they allow for regional pole placement. Also, the performances of the DLMI20 and CLMI10 controllers are similar at this base case.

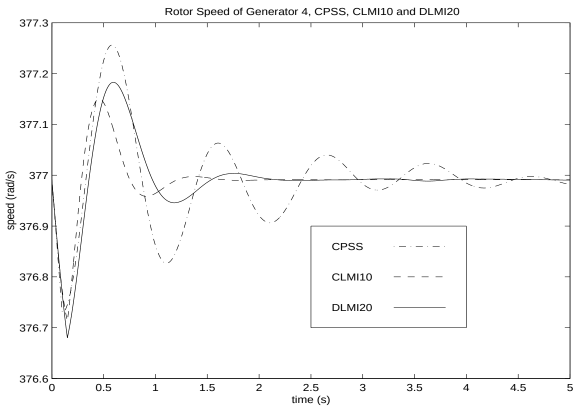

Fig. 4. Comparison among CPSS, CLMI10 and DLMI20, base case system.

The robustness of the DLMI20 controller is demonstrated by the simulations shown in figure 5. The conditions of operation for this simulation are inside the range of variation covered by the ±20% polytope. The load levels are P_{L1} = 822 MW, Q_{L1} = 85 MVAr, P_{L2} = 2032 MW and Q_{L2} = 115 MVAr, corresponding to a tie line power flow of P_{tie} = 524 MW. Note that the DLMI20 controller maintains its performance characteristics. In contrast, the response of the system with the CLMI10 is slightly more oscillatory, because the operating condition is outside the region of the ±10% polytope. The variable shown is the rotor speed of generator 2.

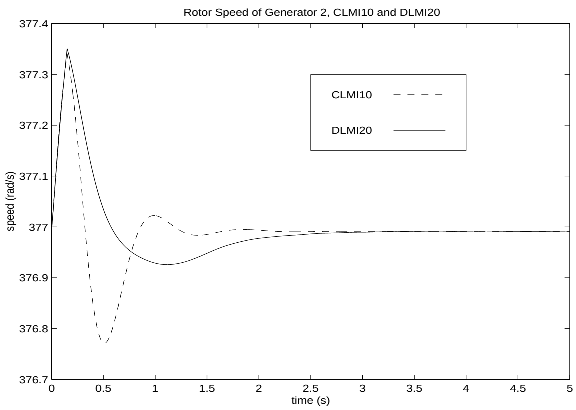

Fig. 5. Comparison between DLMI20 and CLMI10, P_{tie} = 524 MW.

The same load levels of the previous simulation (corresponding to P_{tie} = 524 MW) were used to generate the next figures. A number of other operating points were tested, but this case was chosen for the next comparisons due to the high power flow through the tie lines, which makes the open loop system strongly unstable.

Figure 6 shows the effect of the smaller feedback gains of the DLMI20 controller. From the plots of the terminal bus voltages of generator 2, it can be seen that the CLMI10 controller provides stronger control actions than those provided by the DLMI20 controller.

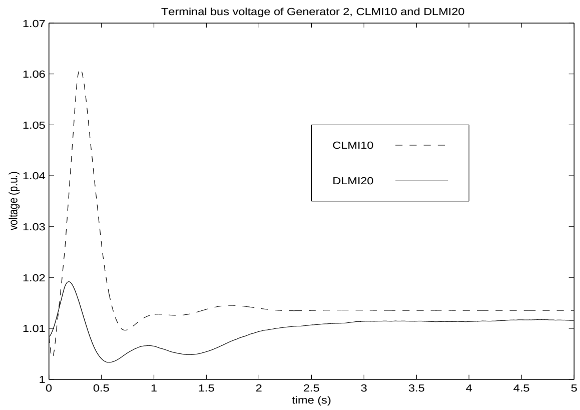

Fig. 6. Control actions of DLMI20 and CLMI10, P_{tie} = 524 MW.

The difference in the feedback gains becomes even more evident in figure 7. This figure shows the AVR outputs of generator 2. The actual input to the field circuit (E_{fd}), in the case of the CLMI controller, corresponds to the dashed curve limited by the solid line, due to the exciter limiters. The higher gains of the CLMI10 controller saturate the AVR, while the DLMI20 controller overcomes this problem.

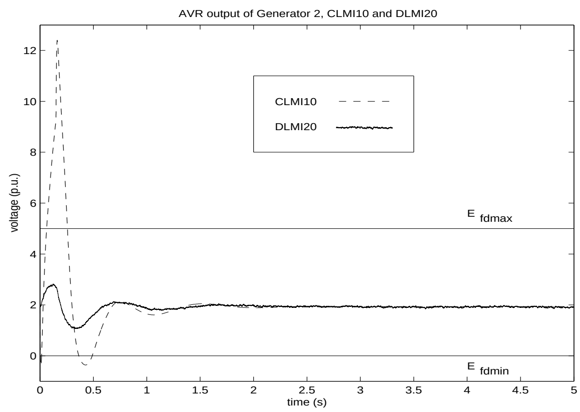

Fig. 7. Saturation of the AVR and cancellation error rejection.

In addition, the effect of the cancellation error, modeled as a weighted white noise in the AVR input, is seen in figure 7. From figures 5, 6 and 7, it is clear that the DLMI20 controller is able to reject this cancellation error.

Finally, the coordination of the DLMI20 controller is demosntrated in figure 8, where the rotor angle responses for generators 1, 2 and 4 are shown. It can be seen that the strongly coupled generators 1 and 2 are damped coherently.

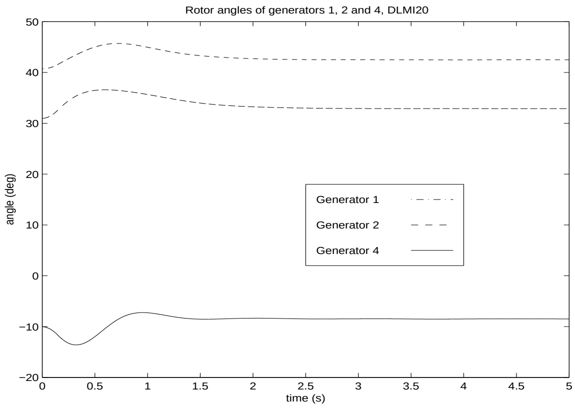

Fig. 8. Coordination of the DLMI20 controller.

6. CONCLUSIONS

This paper presents an effective way of improving an LMI-based design of damping controllers for power systems. With the cancellation of nonlinearities and a corresponding uncertainty reduction in the model, the design procedure yields a less conservative controller, compared to a conventional LMI design based on a polytopic representation of the system.

The proposed combination of the DFL technique with an H_2/H_∞ design (with regional pole placement) makes the controller able to deal with the cancellation error inherent to all kinds of feedback linearization schemes. Also, this DFL can be implemented with local measurements and does not change the AVR structure.

Comparisons of a controller designed by the proposed procedure to both classical and conventional LMI controllers demonstrated the effectiveness of the DFL/LMI combination presented. The DFL/LMI controller retained the robustness and coordination characteristics of the LMI designs, but its smaller feedback gains provided smoother control actions, making this controller more suited to the power system damping problem.

7. REFERENCES

Anderson, P. M. and Fouad, A. A. (1994). *Power System Control and Stability*, IEEE Press.

Boukarim, G. E., Wang, S., Chow, J., Taranto, G. N. and Martins, N. (2000). A comparison of classical, robust, and decentralized control designs for multiple power system stabilizers, *IEEE Transactions on Power Systems* **15**(4): 1287–1292.

Boyd, S., El Gahoui, L., Feron, E. and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia.

DeMello, F. P. and Concordia, C. (1969). Concepts of synchronous machine stability as affected by excitation control, *IEEE Transactions on Power Apparatus and Systems* **PAS-88**(4): 316–329.

Fischman, A., Bazanella, A. S., Silva, A. S., Dion, J. M. and Dugard, L. (1997). Application of linear matrix inequalities techniques to the design of robust coordinated controllers for power systems, *Proceedings of the 3rd IFAC Conference on System Structure and Control*, Bucareste, Romania.

Gahinet, P., Nemirovski, A., Laub, A. J. and Chiali, M. (1995). *LMI Control Toolbox User's Guide*, The MATHWORKS Inc.

Guo, G., Wang, Y. and Hill, D. J. (2000). Nonlinear output stabilization control for multimachine power systems, *IEEE Transactions on Circuits and Systems, Part I* **47**(1): 46–53.

Guo, Y., Hill, D. J. and Wang, Y. (2000). Nonlinear decentralized control of large-scale power systems, *Automatica* **36**: 1275–1289.

Kundur, P. (1994). *Power System Stability and Control*, McGraw-Hill.

Larsen, E. V. and Swann, D. A. (1981). Applying power system stabilizers, parts i, ii and iii, *IEEE Transactions on Power Apparatus and Systems* **PAS-100**(6): 3017–3046.

Wang, Y., Guo, G. and Hill, D. J. (1997). Robust decentralized nonlinear controller design for multimachine power systems, *Automatica* **33**: 1725–1733.

Zhu, C., Zhou, R. and Wang, Y. (1997). A new nonlinear voltage controller for power systems, *Electrical Power and Energy Systems* **19**(1): 19–27.