15th Triennial World Congress, Barcelona, Spain

ADAPTIVE NARROW BAND DISTURBANCE REJECTION IN ACTIVE VIBRATION CONTROL

Aurelian Constantinescu *,[1] **Ioan Doré Landau** *

* *Laboratoire d'Automatique de Grenoble*
ENSIEG, BP 46, 38402 Saint Martin d'Hères, France
fax: + 33 (0)4 76 82 62 44
e-mail: landau@lag.ensieg.inpg.fr

Abstract: This paper presents a methodology for feedback adaptive control of active vibration attenuation systems in the presence of unknown narrow band disturbances. The proposed methodology consists in two algorithms based on the Internal Model Principle. The first one deals with an indirect adaptive control, the second one with a direct adaptive control. The feasibility of the two algorithms is illustrated in real-time on an active suspension system.

Keywords: Active control, adaptive control, active suspension, feedback control.

1. INTRODUCTION

One of the basic problems in active vibration control is the attenuation (rejection) of narrow band disturbances of unknown or varying frequency.

Perturbations rejection has been studied for a long time. An efficient method to eliminate their effect is to introduce the perturbation's dynamics into the closed loop system. This approach is known as the Internal Model Principle.

The application of the Internal Model Principle is simplified when both the structure and the parameters of the perturbation are known. It is the case of what we call completely known perturbations, discussed, for example, in (Francis and Wonham, 1976; Johnson, 1976; Bengtsson, 1977; Tsypkin, 1997). However, complete knowledge of the perturbation is not often possible. Hence, the perturbation is usually unknown and may be time-varying. In the case of time-varying perturbations, the controller parameters have to be adapted in order to verify the desired specifications. Closed loop adaptive control methods can be used to solve this problem. An indirect and a direct adaptive control schemes are proposed in this paper and comparatively evaluated.

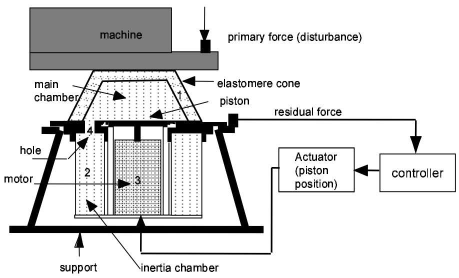

Fig. 1. Active suspension system

The structure of the system is presented in fig. 1. The controller will act upon the piston (through a power amplifier) in order to reduce the residual force. The sampling frequency is $800Hz$.

The equivalent scheme is shown in fig. 2. The system input, $u(t)$ is the position of the piston (see figs. 1, 2), the output $y(t)$ being the residual force measured by a force sensor.

The principle of the active suspension is to vary the system's stiffness in order to attenuate the vibrations generated by the part that we want to isolate (primary force - disturbance). In our case (for testing purposes),

[1] Aurelian Constantinescu is currently at CNES, Launcher Directorate.

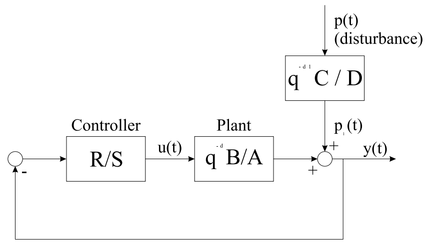

Fig. 2. Block diagram of the active suspension system

the primary force is generated by a shaker controlled by a signal given by the computer.

The transfer function ($q^{-d_1}\frac{C}{D}$), between the signal sent to the shaker, p, and the residual force $y(t)$ is called primary path. The transfer function ($q^{-d}\frac{B}{A}$) between the input of the system, $u(t)$ and the residual force is called secondary path. The input of the system being a position and the output a force, the secondary path transfer function has a double differentiator behavior. In our case, treating narrow band perturbations, we shall model them using an AR (auto regressive) model.

The control objective is to reject the effect of unknown narrow band disturbances on the output of the system (residual force).

2. PLANT REPRESENTATION AND CONTROLLER STRUCTURE

The structure of a linear time invariant discrete time model of the plant (on which is based the design of the controller) is

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})}$$

where:

$$\begin{aligned} d &= \text{number of sampling periods} \\ &\quad \text{on the plant pure time delay;} \\ A &= 1 + a_1 z^{-1} + \ldots + a_{n_A} z^{-n_A}; \\ B &= b_1 z^{-1} + \ldots + b_{n_B} z^{-n_B}. \end{aligned}$$

The controller to design is a RS-type controller (see fig. 2). The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p_1(t); \quad (1)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t). \quad (2)$$

The output sensitivity function (the transfer function between the perturbation $p_1(t)$ and the output $y(t)$) is:

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})},$$

where

$$P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \quad (3)$$

are the poles of the closed loop.

The polynomials $R(z^{-1})$ and $S(z^{-1})$ are expressed as:

$$\begin{aligned} R(z^{-1}) &= R'(z^{-1}) \cdot H_R(z^{-1}); \\ S(z^{-1}) &= S'(z^{-1}) \cdot H_S(z^{-1}), \end{aligned} \quad (4)$$

where H_R and H_S are fixed parts of the controller.

Suppose that $p_1(t)$ is a stochastic perturbation, so it can be written as $p_1(t) = \frac{1}{D_p(q^{-1})} \cdot e(t)$, where $e(t)$ is a gaussian white noise. The effect of the perturbation $p_1(t)$ on $y(t)$ is given by:

$$y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})} \cdot \frac{1}{D_p(q^{-1})} \cdot e(t). \quad (5)$$

Hence, in order the eliminate (minimize) the effect of the perturbation on the output $y(t)$, we have to take $H_s(z^{-1}) = D_p(z^{-1})$ (Internal Model Principle).

3. INDIRECT ADAPTIVE CONTROL FOR NARROW BAND PERTURBATION ATTENUATION

The methodology proposed in this section concerns the indirect adaptive control for the attenuation of narrow band perturbations and consists in two steps:

(1) On-line identification of the perturbation model.
(2) Computation of a digital controller using the identified perturbation model in order to reject the perturbation effect on the output of the system.

Knowing that we deal only with narrow band perturbations, let consider an AR (auto regressive) perturbation model:

$$y(t) = \frac{1}{D_p(q^{-1})} \cdot e(t), \quad (6)$$

where $e(t)$ is a gaussian white noise and

$$\begin{aligned} D_p(z^{-1}) &= 1 + d_{p_1} z^{-1} + \ldots + d_{p_{n_D}} z^{-n_D} \\ &= 1 + z^{-1} D_p^*(z^{-1}); \\ n_D &= \text{the order of } D_p(z^{-1}). \end{aligned}$$

Remark: For the case of a damped sinusoid we can consider $n_D = 2$. For the case of not-damped sinusoids $d_{p_2} = 1$.

From equation (6) we obtain:

$$y(t+1) = -\sum_{i=1}^{n_D} d_{p_i} y(t-i+1) + e(t+1). \quad (7)$$

The problem is, in fact, an on-line adaptive estimation of a signal in presence of noise.

One construct an adjustable predictor for $y(t)$ given by (7).

The *a priori* adjustable predictor is

$$\hat{y}^0(t+1) = -\sum_{i=1}^{n_D} \hat{d}_{p_i}(t) y(t-i+1)$$
$$= \hat{\theta}^T(t)\phi(t),$$

where

$$\hat{\theta}^T(t) = [\hat{d}_{p_1}(t) \ \dots \ \hat{d}_{p_{n_D}}(t)];$$
$$\phi^T(t) = [-y(t) \ \dots \ -y(t-n_D+1)].$$

The *a posteriori* adjustable predictor is:

$$\hat{y}(t+1) = -\sum_{i=1}^{n_D} \hat{d}_{p_i}(t+1) y(t-i+1)$$
$$= \hat{\theta}^T(t+1)\phi(t).$$

The *a priori* prediction error is

$$\varepsilon^0(t+1) = y(t+1) - \hat{y}^0(t+1).$$

The *a posteriori* prediction error is

$$\varepsilon(t+1) = y(t+1) - \hat{y}(t+1).$$

The parameter adaptation algorithm is (Landau *et al.*, 1997):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1); \quad (8)$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1+\phi^T(t)F(t)\phi(t)}; \quad (9)$$

$$\varepsilon^0(t+1) = y(t+1) - \hat{\theta}^T(t)\phi(t); \quad (10)$$

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi^T(t)F(t)\phi(t)}\right] \quad (11)$$

where $F(t)$ is the adaptation gain matrix.

Once the perturbation model is identified, one computes the controller containing the perturbation dynamics by solving the diophantine equation (3), and using (4):

$$A(z^{-1})H_S(z^{-1})S'(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$$
$$= P(z^{-1}), \quad (12)$$

where $H_S(z^{-1}) = \hat{D}_p(z^{-1}) = 1 + \sum_{i=1}^{n_D} \hat{d}_{p_i} z^{-i}$, $B(z^{-1})$ and $A(z^{-1})$ represent the plant model.

In order to apply this methodology we suppose that the plant model $z^{-d}B(z^{-1})/A(z^{-1})$ is known (the model is obtained by identification (Landau *et al.*, 2001)). We suppose also that the degree of $D_p(z^{-1})$, n_D is fixed.

4. DIRECT ADAPTIVE CONTROL FOR NARROW BAND PERTURBATION ATTENUATION

In the literature it has been proposed to add a supplementary degree of freedom into the RS controller, presented in section 2, in order to explicitly take into account the perturbation (see (Tsypkin, 1991), (Tsypkin, 1997)). Using this supplementary degree of freedom, the attenuation problem may me treated independently.

4.1 *Q-parameterization*

Let $[R_0(z^{-1}), S_0(z^{-1})]$ be a nominal controller, verifying the diophantine equation (3) and satisfying the robustness constraints. Using the Q-parameterization (known also as Youla-Kucera parameterization)

$$R(z^{-1}) = R_0(z^{-1}) + A(z^{-1})Q(z^{-1}); \quad (13)$$
$$S(z^{-1}) = S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1}), \quad (14)$$

one obtains the family of all stabilizable controllers, $[R(z^{-1}), S(z^{-1})]$, where $Q(z^{-1})$ is a polynomial of degree n_Q. The polynomial $Q(z^{-1})$ will be designed such that the equivalent $R(z^{-1})/S(z^{-1})$ controller contains the internal model of the disturbance.

The controller equation becomes:

$$S_0(q^{-1}) \cdot u(t) = -R_0(q^{-1}) \cdot y(t) - Q(q^{-1}) \cdot w(t),$$

where $w(t) = A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot u(t)$. The new scheme of the closed loop, with the Q-parameterized controller, is presented in figure 3.

If we take into account the equations (13) and (14), the equation defining the closed loop poles is the same as (3), therefore the closed loop poles remain unchanged.

Having the controller parameterized as in (13) and (14), the perturbation effect on the output of the system, (5), is:

$$y(t) = \frac{S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})}{P(q^{-1})} \cdot w(t),$$

with

$$w(t) = \frac{A(q^{-1})}{D_p(q^{-1})} \cdot e(t)$$
$$= A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot u(t).$$

The output sensitivity function in this case is

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})[S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1})]}{P(z^{-1})}.$$

In order to reject the disturbance, $Q(z^{-1})$ should be selected such that $S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1}) =$

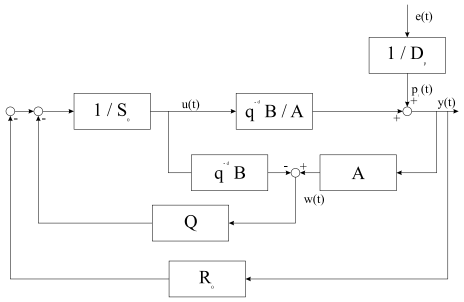

Fig. 3. Structure of the Q-parameterized controller - closed loop scheme

$M(z^{-1})D_p(z^{-1})$, which implies to solve the diophantine equation

$$M(z^{-1})D_p(z^{-1}) + z^{-d}B(z^{-1})Q(z^{-1}) = S_0(z^{-1}), \tag{15}$$

where $D_p(z^{-1})$, $B(z^{-1})$ and $S_0(z^{-1})$ are known.

Defining $\varepsilon(t) = y(t)$ we obtain:

$$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t). \tag{16}$$

Define $\hat{Q}(t, z^{-1})$ as the estimation of $Q(z^{-1})$ at instant t. Using (15), equation (16) becomes:

$$\varepsilon(t) = [Q(q^{-1}) - \hat{Q}(t, q^{-1})] \cdot \frac{q^{-d}B(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t), \tag{17}$$

where $v(t)$ is a filtered white noise type perturbation, which to a large extend takes care of the differences between the true model of the plant and the identified one.

If we consider $\hat{Q}(t, q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \ldots + \hat{q}_{n_Q}(t)q^{-n_Q}$, note $\hat{\theta}^T(t) = [\hat{q}_0(t)\, \hat{q}_1(t) \ldots \hat{q}_{n_Q}(t)]$ the parameters vector.

Equation (17) becomes

$$\varepsilon(t) = [\theta - \hat{\theta}(t)] \cdot w_2(t) + v(t),$$

where $w_2(t) = \dfrac{q^{-d}B(q^{-1})}{P(q^{-1})} \cdot w(t)$. One sees that $\varepsilon(t)$ corresponds to an adaptation error.

The "a priori" adaptation error is:

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t) w_2(t+1),$$

with

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1); \tag{18}$$

$$w_2(t+1) = \frac{q^{-d}B(q^{-1})}{P(q^{-1})} \cdot w(t+1); \tag{19}$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - B(q^{-1}) \cdot u(t+1).$$

For the estimation of the parameters of $\hat{Q}(z^{-1})$ we use the following parametric adaptation algorithm:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1); \tag{20}$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)}; \tag{21}$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t); \tag{22}$$

where $F(t)$ is given by (11) and

$$\phi(t) = w_2(t+1).$$

In order to apply this methodology (for rejecting the perturbation $p_1(t)$ (see figure 3)), we suppose that the plant model $\dfrac{z^{-d}B(z^{-1})}{A(z^{-1})}$ is known (it is identified) and that there exist a controller $[R_0(z^{-1}), S_0(z^{-1})]$ who verifies the desired specifications in the absence of the disturbance. We also suppose that the degree n_Q of the $Q(z^{-1})$ polynomial is fixed, $n_Q = n_D - 1$ for the case when the perturbation structure is known.

5. RESULTS OBTAINED ON AN ACTIVE SUSPENSION

The procedure for narrow band perturbation rejection using the methodologies proposed in this article will be illustrated in real time for the case of the control of an active suspension (see figures 1 and 2). In our case the perturbation $p(t)$ will be a varying frequency sinusoid. The objective of the control is to reject the effect of this perturbation on the output of the system, $y(t)$, by adapting the controller parameters on-line as a function of the perturbation's frequency.

The frequency characteristic of the identified primary path model (open loop identification) $\dfrac{q^{-d_1}C(q^{-1})}{D(q^{-1})}$ (see figure 2), between the perturbation $p(t)$ and the residual force $y(t)$, is presented in figure 4. The first vibration mode of the primary path is near $32Hz$.

The frequency characteristic of the identified secondary path model (closed loop identification), between the control $u(t)$ and the residual force $y(t)$, is presented in figure 5. This model has the following complexity: $n_B = 14, n_A = 16, d = 0$.

There exist several vibration modes on the secondary path, the first one being at $31.8Hz$ with a damping factor 0.07. The system contains a double differentiator.

The nominal controller (without the internal model of the perturbation) has been designed using the pole placement method. A pair of dominant poles has been

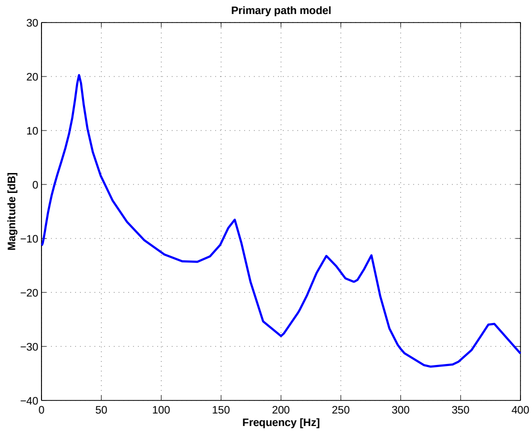

Fig. 4. Frequency characteristic of the primary path model

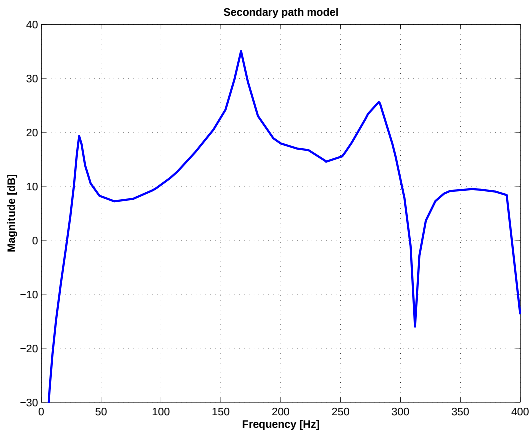

Fig. 5. Frequency characteristic of the secondary path model

fixed at the frequency of the first vibration mode with a damping $\xi = 0.8$. In addition a fixed part $H_R = 1 + q^{-1}$ $(R = H_R R')$ which assures the opening of the loop at $0.5 f_s$ and 10 auxiliary poles at 0.7 have been introduced into the controller . The resulting nominal controller has the following complexity: $n_R = 14, n_S = 16$ and it satisfies the imposed robustness constraints.

Real-time experiments of vibration-canceling performance were performed with sinusoidal signals of varying frequency as perturbation $p(t)$ (see figure 2). The experiments use the secondary path model identified on the real system. We considered sinusoids of frequencies varying between 25 and $47Hz$, the first vibration mode of the primary path being near $32Hz$.

The implementation protocol used consists in a self-tuning operation. It starts in open loop without perturbation. After 4000 samples we apply a sinusoidal perturbation at $32Hz$. After the algorithm converges we compute the controller and we apply it, passing in closed loop. As soon as the controller is applied, the adaptation algorithm is stopped and we wait for a change of frequency. When such a change is detected we restart the algorithm, letting the last controller applied on the system. When the algorithm converges, we compute a new controller and apply it on the system.

The measured residual force obtained with both methodologies, the indirect and direct one, are presented in figures 6 and 7, respectively. We can see the $32Hz$ sinusoid applied after 4000 samples, a $25Hz$ one applied after 12000 samples, again a $32Hz$ one applied after 20000 samples, a $47Hz$ one applied after 28000 samples and again a $32Hz$ one after 36000 samples.

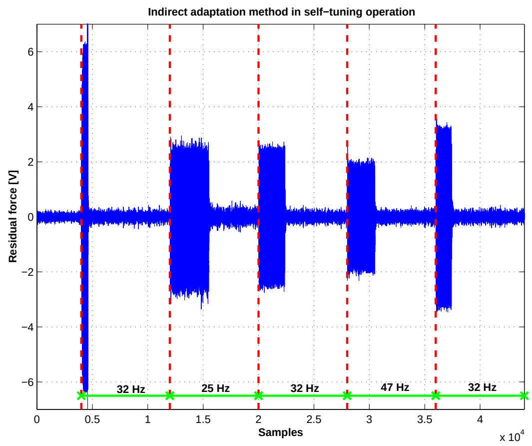

Fig. 6. Temporal results using the indirect adaptive method

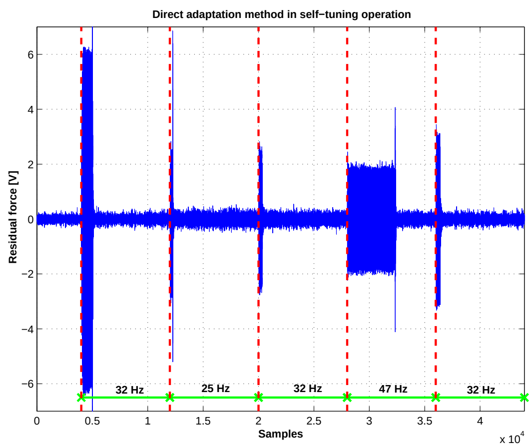

Fig. 7. Temporal results using the direct adaptive method

From figures 6 and 7 we can see that the convergence speed of the direct algorithm is bigger that the convergence speed of the indirect algorithm, except the $47Hz$ frequency.

The residual force spectrum before (in open loop) and after convergence for the indirect and direct methods are presented in figures 8 and 9 respectively, for the three frequencies that we have chosen: $32Hz$, $25Hz$ and $47Hz$. We can remark that the attenuations are bigger than $25dB$ for all the frequencies used and for both algorithms. The attenuations obtained with the direct algorithm are better than those obtained with

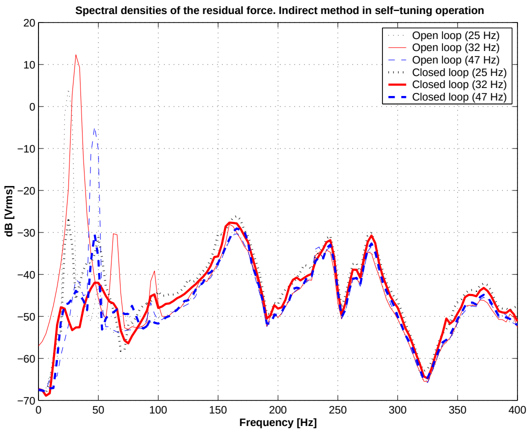

Fig. 8. Spectrum of the residual force in open and in closed loop, using the indirect adaptive method

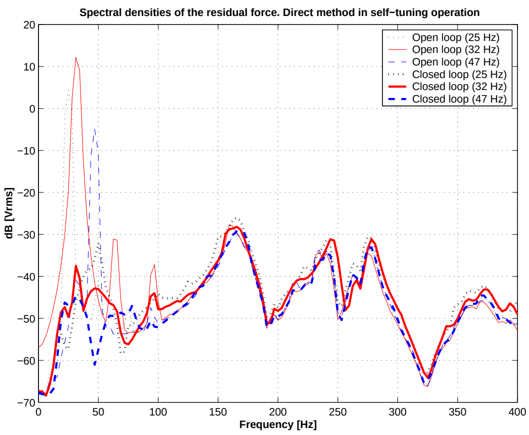

Fig. 9. Spectrum of the residual force in open and in closed loop, using the direct adaptive method

the indirect one at 25 and $47Hz$, while at $32Hz$ the performances of both algorithms are similar.

In the case presented upper, of a pure sine wave, we used $n_D = 2$ for the indirect method and $n_Q = 1$ for the direct one.

6. CONCLUSIONS

Two adaptive methodologies for active vibration control systems in the presence of unknown narrow band disturbances have been presented. The first approach is an indirect adaptive method, the second a direct one. Real time experiments on an active suspension have been carried out. The results obtained lead us to conclude that the direct adaptive algorithm converges faster than the indirect one. Moreover, the direct algorithm is much simpler than the indirect one.

7. REFERENCES

Bengtsson, G. (1977). Output regulation and internal models - a frequency domain approach. *Automatica* **13**, 333–345.

Francis, B. A . and W. M. Wonham (1976). The internal model principle of control theory. *Automatica* **12**, 457–465.

Johnson, C. D. (1976). Theory of disturbance-accomodating controllers. In *Control and Dynamical Systems* (C. T. Leondes, Ed.). Vol. 12, pp. 387-489.

Landau, I. D., A. Constantinescu, P. Loubat, D. Rey and A. Franco (2001). A methodology for the design of feedback active vibration control systems. *Proceedins of the European Control Conference ECC 2001*.

Landau, I. D., R. Lozano and M. M'Saad (1997). *Adaptive control*. Springer. London.

Tsypkin, Y. Z. (1991). Adaptive-invariant discrete control systems. In *Foundations of Adaptive Control* (M. Thoma and A. Wyner, Eds.). Vol. 160 of *Lecture Notes in Control and Information Science*, pp. 239-268, Springer Verlag.

Tsypkin, Y. Z. (1997). Stochastic discrete systems with internal models. *Journal of Automation and Information Sciences* **29**(4&5), 156–161.