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STATE-DEPENDENT PARAMETER NONLINEAR SYSTEMS: IDENTIFICATION, ESTIMATION AND CONTROL

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Abstract: A control system design procedure is proposed for a widely applicable class of discrete-time, non-linear systems in which the system nonlinearities are incorporated into a linear model structure in the form of State-Dependent Parameter (SDP) functions. The identification and estimation of both non-parametric and parametric SDP models is discussed briefly. The SDP NMSS model structure is defined, and the SDP Proportional-Integral-Plus (PIP) control algorithm is derived using an optimal LQ technique. The practical utility of the design methodology is illustrated by numerical example. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The control system design procedure described in this paper is a novel nonlinear extension of the linear Proportional-Integral-Plus (PIP) controller design proposed by Young *et al.* (1987). This linear PIP controller derives from a Non-Minimal State Space (NMSS) formulation of the linear system equations (Hesketh 1982, Young *et al.* 1987). In the case of single input systems described using the discrete-time backward-shift operator, the non-minimal state variables consist entirely of measurable variables: the present and past values of the output variable; the past values of the input variable; and an integral-of-error state variable included to ensure type-one servomechanism performance. This basic non-minimal state vector can also be augmented with other measured variables, such as disturbance inputs, if these have an appreciable effect on the system dynamics.

A major advantage of the PIP control design method is that the NMSS variables are all measurable, permitting the use of any state variable feedback (SVF) technique without recourse to the implementation of a state reconstruction or observer system, such as the Kalman Filter. This avoids unwarranted complexity in the PIP system design and reduces the dependency of the controller on the model of the system, with obvious advantages in terms of robustness (see e.g. Bitmead *et al*, 1990). The PIP controller in this form can be regarded as a logical extension of the well-known classical multi-term Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers, to which additional feedback and forward-path compensators have been added to improve the PI/PID action. These extra compensators are derived automatically from the NMSS state feedback control law. PIP controllers have been successfully implemented in a wide range of applications (Chotai, *et al*. 1991, Lees, *et al*. 1995, Chotai, *et al*. 1998, Taylor *et al*. 2000).

State-dependent parameter (SDP) models (Young, 1993, 1998, 2000, 2001a; Young *et al*, 2001) have the same basic structure as linear Time Variable Parameter (TVP) transfer function models. However, the temporal variation of the parameters is not caused by some slow variations in the system characteristics but is assumed to arise because the parameters are actually functions of other variables, such as the non-minimal states. In this manner, the SDP model is able to provide a description for a widely applicable class of nonlinear systems that includes chaotic processes (see examples in the above references).

As we shall see, when SDP models are applied within a control context they lead to control system gains that are also state dependent, revealing an obvious connection with certain gain scheduling design methods. The value of using linear parameter-varying systems as the basis for gain-scheduled control has been expounded in various references (e.g. Becker and Packard 1994, Apkarian and Adams

1998). The main advantage of such systems is the applicability of many aspects of linear systems theory. This is clearly also relevant to SDP modelled systems and it means that conventional SVF design procedures can be exploited in the design of the nonlinear SDP-PIP controller, resulting in a closed-loop system with state-dependent SVF gains.

Section 2 discusses briefly the statistical identification and estimation of SDP models and the application of these methods to the modelling of a typical nonlinear system that can be modelled in SDP terms. The development of the SDP non-minimal state space description is described in Section 3, where the design principles are demonstrated by application to the SDP model derived in Section 2.

2. SDP MODELS: IDENTIFICATION AND ESTIMATION

The idea of using SDP models to represent nonlinear dynamic systems goes back to Young (1978), who showed how the forced logistic growth equation could be represented, identified and estimated in SDP form. However, the practical development of these ideas is of a more recent origin (Young, 1993, 1998, 2000, 2001a; Young et al, 2001). The simple SDP transfer function (SDTF) model considered in the present paper can be written in the following TVP form:

$$y_k = \frac{B_k(z^{-1})}{A_k(z^{-1})} u_{k-\delta} + \xi_k \qquad (1)$$

where z^{-i} is the backward shift operator, i.e. $z^{-i} y_k = y_{k-i}$; δ is a pure time delay; ξ_k is a general, zero mean noise signal; and $A_k(z^{-1})$, $B_k(z^{-1})$ are the following TVP polynomials in the backward shift operator z^{-1}

$$\begin{aligned} A_k(z^{-1}) &= 1 + a_1(w_k)z^{-1} + \ldots + a_n(w_k)z^{-n} \\ B_k(z^{-1}) &= b_1(w_k)z^{-1} + \ldots + b_m(w_k)z^{-m} \end{aligned} \qquad (2)$$

where the notation $\alpha_i(w_k)$ indicates that any SDP α_i is a nonlinear function of a variable w_k: Below, the $\alpha_i(w_k)$ are denoted as simply $\alpha_{i,k}$, for convenience of notation. However, it is important to emphasise that they are state dependent and *not* 'slowly variable' functions of time, *k*. In general, w_k is defined in terms of any variables on which the parameters are identified as being dependent and so it could be defined as any element of the NMSS vector $\boldsymbol{x}_k$ (see later). In the present context, however, each SDP is assumed to be a nonlinear function its associated past input or output variable, i.e.,

$$\begin{aligned} A_k(z^{-1}) &= 1 + a_1(y_{k-1})z^{-1} + \ldots + a_n(y_{k-n})z^{-n} \\ B_k(z^{-1}) &= b_1(u_{k-1})z^{-1} + \ldots + b_m(u_{k-m})z^{-m} \end{aligned} \qquad (3)$$

Young (2000, 2001a) and Young et al (2001) describe an identification and estimation strategy for SDP models of this general type. The details of this strategy are given in these references and it will suffice here to outline the main features of the approach.

The SDP model identification and estimation process is based on analysis of the input-output data set and consists of three main stages:

1. First, identification of the most appropriate SDP model structure and order based on a special form of non-parametric estimation that exploits recursive fixed interval smoothing and special data sorting utilised within an iterative 'back-fitting' algorithm (see above references). This yields an appropriate minimal model order and an estimate of each constituent SDP which, when plotted against the associated dependent variable, reveals the nature of the associated nonlinearity. In this non-parametric (graphical) form, the SDTF model can be simulated in SIMULINK™ using look-up tables to represent the graphically defined SDP nonlinearities.

2. Second, an initial parametric identification and estimation stage, where the non-parametrically defined nonlinearities obtained in stage 1. are parameterised in some manner in terms of their associated dependent variable: for example by defining an appropriate parametric model in some convenient form, such as a polynomial or trigonometric function; a radial basis function or a more general neuro-fuzzy relationship; or a neural network. Initial identification and estimation of this parametric function is then based, for example, on least squares optimization of the function against the non-parametric SDP estimate, as in the example below.

3. Third, a final estimation stage in which the parameters of the parametric nonlinear model obtained in stage 2. are estimated directly from the input-output data using some method of dynamic model optimization: e.g. deterministic nonlinear least squares or, preferably, a more statistically efficient stochastic method, such as maximum likelihood based on prediction error decomposition, where the nature of the noise processes are also taken into consideration (see the above references).

2.1 *An Illustrative Example*

A typical example of an SDP model is the following forced logistic equation with a soft limited input nonlinearity based on a tanh function:

$$\begin{aligned} x_k &= (2 - 2x_{k-1})x_{k-1} + 0.01(\tanh(u_{k-1}) + 0.02u_{k-1}) \\ y_k &= x_k + e_k \qquad e_k = N(0,\sigma^2) \end{aligned} \qquad (4)$$

The variance of u_k is $\sigma_u^2 = 8$ and the output noise e_k is $\sigma^2 = 4\text{x}10^{-6}$, producing a 20% level of noise by standard deviation. The SDP estimation results are shown in figure 1. The estimated SDP $\hat{a}_1(y_{k-1})$ for the state (internal) nonlinearity, as shown in the left panel of figure 1, is clearly a linear function of y_{k-1}, as expected. Consequently, simple least squares estimation yields the following linear relationship $a_1(y_{k-1}) = \alpha_1 + \beta_1 y_{k-1}$, with associated parameter estimates,

$$\hat{\alpha}_1 = 1.994(0.009) \text{ and } \hat{\beta}_1 = -1.988(0.0019)$$

where the standard errors on the estimates are shown in parentheses. The estimated input nonlinearity, as plotted in the right panel of figure 1, is clearly a soft limiting function and simple least squares estimation of the linear-in-the parameters tanh-type relationship, $b_0(u_{k-1}).u_{k-1} = \alpha_0 \tanh(u_{k-1}) + \beta_0 u_{k-1}$, yields a good representation of the nonlinearity, with the following parameter estimates and standard errors:

$$\hat{\alpha}_0 = 0.010(0.001)\,;\; \hat{\beta}_0 = 0.000188(0.0003)\,.$$

If we consider this SDP estimation as an initial identification stage in the modelling, then we could obtain final parametric estimates of the constant parameters in the model (4) using Maximum Likelihood estimation, as mentioned above. For the purposes of the present illustrative example, however, it will suffice to use the above parameter estimates as the basis for SDP-PIP control system design. This has the added advantage that the larger estimation errors will serve to evaluate the sensitivity of this design to such parametric modelling errors.

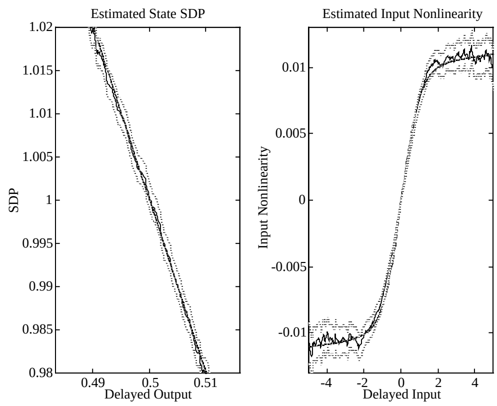

Fig.1 SDP non-parametric estimation results: true functions (dashed lines); estimated functions (full lines); standard errors on estimated functions (dotted lines).

3. SDP-NMSS SYSTEMS

In SDP systems, the variables associated with the state dependency are normally the elements of the NMSS vector and the control input u_k. However, variables other than this may also be incorporated into the state vector, or regarded as 'virtual' states to avoid unnecessary dimensional enlargement.

3.1 *The NMSS Description*

The plant equation represented by transfer function (1) is expressed in NMSS form as:

$$\begin{aligned} \boldsymbol{x}_k &= \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{g}_{k-1}u_{k-1} + \boldsymbol{d}r_k \\ y_k &= \boldsymbol{h}\boldsymbol{x}_k \end{aligned} \qquad (5)$$

where r_k is the reference input variable, and $\boldsymbol{x}_k$ is the non-minimal state vector,

$$\boldsymbol{x}_k = [y_k \;\, y_{k-1} \,\ldots\, y_{k-n+1} \;\, u_{k-1} \;\, u_{k-2} \,\ldots\, u_{k-m+1} \;\, z_k]^T$$

with the integral-of-error state, z_k, defined as:

$$z_k = z_{k-1} + \{r_k - y_k\}$$

The state transition matrix $\boldsymbol{F}_k$ and the vectors $\boldsymbol{g}_k$, $\boldsymbol{h}$ and $\boldsymbol{d}$ are defined as

$$\boldsymbol{F}_k = \begin{bmatrix} -a_{1,k} & -a_{2,k} & \cdots & -a_{n-1,k} & -a_{n,k} & b_{2,k} & \cdots & b_{m-1,k} & b_{m,k} & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ a_{1,k} & a_{2,k} & \cdots & a_{n-1,k} & a_{n,k} & -b_{2,k} & \cdots & -b_{m-1,k} & -b_{m,k} & 1 \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{g}_k &= [b_{1,k} \quad 0 \quad 0 \quad \cdots \quad 0 \quad 1 \quad 0 \quad \cdots \quad 0 \quad -b_{1,k}]^T \\ \boldsymbol{d} &= [0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 1]^T \\ \boldsymbol{h} &= [1 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0]\,. \end{aligned}$$

3.2 *SDP-PIP Non-linear Control*

Assuming that the SDP-NMSS system, $[\boldsymbol{F}_k, \boldsymbol{g}_k, \boldsymbol{d}, \boldsymbol{h}]$, is fully controllable for all k, then the vector of state-dependent feedback gains, $\boldsymbol{v}_k$, that will form the SVF control law associated with the SDP-NMSS model (5) is defined in the usual fashion:

$$u_k = -\boldsymbol{v}_k \boldsymbol{x}_k \qquad (6)$$

The vector, $\boldsymbol{v}_k$, is derived from:

$$\boldsymbol{v}_k = (R + \boldsymbol{g}_k^T \boldsymbol{P} \boldsymbol{g}_k)^{-1} \boldsymbol{g}_k^T \boldsymbol{P} \boldsymbol{F}_k \qquad (7)$$

in which R is a positive scalar for a SISO system, and $\boldsymbol{P}$ is taken to be the invariant symmetrical positive-definite matrix solution to an SDP algebraic Riccati equation (ARE). The 'frozen-parameter' system, $[\boldsymbol{F}'_k, \boldsymbol{g}'_k, \boldsymbol{d}, \boldsymbol{h}]$, is defined as a single sample member of the family of systems, $[\boldsymbol{F}_k, \boldsymbol{g}_k, \boldsymbol{d}, \boldsymbol{h}]$, and may be used with the linear quadratic (LQ) cost function:

$$J = \sum_{i=0}^{\infty} \left\{ \boldsymbol{x}_i^{\;T} \boldsymbol{Q} \boldsymbol{x}_i + R u_i^{\;2} \right\}$$

in which $\boldsymbol{Q}$ is an invariant symmetrical positive-definite matrix, to produce the discrete-time matrix ARE:

$$P = F_k'^T P F_k' - F_k'^T P g_k' (R + g_k'^T P g_k')^{-1} g_k'^T P F_k' + Q$$

from which the feedback vector (7) is derived. Incorporating the control law (6), the SDP-NMSS closed-loop system becomes

$$\begin{aligned} \boldsymbol{x}_k &= (\boldsymbol{F}_{k-1} - \boldsymbol{g}_{k-1}\boldsymbol{v}_{k-1})\boldsymbol{x}_{k-1} + \boldsymbol{d}r_k \\ y_k &= \boldsymbol{h}\boldsymbol{x}_k \end{aligned} \qquad (8)$$

If the NMSS system (5), is fully controllable for all k, then the stability of the closed-loop system (8) can be demonstrated using Liapunov's stability theorem. The vector of varying gains, $\boldsymbol{v}_k$, defined in (7), can also be expressed as:

$$\boldsymbol{v}_k = [l_{0,k} \;\; l_{1,k} \;\; \ldots \;\; l_{n-1,k} \;\; m_{1,k} \;\; \ldots \;\; m_{m-1,k} \;\; -k_{I,k}]$$

in which: $l_{i,k}$ are the feedback gains associated with the present and past output variables; $m_{i,k}$ are the feedback gains associated with the past input variables; and $k_{I,k}$ is the integral gain. Defining the feedback polynomials as:

$$L_k(z^{-1}) = l_{0,k} + l_{1,k}z^{-1} + \ldots + l_{n-1,k}z^{-(n-1)}$$

and

$$M_k(z^{-1}) = m_{1,k}z^{-1} + \ldots + m_{m-1,k}z^{-(m-1)}$$

then the general arrangement of the closed loop is as depicted in figure 2.

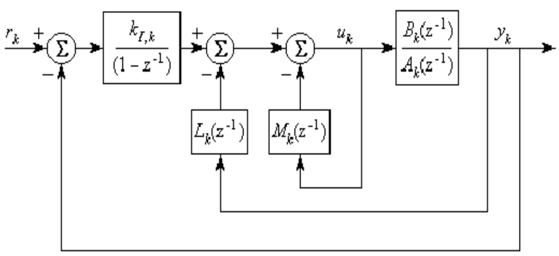

Figure 2: The Nonlinear SDP-PIP-controlled closed loop in feedback configuration.

3.3 *Illustrative Example Continued*

The deterministic part of the identified SDP model (4) can be written in the form:

$$\begin{aligned} x_k &= f(x_{k-1})x_{k-1} + g(u_{k-1})u_{k-1} \\ y_k &= x_k \end{aligned}$$

where,

$$f(x_{k-1}) = 1.994 - 1.988x_{k-1}; \quad \text{and}$$

$$g(u_{k-1}) = \frac{0.01\tanh u_{k-1} + 0.000188u_{k-1}}{u_{k-1}}$$

Then the NMSS description takes the form:

$$\begin{bmatrix} x_k \\ z_k \end{bmatrix} = \begin{bmatrix} f(x_{k-1}) & 0 \\ -f(x_{k-1}) & 1 \end{bmatrix}\begin{bmatrix} x_{k-1} \\ z_{k-1} \end{bmatrix} + \begin{bmatrix} g(u_{k-1}) \\ -g(u_{k-1}) \end{bmatrix}u_{k-1} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}r_k$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_k \\ z_k \end{bmatrix}$$

'Freezing' the NMSS system at the equilibrium point: $x_k = u_k = 0$ (k = 0, 1, 2, …), the solution to the ARE is:

$$\boldsymbol{P} = \begin{bmatrix} 40 & 0 \\ 0 & 6.8443 \end{bmatrix}, \text{ with } \boldsymbol{Q} = \begin{bmatrix} 40 & 0 \\ 0 & 1 \end{bmatrix}, \; R = 1\times10^{-9}$$

from which the vector of state-dependent gains is derived as:

$$\boldsymbol{v}_k = [v_{1,k} \quad v_{2,k}]$$

in which:

$$v_{1,k} = \left[\frac{46.8443f(x_k)g(u_k)}{(1\times10^{-9}) + 46.8443[g(u_k)]^2}\right]$$

$$v_{2,k} = \left[\frac{-6.8443g(u_k)}{(1\times10^{-9}) + 46.8443[g(u_k)]^2}\right]$$

The SDP-PIP controller is simulated by applying it to the control of the *actual* system (4), initially without the additive noise, so evaluating the system performance in the presence of the parameter estimation errors.

The responses of closed-loop system to a repeated step reference input are shown in figure (3). The output displays a small amount of overshoot as transient effects are overcome, but settles into a excellent tracking response by the third change in the reference input. Note that the control input response is negative where the output response is positive, and vice versa. This is due to the existence of a second equilibrium point in the original nonlinear system at the point: $x_k = 0.5$ (k = 0, 1, 2, …): this requires the control input to be negative for $0 \le x_k \le 0.5$ and positive elsewhere. If the additive noise e_k in (4) is introduced, the response shown in figure 3 is not affected very much: there are just additional small, noise-induced perturbations about the output response.

The variation in the controller gains is shown in figure 4. Note the large changes that occur in the gains during rapid variation of the states, compared to the constant values obtained when steady-state conditions arise. Of course, the gains are constantly variable for acontinuously perturbing command input, such as a sinusoid or random signal (see below).

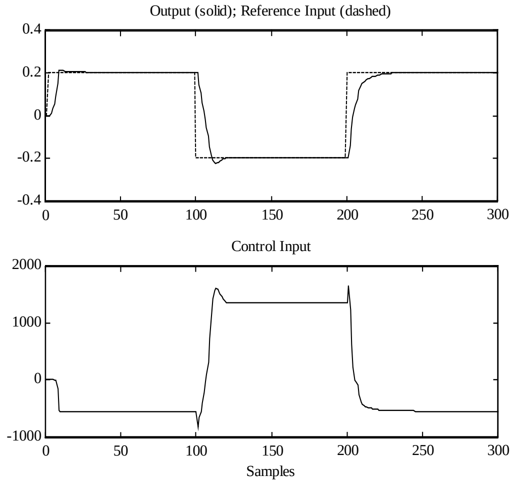

Figure 3: SDP-PIP-controlled closed loop responses.

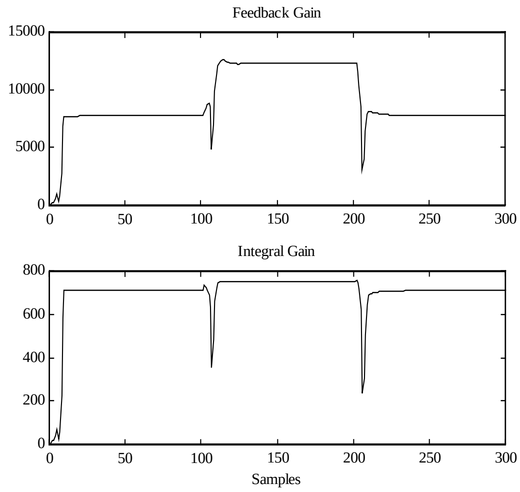

Figure 4: SDP controller gains.

As a final example, the SDP-PIP system was re-designed to provide closed loop, dead-beat response. This is quite a challenge in the case of this nonlinear system because of the required rapid response of the system. Even so, as shown in figure 5, the type-one SDP-PIP controlled system is able to track a sinusoidal reference input with the one sample delay required for dead-beat response in this case. The nonlinearity of the system is obvious from the nature of the control input.

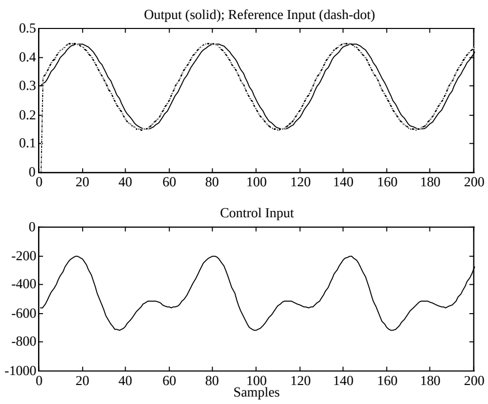

Figure 5: SDP-PIP dead-beat control.

4. CONCLUSIONS

The *State Dependent Parameter* (SDP) transfer function model is able to represent a wide range of nonlinear stochastic, dynamic systems and yet it retains some of the advantages of its constant or slowly time variable parameter progenitors. As such, it opens up the possibility of a whole new range of SDP-based control and estimation theory developments. In the present paper, we have addressed just one of these: the development of a nonlinear version of the *Proportional-Integral-Plus* (PIP) controller and shown how it can be used to control a typical nonlinear system with both internal (state) and input nonlinearities. By its explicit incorporation of state-dependent parameters, this SDP approach unifies and extends earlier feedback linearization and linear parameter varying ideas in a fundamental and very useful manner.

Moreover, the results obtained in this paper suggest that other SDP-based control systems are worthy of investigation, such as SDP versions of delta operator and robust PIP systems, as well as SDP versions of other more automatic control procedures, from conventional three term PID to H_∞ control.

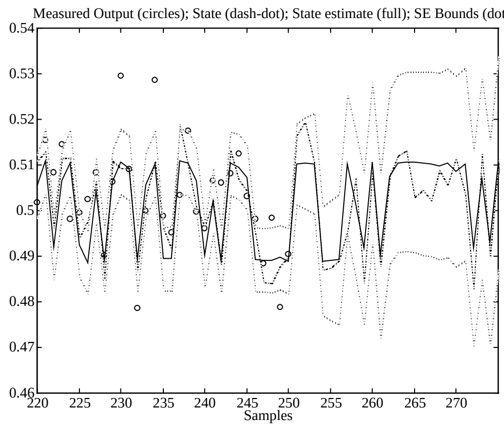

Figure 6 SDP-KF estimation and multi-step-ahead forecasting

It also seems possible that conventional state estimation (Kalman Filter: KF) theory can be extended by incorporating SDP models. The KF is inherently a nonstationary form of estimation theory and it can accommodate time variable parameters in the model state equations, as well as associated time variable hyper-parameters (i.e. noise variance-covariance parameters). As a result, it is possible to incorporate SDPs into the KF in a similar manner (see Young, 2000). Figure 6 shows the results of such an SDP-KF approach in the case of the nonlinear model (4) but with, $\sigma^2 = 1\text{x}10^{-4}$ and added system noise with $\sigma_x^2 = 2.5\text{x}10^{-5}$. Here, the one-step-ahead predictions (full line) are compared with measured output (circular points) and the noise-free state (dashed-dot line) up to sample 250 and, thereafter, the SDP-KF generates a very good, true multi-step-ahead forecast up to sample 275. The standard error bounds are shown dotted.

As in SDP-PIP control, the SDP-KF will not be globally applicable and its confident use in practical terms will require considerable further research. If successful, however, it could provide a method of state estimation for a certain class of nonlinear systems that does not require re-linearization. Like the re-linearized KF, however, such a SDP-KF could only provide information on the first two statistical moments (mean and covariance) associated with the state estimation errors, even though the SDP model is capable of producing state behaviour with highly non-normal probability distributions.

Finally, SDP models can also be developed for continuous-time systems (Young, 1993, 1998) and so it is clearly possible to extend the continuous-time linear PIP controller (Chotai *et al*., 1994; Chotai *et al*., 1998) into a nonlinear SDP-PIP form. Research on this and other SDP related systems and time series analysis techniques is proceeding at Lancaster.

5. ACKNOWLEDGEMENT

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