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# ACTIVE CONTROL ACCOMMODATION OF PLANTS WITH LARGE PARAMETRIC UNCERTAINTIES

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Abstract: In this paper, a strategy of an active control accommodation is proposed to deal with the regulation of SISO plants with large parametric uncertainties. The model of the plant is partitioned into a family of a finite number of LTI models. Then, the associated controllers are designed such that they achieve the same performances for a given control objective. The selection of the adequate controller is based on an online detection algorithm of these models. The stability and bumpless transfer issues linked to the switching of controllers are discussed. The results of a real-time experimentation of this strategy on a laboratory thermal process demonstrate its effectiveness. *Copyright © 2002 IFAC*

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## 1. INTRODUCTION

The mathematical models of plants are just an approximation of what should be their behavior under some specific operating conditions. They do not approximate the plant accurately when the operating conditions change strongly. For models with large parametric uncertainties resulting from these changes, some examples have shown that it does not exist a linear controller with constant parameters which can perform the control task in a satisfactory manner (Morse, 1996). The control law, in this case, must be able to insure satisfactory performances even though the plant model is subject to these uncertainties.

In this paper, an active control accommodation strategy which takes into account these large parametric uncertainties is proposed. It is based on the design of an additional detection loop and a switching control technique. The concept of this accommodation based on a switching control strategy is introduced in section 2. The controller design and the stability analysis relating to the controllers switching are presented and discussed in section 3. The section 4 deals with the experimental results of a real-time implementation of this strategy applied to a laboratory thermal process.

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## 2. ACTIVE CONTROL ACCOMMODATION BY SWITCHING CONTROL

In our approach, a supervised control structure integrating an operating mode detection and active accommodation loop is designed. The active control accommodation is based on an indirect switching control since it depends on the detection of the actual process model. The operating mode detection and accommodation (OMDA) structure is depicted in figure (1). A multi-controller struc-

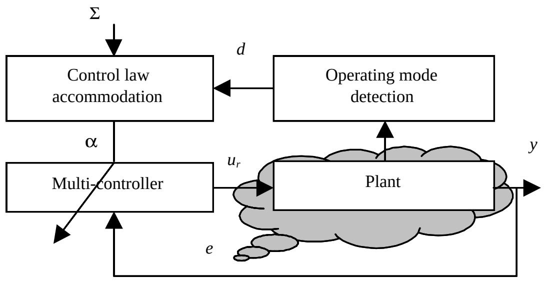


Fig. 1. OMDA structure of a feedback system.

ture is proposed to elaborate a control, denoted in the following by $u_r$, for a process with large parametric uncertainties that are denoted by $e$. The supervision setpoint $\Sigma$ describes the control objectives (e.g., setpoint control change, regulation objectives change) to be achieved. The accommodation signal $\alpha$ selects the controller corresponding to the detected model by the detection signal $d$.

Let us denote the plant model as $G$ and assume that its parameters are piecewise constant and belong to a bounded compact domain $D$. To handle the control task by taking into account these uncertainties, one should partition the domain $D$ into a finite number of compact sub-domains $D_i$, $i \in I$ and $I = \{1, 2, ..., g\}$, such that $D = \bigcup\limits_{i \in I} D_i$ and design a controller $C_i$ for each nominal model $G_i$ corresponding to the center of the sub-domain $D_i$. This sub-domain can be taught as the small parametric uncertainties (e.g., measurement noise, modelling error...) around the nominal model $G_i$. From the families of the process models and the controllers, it can be defined an Operating Mode (OM) matrix which relates each controller $C_i$ to each process model $G_j$, given by:

$$M = \{M_{i,j} = (C_i, G_j) \mid (i, j) \in I^2\} \qquad (1)$$

Each element in the diagonal of $M$ achieves the desired control objectives and performances. To determine which controller must be used in the $k$th sampling period, the proposed detection method proceeds in three steps: the online simulation of the models $G_j$; the evaluation of the residual $\varepsilon_j = y(k) - y_j(k)$, associated to the $j$th model, where $y_j(k)$ is the output of the $j$th model; and the model isolation which is based on the search of the least of the $J_j(k)$ given by:

$$J_j(k) = J_j(k-1) + \frac{1}{N-1}\,(\varepsilon_j^2(k) - \varepsilon_j^2(k-N)) \qquad (2)$$

where $N$ is the size of the sliding window.

Following these guidelines, a pair $(d,\ t_d)$ is defined as the detection test which describes each detected model $d$ and the detection time $t_d(k)$. The detection rule is computed online by:

$$d(k) = \{P = G_m,\quad m = \arg\min_{1 \le j \le g} J_j(k)\} \qquad (3)$$

This rule decides that the process is operating in the $m$th model $G_m$. At each sampling period $T_d$ a minimization search is carried out. The selected controller corresponds to the argument of the smallest $J_j$. The detection time $t_d$ is the instant $kT_d$ where $k$ is the sample when the detection changes, i.e., $d(k) \neq d(k-1)$ and $T_d$ is the detection-accommodation loop sampling period. $t_d$ is expressed by $t_d = \{kT_d,\ d(k) \neq d(k-1)\}$.

The control input $u_r$ is evaluated at each instant $t = k'T_c$, where $T_c$ is the control sampling period. The detection-accommodation loop sampling period is chosen such that $T_d = \frac{1}{l}T_c$, $l \in \mathbb{N}^*$ such that the accommodation function anticipates the control input in order to minimize the detection time delay with respect to the control sampling period $T_c$. For a correct signal to noise ratio a good choice of $l$ and $T_d$ allows a good tuning of the multi-model based detector.

The objective of supervision is to ensure a safe behavior of the system. When a fault occurs during the process operation, there would be no model $G_j$ approximating the model of the process and hence no one of the controllers would achieve the control task in a satisfactory manner. This problem is handled by adding a vector of performance levels $\Pi = [\pi_1, \pi_2, ..., \pi_g]$, $\Pi \in \mathbb{R}^g$, where $\pi_j$ is the performance level associated to the process model $G_j$, to the supervision set point. Then, the accommodation signal $\alpha$ is a piecewise continuous switching function which represents the series of the successive activated controllers. It selects the $j$th controller corresponding to the detected model $G_j$ whose criterion $J_j$ is smaller than the performance threshold $\pi_j$. It is expressed by:

$$\alpha(k) = \{j,\ [d(k) = G_j] \wedge [J_j < \pi_j]\} \qquad (4)$$

If the condition $J_j < \pi_j$ is not satisfied, an emergency shutdown procedure is activated.

## 3. MULTI-CONTROLLER DESIGN AND STABILITY ANALYSIS

### 3.1 *Multi-controller Structure*

Let us denote the process transfer function, associated to the sub-domain $D_j$, by $G_j(q)$:

$$G_j(q) = \frac{B_j(q)}{A_j(q)} \qquad (5)$$

for which a control $u_j$ is designed. Here, $q$ stands for the shift forward operator. The corresponding control input $u_j$ and the process output $y$ are related by:

$$y(k')\ A_j(q) = u_j(k')\ B_j(q), \qquad (6)$$

and each control is designed with respect to each operating mode, by the method of pole placement presented in (Aström and Wittenmark, 1997). Namely, the $j$th control corresponding to the $j$th model is given by the following relation:

$$R_j(q)\ u_j(k') = T_j(q)\ y_{ref}(k') - S_j(q)\ y(k') \qquad (7)$$

where $R_j(q)$, $S_j(q)$ and $T_j(q)$ are polynomials to be determined as indicated in (Aström and Wittenmark, 1997). This leads to the multi-controller structure shown on figure (2). The polynomial

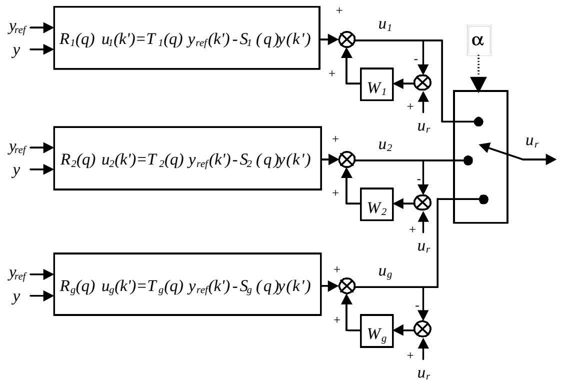


Fig. 2. Multi-controller structure.

$R_j(q)$ can be chosen as monic and its degree must be greater than or equal to those of $S_j(q)$ and $T_j(q)$, so that to design a realizable controller. The design of these polynomials depends on the desired closed-loop specifications given in terms of the characteristic polynomial. By eliminating $u_j(k')$ in the relations (6) and (7), it follows:

$$y_{ref}(k')B_j(q)\ T_j(q) = y(k')(A_j(q)\ R_j(q) + B_j(q)\ S_j(q)) \qquad (8)$$

The closed-loop characteristic polynomial is:

$$P_{cl}(q) = A_j(q)\ R_j(q) + B_j(q)\ S_j(q) \qquad (9)$$

The problem of pole placement consists in determining the polynomials $R_j(q)$ and $S_j(q)$ satisfying the Diophantine equation (9) for given polynomials $A_j(q)$, $B_j(q)$ and $P_{cl}(q)$. Then, the closed-loop transfer function, relating the reference input $y_{ref}(k')$ to the process output $y(k')$, is:

$$G_{j,j}(q) = \frac{B_j(q)\ T_j(q)}{A_j(q)\ R_j(q) + B_j(q)\ S_j(q)} \qquad (10)$$

The choice of $T_j(z)$ is done such that to simplify the auxiliary poles of the characteristic polynomial. In figure 2, $W_i$ is a bumpless transfer compensator which will be introduced in section 5.

### 3.2 *Stability Analysis*

The design of a given controller $C_j$, for a model $G_j$, does not insure the same performances for the remaining models $G_{i \neq j}$, $i = \{1, ..., g\}$. Since the detection time delay is never equal to zero, the stability of the feedback loop, in the presence of an operating mode which is not one of the diagonal elements of the matrix $M$, must be insured. The stability analysis of the controllers switching proceeds in two steps (Liberzon and Morse, 1999): the first one consists in checking the stability of each subsystem, i.e., each controller must asymptotically stabilizes each process model; the second one consists in analyzing the stability of the overall system for arbitrary switching signals. The first step is easily checked by computing all the closed-loop poles of the operating mode transfer functions $G_{i,j}(q)$, $(i,\ j) \in I^2$ and $i \neq j$, located out of the diagonal of the matrix $M$, where:

$$G_{i,j}(q) = \frac{B_j(q)\ T_i(q)}{A_j(q)\ R_i(q) + B_j(q)\ S_i(q)} \qquad (11)$$

If all the roots of the characteristic polynomials are within the unit circle, then, the first condition is satisfied. For the second condition, the state space representation relates the process output $y(k')$ to the reference input $y_{ref}(k')$ by:

$$\begin{cases} x(k'+1) = A_{cl}(k')x(k') + B_{cl}(k')y_{ref}(k') \\ y(k') = C_{cl}(k')x(k') \end{cases} \qquad (12)$$

This system is called a differential inclusion (ElGhaoui and Niculescu, 2000). ($A_{cl}(k')$, $B_{cl}(k')$, $C_{cl}(k')$) is one of the vertices of the polyhedral formed by the elements belonging to the set $\{(A_{i,j},\ B_{i,j},\ C_{i,j}),\ (i,j) \in I^2\}$. ($A_{i,j}$, $B_{i,j}$, $C_{i,j}$) represents the state space realization of the closed-loop transfer function given by $G_{i,j}(q)$. The matrices of system (12) are expressed as:

$$\begin{cases} A_{cl}(k') = \sum_{i=1}^{g}\sum_{j=1}^{g} \lambda_{i,j}(\sigma(k'))A_{i,j} \\ B_{cl}(k') = \sum_{i=1}^{g}\sum_{j=1}^{g} \lambda_{i,j}(\sigma(k'))B_{i,j} \\ C_{cl}(k') = \sum_{i=1}^{g}\sum_{j=1}^{g} \lambda_{i,j}(\sigma(k'))C_{i,j} \end{cases} \qquad (13)$$

where $\sigma$ is the current value, at the instant $t = k'T_c$, of the switching function describing all the possible switching sequences of the accommodation signal $\alpha$, $\lambda_{i,j}(\sigma(k')) \in \{0, 1\}$ such that $\sum_{i=1}^{g}\sum_{j=1}^{g} \lambda_{i,j}(\sigma(k')) = 1$.

In the literature, such system is also called a hybrid system and its behavior can be modeled by a hybrid automaton (Branicky, 1995). The methods that allow its study of stability have been reviewed in (Liberzon and Morse, 1999). They are based on the computation of candidate Lyapunov functions using the LMIs formulation like in (Pettersson, 1999). In (Charbonnaud *et al.*, 2001), the stability test did not take into account the structure of switching and resulted in a conservative set of LMIs where every possible switch has been considered. Let us consider an example of a switching system whose operating mode matrix is of dimension 3, i.e., $g = 3$. To satisfy the second condition, a common Lyapunov function $V(x) = x^T Rx$, $R \in \mathbb{R}^{n\times n}$, $R = R^T$, $R > 0$ and a matrix $Q \in \mathbb{R}^{n\times n}$, $Q = Q^T$, $Q > 0$, must be found such that:

$$A^T_{i,j} RA_{i,j} - R \leqslant -Q, \ \forall (i,j) \in I^2 \qquad (14)$$

This is a system of nine linear matrix inequalities (LMIs), which takes into account all the possible switches. It is tractable and can be resolved by the interior-point algorithm of Nesterov and Nemirovski (ElGhaoui and Niculescu, 2000). It was suggested in (He and Lemmon, 1998) that the switching structure does account in the stability analysis of switching systems. Therein, the stability test is based on the determination of Lyapunov-like functions associated to the fundamental cycles of the hybrid automaton describing it. Our problem is not tractable by this method since, there is no specification on the guard sets. However, the idea of the switching structure is very interesting in the sense that the set of LMIs can be reduced in a less conservative set. Let us assume that the behavior of the considered switching system is modeled by the hybrid automaton depicted in figure 3.

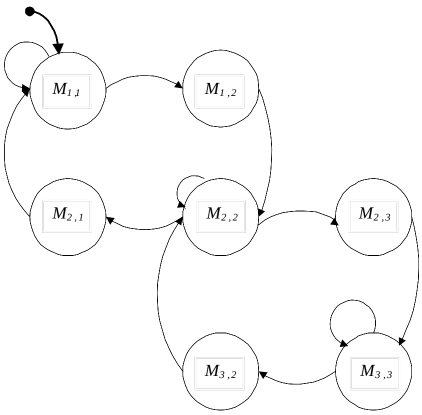


Fig. 3. The hybrid automaton describing the attainable states.

The behavior of this automaton is roughly as follows. The initial state is $M_{1,1}$, i.e., without loss of generality we assume the process to be in the first OM at the starting time. Whenever there exists a model $G_i$ whose corresponding criterion is the smallest among the $J_i$ and satisfies the performance level $\pi_i$, at a time $k$, the state of the system switches to $M_{i,i}$, otherwise, if it does not satisfy the performance level $\pi_i$, the state of the system switches to the stop state which is not shown in this automaton for the sake of space. In this example, the process can not jump from $M_{1,1}$ to $M_{1,3}$ or from $M_{3,3}$ to $M_{3,1}$ because there is always a detection time delay at the switching instants, since the process has to pass by the second operating point before reaching the third one. Therefore, the states $M_{1,3}$ and $M_{3,1}$ are not attainable. Thus, they can be removed from the hybrid automaton. Therefore, the system of LMIs (14) is reduced to a set (less conservative) of seven LMIs as follows:

$$A^T_{i,j} RA_{i,j} - R \leqslant -Q, \\ \forall (i,j) \in I^2 - \{(1,3),(3,1)\} \qquad (15)$$

## 4. EXPERIMENTAL RESULTS

This active control accommodation strategy was implemented in real-time (AD RTI 815) on the thermal process depicted in figure 4. The control objective is to maintain the output temperature $T_s$ at 32 $°C$ corresponding to a 3 Volts thermistor output. The ambient temperature $T_e$ is around 20 $°C$. For this objective, two experiments have been considered to prove the performances of the proposed strategy. In the first one, the OMDA structure, without using bumpless-transfer compensators, was used. The switching transient response has been improved by adding a bumpless-transfer compensator to each controller. We have also compared the obtained results with those obtained with a unique robust RST controller whose design is based on the shaping of the sensitivity function (Landau *et al.*, 1998).

The thermal process is composed of a tube with a constant volume $V$, a heating resistor $R$ connected to a direct current power supply $u(t)$. Here, $C$ is the specific heat constant for the air. The process consists in heating the air flowing into the tube, with a flow rate $f_j$, to a desired temperature level. The flow rate signal is assumed piecewise constant and can vary by changing the throttle position $j$.

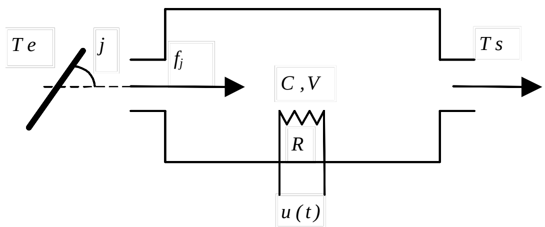


Fig. 4. Thermal process schematic.

The process can be modeled by a first order system with time delay as follows:

$$G_j(s) = \frac{k_j \ e^{-\tau_j s}}{(1 + T_j s)} \qquad (16)$$

where: $k_j$, $\tau_j$ and $T_j$ are, respectively, the static gain, the time delay and the time constant corresponding to the $j$th throttle position. Three models associated to three different air flow rate levels, i.e., low, medium and high, have been identified. The corresponding transfer functions sampled at $T_c = 0.3\ s$, $T_c > \tau_j,\ \ j \in \{1, 2, 3\}$ are:

$$\begin{aligned} G_1(q) &= \frac{0.0387\ q + 0.2791}{q^2 - 0.6303\ q} \\ G_2(q) &= \frac{0.1213\ q + 0.2072}{q^2 - 0.5282\ q} \\ G_3(q) &= \frac{0.1633\ q + 0.1133}{q^2 - 0.4811\ q} \end{aligned} \tag{17}$$

The desired closed-loop transfer function corresponds to a continuous second order system with a natural frequency $\omega_0 = 3\,\text{rad.s}^{-1}$ and a damping factor $\xi = 0.8$. Its discrete time equivalent is given by the following transfer function:

$$G_{cl}(q) = \frac{0.2488\ q + 0.1531}{q^2 - 0.8350\ q + 0.2369} \tag{18}$$

The method is illustrated through the design of the controller associated to the first model $G_1$. A simple controller with an integral action can be written as:

$$\begin{aligned} R_1(q) &= (q-1)\ (q + r_2) \\ S_1(q) &= s_0\ q^2 + s_1\ q + s_2 \end{aligned} \tag{19}$$

The degree of the denominator of $G_{cl}(q)$ must be augmented by a dominant pole at $(q - 0.521)$ and an auxiliary pole at the origin to resolve the Diophantine equation. It follows that:

$$P_{cl}(q) = q(q - 0.521)(q^2 - 0.8350\ q + 0.2369) \tag{20}$$

Also, $P_{cl}(q)$ is given by the relation (9):

$$\begin{aligned} P_{cl}(q) &= (q^2 - 0.63031q)\ R_1(q) \\ &+ (0.03879q + 0.27914)\ S_1(q) \end{aligned} \tag{21}$$

Equating the two precedent equations gives: $s_0 = 1.5017$, $s_1 = -0.9113$, $s_2 = 0$, $r_2 = 0.2160$. To determine the polynomial $T_1(q)$, the polynomial $P_{cl}(q)$ can be factorized under the form $P_{cl}(q) = A_o(q)\ A_c(q)$, where $A_o(q)$ corresponds to the augmented factor and $A_c(q)$ corresponds to the denominator $den(G_{cl}(q))$. Thus, $T_1(q)$ can be chosen such that $T_1(q) = t_0\ A_o(q)$, with $t_0 = A_1(1)/B_1(1)$ which leads to a static gain equals to 1. Thus, $T_1(q) = 1.2326\ q^2 - 0.6422\ q$. The controllers $C_2$ and $C_3$ are designed in the same way:

$$\begin{aligned} R_2(q) &= q^2 - 0.9743\ q - 0.0256 \\ S_2(q) &= 1.2070\ q^2 - 0.6358\ q \\ T_2(q) &= 1.1926\ q^2 - 0.6213\ q \\ R_3(q) &= q^2 - 1.1182\ q + 0.1182 \\ S_3(q) &= 1.4166\ q^2 - 0.7380\ q \\ T_3(q) &= 1.3302\ q^2 - 0.6517\ q \end{aligned} \tag{22}$$

The stability of the resulting switched system is insured by resolving the LMIs given by the relation (15). The optimization problem is found to be feasible and the common Lyapunov function is $V(x) = x^T Rx$, where $R$ is definite positive and given by:

$$R = \begin{bmatrix} 5.323 & -1.944 & -3.916 & 0.176 \\ -1.944 & 43.860 & -18.996 & -33.654 \\ -3.916 & -18.996 & 42.075 & 5.608 \\ 0.176 & -33.654 & 5.608 & 41.575 \end{bmatrix}$$

The experiment results are shown on figure (5). The regulation is satisfactory even in the presence of abrupt operating conditions changes that are shown on figure (5.d). The measurement noise variance is $\sigma^2 = 1.0436 \times 10^{-3}$. The size of the sliding window is $N = 9$. However, the switching between controllers produce large overshoots on the process output and make the settling time longer than what it is supposed to be. These effects are due to the control input discontinuities. Large bumps on the control input (see fig. 5.b) and on the process output (see fig. 5.a) can be observed at the switching instants. At $t \simeq 27.5$ sec, one can observe the effect of the second controller activation when the process evolves from $M_{3,3}$ to $M_{1,1}$. The activation of the second controller has an effect of jerking on the control signal and hence on the process output signal. The same phenomenon is repeated at each switching. To attenuate these bumps, a bumpless-transfer compensator (see Fig. 2) was added to each controller to improve the transient switching response. The method proposed in (Aström and Wittenmark, 1997) is used to reduce these bumps. Then, an integrator is introduced in each controller, i.e., $W_i = T_c\frac{1}{q-1}$, $i = 1, 2, 3$. Figure (6) shows the results of the second experience. The bumps pointed out in the precedent experience are attenuated and the settling time is reduced.

These results are compared to those obtained by a robust RST controller. This controller is designed for the model of the process corresponding to a medium air flow rate, i.e., the model $G_2$. Its robustness margins are: 2.757 gain margin, 58° phase margin, 0.65 sec delay margin and 0.598 modulus margin. The results obtained with this controller are shown on figure (7). The process evolves according to the same switching sequence as in the first and second experiment, i.e., $j = 3, 1, 2, 3, 2, 1, 3$. This figure shows that the RST controller insures satisfactory control performances. However, when the air flow rate switches to the low level (between $t \simeq 27$ sec and $t \simeq 47$ sec, $t \simeq 120$ sec and $t \simeq 130$ sec), the control input shows some chattering that did not happen when the OMDA structure had been used. Also, the noise perturbations are less attenuated on the output than with the OMDA structure.

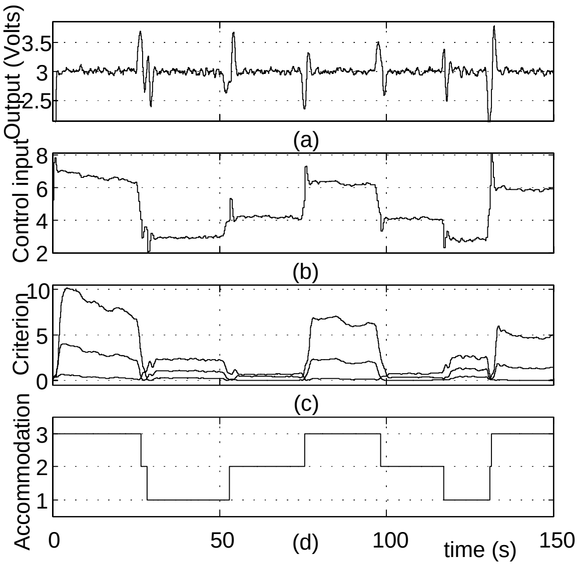


Fig. 5. Accommodation without a bumpless-transfer compensator.

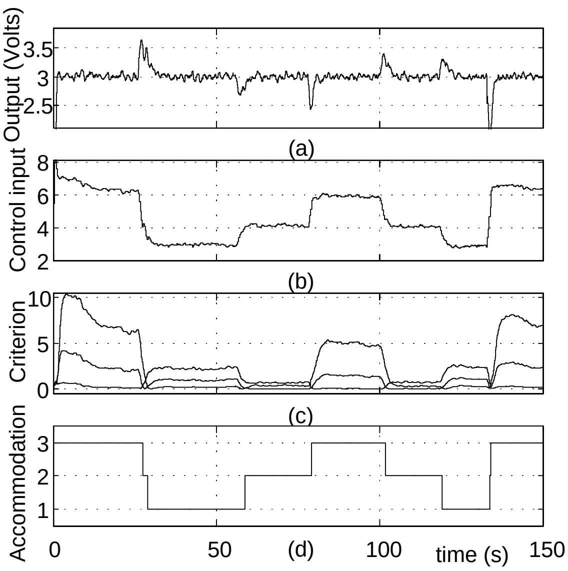


Fig. 6. Accommodation with a bumpless-transfer compensator.

## 5. CONCLUSION

The OMDA structure is proposed to detect the different operating modes of a process and to accommodate its control input by the selection of the adequate controller. The implementation of this structure was illustrated by the active control accommodation of a thermal process in presence of large variations in the operating conditions. The experimental results show that this structure is easy to implement and gives satisfactory results.

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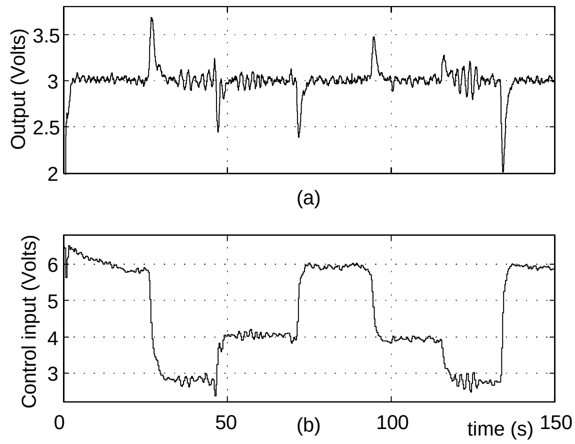


Fig. 7. Regulation by a robust RST controller.


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