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ON STRUCTURAL IDENTIFIABILITY OF SYSTEM PARAMETERS OF LINEAR MODELS

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Abstract: Investigation of structural identifiability of even linear dynamical models requires complicated algebraic manipulations with use of computer algebra methods. In present work very efficient approach to testing both local and global identifiability of system parameters is offered which does not require too complex algebraic computations and therefore allows to increase the dimension of testable models. In addition to consideration of general case two widely used classes of model structures are studied. Illustrative example is given to show advantages of the approach.

Keywords: Parameter identification, identifiability, linear systems, parametrization

1. INTRODUCTION

The first step of parameter identification procedure is formulating the model structure. As a result we obtain a set of models $M = \{M(\theta) : \theta \in \Omega \subset R^p\}$ parametrized with vector of unknown parameters θ from admissible parametric space Ω, see (Ljung, 1987). The next step is determining "the best" model in the set M. On this step the situation may take place when there are several such "best" models equally well describing the process under study. In other words the problem of estimating the unknown parameters of the model has non-unique solution. If that's the case the model structure is called nonidentifiable.

Let us give stronger definitions of structural identifiability borrowed from (Walter and Pronzato, 1990). Denote the equality of the model input/output maps obtained for two values θ and θ^* of the parameter vector by $M(\theta) \approx M(\theta^*)$. This property is also called indistinguishability of the models from input/output observations. The parameter θ_i is called structurally globally identifiable (s.g.i.) if for almost any $\theta^* \in \Omega$ (with the exception of subsets of Ω of measure zero)

$$M(\theta) \approx M(\theta^*) \Rightarrow \theta_i = \theta_i^*;$$

it is structurally locally identifiable (s.l.i.) if for almost any θ^* there exists a neighbourhood $v(\theta^*)$ such that if $\theta \in v(\theta^*)$, then

$$M(\theta) \approx M(\theta^*) \Rightarrow \theta_i = \theta_i^*.$$

A parameter that is not s.l.i. is structurally non-identifiable (s.n.i.). A model is s.g.i.(s.l.i.) if all its parameters are s.g.i.(s.l.i.). A model is s.n.i. if any of its parameters is s.n.i.

Substantial literature was devoted to the problem of structural identifiability of linear and nonlinear dynamical models during last three decades. The most important results were combined in book (Walter(Ed.), 1987). At present one could say about existence of sufficiently complete theory and methodology giving a variety of methods and algorithms for identifiability analysis. However, there is a certain difficulty in practical application of suggested methods. The point is that identifiability analysis is executed before parameter estimation (a priori) and independently of particular values of the parameters (structurally). Therefore all computations must be carried out in the symbolic (analytic) form. Program systems for symbolic computations are a good tool. However,

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to rely only on the computing power could be disappointing because most suggested algorithms generate very complicated computations resulted in the impossibility to obtain an answer even for models of low dimensions.

We propose an approach to testing structural local and global identifiability of linear dynamical models which does not generate such complicated computations. The approach is based on the conditions for identifiability analysis presented in (Glover and Willems, 1974), but it is not simply development of these conditions in the direction of practical application. First, we substantially reduce dimensions of matrices in the identifiability conditions, which is very significant for symbolic computations. Second, imposing some restrictions on the model structure we obtain identifiability conditions for two important classes of model structures. The more restrictions we impose, the easier conditions we have. In addition, for each class we propose necessary conditions (order conditions) being very simple for testing that can be very useful at the beginning of the identification procedure to reject a great number of non-identifiable model structures. Third, in the majority of papers the result of testing model structure identifiability is simply an answer: yes or no. But what to do in case of non-identifiability? Within the bounds of our approach possibility of obtaining additional information about non-identifiable model structure, which is useful in order to eliminate non-identifiability, is considered.

It is worth to mention of promising differential algebra approach (Ljung and Glad, 1994) and recently developed computer algebra algorithm (Audoly *et al.*, 1998). The first method is not particularly efficient for linear models. The second one has advantages mostly when applied to linear compartmental models. Moreover, implementation of the algorithm has been done by starting from numerical point θ. Approach presented in our paper allows to obtain the same results purely in the symbolic form and can be applied to any linear state space model.

2. RANK AND ORDER CONDITIONS FOR TESTING IDENTIFIABILITY

Consider the following model structure in state space form:

$$\begin{cases} \dot{x}(t) = A(\theta)x(t) + B(\theta)u(t), \\ y(t) = C(\theta)x(t) + D(\theta)u(t), \end{cases} \tag{1}$$

where t is time variable, $x \in R^n$, $u \in R^k$, $y \in R^m$ are state, control and observation vectors ($x(0) = 0$), $\theta \in \Omega$ is vector of unknown parameters of the model, $A \in R^{n\times n}, B \in R^{n\times k}, C \in R^{m\times n}, D \in R^{m\times k}$ are system matrices.

In this paper we consider natural parametrization when elements of system matrices are unknown parameters of the model. Such parameters are called system parameters. There are several reasons to choose system parameters. First, this allows us to decrease the dimensions of symbolic matrices in the identifiability conditions and to derive different conditions for several classes of model structures. Second, system parameters frequently have physical significance and their estimation is of interest for the appropriate specialist. Third, testing identifiability of system parameters can be an intermediate stage in making identifiable final parametrization of system matrices. We offer to divide one complex task of testing structural identifiability into two easier tasks: at first testing identifiability of system parameters, then special task of studying possibility of determining final parameters from identifiable system parameters. For deciding the first task we develop general approach presented in this paper.

Let $s^T = (\bar{A}^T, \bar{B}^T, \bar{C}^T, \bar{D}^T)$ - vector composed of all elements of system matrices row by row - be the vector of system parameters. As it will be clear later, to achieve identifiability of model structure we have to impose constraints on s. We propose the following form of presenting constraints convenient for further consideration

$$\Gamma s = \Gamma_0, \tag{2}$$

where Γ and Γ_0 are numerical matrix and vector of dimensions $r \times N$ and $r \times 1$, $N = n(n + m + k) + mk$. Let $rank\Gamma = r, r < N$, then r components of vector s depend on other elements. Let in (1) vector θ of unknown parameters be vector of dimension $(N - r)$ consisting of independent system parameters. Note that choice of independent elements from vector s is not unique.

Our method is based on similarity transformation approach for linear models. In accord with this approach we search for the set of all state-space models with the same input/output map as (1):

$$\begin{cases} \dot{x}^*(t) = A^*x^*(t) + B^*u(t), \\ y(t) = C^*x^*(t) + D^*u(t), \end{cases} \tag{3}$$

and the same model structure (the same structural constraints)

$$\Gamma s^* = \Gamma_0, \tag{4}$$

where $s^* = (\bar{A^*}^T, \bar{B^*}^T, \bar{C^*}^T, \bar{D^*}^T)^T$ is vector of system parameters for the model (3). From the system (4) we also can determine vector θ^* of independent system parameters.

It is known that under assumption of controllability and observability of models (1) and (3) the state vectors x and x^* are related by nonsingular similarity transformation $x^* = Tx$, where transformation matrix $T \in GL(n) = \{T : det\,T \neq 0\}$ is determined by the following system:

$$\begin{cases} A^*T - TA(\theta) = 0, \\ B^* - TB(\theta) = 0, \\ C^*T - C(\theta) = 0, \\ D^* - D(\theta) = 0, \\ \Gamma(s^* - s(\theta)) = 0, \end{cases} \quad (5)$$

where dependence on θ means that all elements of vector s are expressed through its independent elements. If the system (5) has only one solution $(T = I, s^* = s)$ (there are no models indistinguishable with the initial model), the model is s.g.i. If we have continuous set of solutions to the system, the model is s.n.i. Otherwise, if several solutions are isolated points in the parametric space, the model is s.l.i. Note, that if there are no constraints on system parameters, the last equation is removed from the system (5). In this case the model is s.n.i., because for each nonsingular T there is a solution for s^*.

Write down the system (5) in general form:

$$F(T, s^*; \theta) = 0. \quad (6)$$

The system (6) gives transformation $F : GL(n) \times R^N \times R^{N-r} \longrightarrow R^{N+r}$. It is clear that F is a C' function. Let us fix an arbitrary point θ in R^{N-r}. Note, that the point $(T = I, s^* = s)$ is solution to the equation (5). Consider Jacobian of the system (5) for variables (T, s^*):

$$F' = \left. \frac{\partial F(T, s^*; \theta)}{\partial (T, s^*)} \right|_{(T=I;\, s^*=s)}.$$

Suppose that all columns of matrix F' are linearly independent, i.e.

$$rank\,(F') = rank\,(F'_T \,|\, F'_{s^*}) = n^2 + N, \quad (7)$$

where F'_T and F'_{s^*} are $(N+r)\times n^2$ and $(N+r)\times N$ submatrices of matrix F'. Then by the implicit function theorem there are neighbourhoods $N_1 \subset R^{N-r}$, of point θ and $N_2 \subset GL(n) \times R^N$ of point (I, s), and also single-valued transformation $N_1 \longrightarrow N_2$. Therefore at point θ there is locally unique solution of the equation (6).

To obtain expression for F' we present the system (5) in the following form:

$$\begin{cases} X^*(s^*, \theta)(\bar{T} - \bar{I}_n) + (s^* - s(\theta)) = 0, \\ \Gamma(s^* - s(\theta)) = 0, \end{cases} \quad (8)$$

where $\bar{T}$ and $\bar{I}_n$ are vectors obtained from elements of appropriate matrices row by row,

$$X^*(s^*, \theta) = \begin{bmatrix} A^* \otimes I_n - I_n \otimes A^T(\theta) \\ -I_n \otimes B^T(\theta) \\ C^* \otimes I_n \\ 0 \end{bmatrix}.$$

Differentiating the system (8) with respect to T and s^* and evaluating the result at point $(T = I, s^* = s)$ we obtain

$$F' = (F'_T | F'_{s^*}) = \begin{bmatrix} X(\theta) & | & I_N \\ 0 & | & \Gamma \end{bmatrix},$$

where

$$X(\theta) = X^*(s(\theta), \theta) = \begin{bmatrix} A(\theta) \otimes I_n - I_n \otimes A^T(\theta) \\ -I_n \otimes B^T(\theta) \\ C(\theta) \otimes I_n \\ 0 \end{bmatrix}.$$

Introduce matrix $\tilde{F}'$:

$$\tilde{F}' = F' \cdot \begin{bmatrix} I_{n^2} & 0 \\ -X(\theta) & I_N \end{bmatrix} = \begin{bmatrix} 0 & I_N \\ -\Gamma X(\theta) & \Gamma \end{bmatrix}.$$

It is evident that

$$rank(F') = rank(\tilde{F}') = N + rank(\Gamma X(\theta)).$$

Now we can formulate our main result.

Theorem 1. (rank condition for s.l.i.).
Necessary and sufficient condition for the model (1) with constraints on model structure (2) to be structurally locally identifiable under assumption of controllability and observability is that for almost all θ excluding sets of measure zero

$$rank\,\Gamma X(\theta) \quad = \quad n^2. \quad (9)$$

Corollary 2. (order condition).
Necessary condition for model structure defined in theorem 1 to be s.l.i. is

$$r \quad \geq \quad n^2.$$

Remark 3. Note, that the result of testing the condition (9) does not depend on choice of independent components from vector of system parameters. Moreover, if we choose arbitrary set of estimated parameters (not system one) connected with set of independent system parameters through one-to-one relation, the results of s.l.i. analysis will be the same (of course, it doesn't concern identifiability of individual parameters).

Remark 4. From the forth equation of system (5) it is clear that all elements of system matrix D are always s.g.i. as distinct from other system matrices. So, as also seen from the structure of matrix $X(\theta)$, restrictions on the elements of matrix D do not influence on the result of analysis.

Therefore in the identifiability analysis one can consider the case $D = 0$ for simplicity.

In case the condition (9) does not satisfied, the model structure is s.n.i. But along with non-identifiable parameters s.n.i. model can contain s.l.i. and even s.g.i. parameters. In (Avdeenko and Je, 2000) we presented development of the approach intended for studying identifiability of each parameter considered separately. If the condition (9) (and hence the condition (7)) is not satisfied, there exist matrices $L(\theta)$ and $\Lambda(\theta)$ composed of null space vectors for the matrix F':

$$F'_T \cdot L(\theta) + F'_{s^*} \cdot \Lambda(\theta) = 0. \qquad (10)$$

Rows of these matrices correspond to the elements of system vector s. If matrix $\Lambda(\theta)$ has zero row, appropriate system parameter is s.l.i. Moreover, this matrix can help to change model structure so as to eliminate nonidentifiability. In (Gorsky, 1993) method of computation of estimable parametric functions on the basis of matrix $\Lambda(\theta)$ is given. Such functions can be chosen as new parameters of the locally identifiable model structure.

To find $\Lambda(\theta)$ we use relation between matrices $\tilde{F}'$ and F'

$$\tilde{F}' \cdot \begin{bmatrix} \tilde{L} \\ \tilde{\Lambda} \end{bmatrix} = F' \cdot \begin{bmatrix} \tilde{L} \\ -X\tilde{L} + \tilde{\Lambda} \end{bmatrix} = F' \cdot \begin{bmatrix} L \\ \Lambda \end{bmatrix} = 0.$$

From the structure of matrix $\tilde{F}'$ one can see that $\tilde{\Lambda}(\theta) = 0$. Thus, matrix $\Lambda(\theta)$ is determined from

$$\Lambda(\theta) = -X(\theta)\tilde{L}, \quad \Gamma X(\theta)\tilde{L} = 0.$$

Now turn to the question of global identifiability. Premultiplying the first equation in (8) by Γ and taking into account the second equation, we have

$$\Gamma X^*(s^*, \theta)(\bar{T} - \bar{I}_n) = 0. \qquad (11)$$

It is evident that if matrix $\Gamma X^*(s^*, \theta)$ has full column rank, then $T = I_n$ and from the system (8) $s^* = s$, i.e. the model structure is s.g.i. Now we can formulate sufficient condition for global identifiability.

Proposition 5. (rank condition for s.g.i.).
Sufficient condition for model structure defined in theorem 1 to be structurally globally identifiable is that the equality

$$rank\, \Gamma X^*(s^*(\theta^*), \theta) \quad = \quad n^2, \qquad (12)$$

be not satisfied solely for θ or θ^* belonging to subsets of measure zero in the parametric space.

The following corollary gives constructive (but not very efficient) method for practical application.

Corollary 6.
Sufficient condition for model structure defined in theorem 1 and satisfying rank condition (9) to be s.g.i. is that factorization

$$\det[(\Gamma X^*)^T \Gamma X^*] = \\ = \prod_i e_i(\theta) \prod_j g_j(\theta^*) \prod_l h_l(\theta, \theta^*)$$

do not contain multipliers of the form $h_l(\theta, \theta^*)$.

Proposition 5 gives only sufficient condition for s.g.i. On the basis of this condition we have developed efficient method for full analysis based on checking compatibility of the equalities $h_l(\theta, \theta^*) = 0$ with the initial system of equations.The approach allows to decrease the degree of polynomials in the initial system of equations (5).

3. IDENTIFIABILITY CONDITIONS FOR TWO CLASSES OF MODEL STRUCTURES

In this section we apply identifiability conditions obtained in previous section for two frequently used classes of model structures and develop easier test for them. First, suppose that $D = 0$ (see Remark 4), i.e. the dimension of vector of system parameters is reduced.

3.1 *First class of model structures*

Consider model structure in which control and observation matrices are numerical and do not depend on the unknown parameters. In this case $B^* = B$, $C^* = C$ and

$$\Gamma = \begin{bmatrix} \psi & 0 & 0 \\ 0 & I_{nk} & 0 \\ 0 & 0 & I_{nm} \end{bmatrix},$$

Matrix ΓX in condition (9) has the form:

$$\Gamma X(\theta) = \begin{bmatrix} \psi[A(\theta) \otimes I_n - I_n \otimes A^T(\theta)] \\ -I_n \otimes B^T(\theta) \\ C(\theta) \otimes I_n \end{bmatrix}. \qquad (13)$$

Let U and V be upper triangular matrices of transformation of matrices B^T and C to the column echelon forms $\tilde{B}^T$ and $\tilde{C}$:

$$\tilde{B}^T = B^T U; \quad \tilde{C} = CV. \qquad (14)$$

Assuming without loss of generality that $rank\, B = k$ and $rank\, C = m$, matrix $\tilde{B}^T$ has $(n - k)$ columns consisting of zeros, and matrix $\tilde{C}$ has $(n - m)$ such columns. Let the numbers of zero columns of $\tilde{B}^T$ and $\tilde{C}$ form sets J^1 and J^2. Define $\tilde{U} = U(J^1)$, $\tilde{V} = V(J^2)$ - submatrices of matrices

U and V consisting of columns with numbers from J^1 and J^2.

Multiply matrix $\Gamma X(\theta)$ in (13) by nonsingular matrix $V \otimes U$. As a result we obtain

$$\Gamma X(V \otimes U) = \begin{bmatrix} \psi[AV \otimes U - V \otimes A^T U] \\ -V \otimes B^T U \\ CV \otimes U \end{bmatrix}.$$

It is not difficult to check that matrix $\Gamma X(V \otimes U)$ has full column rank if and only if its submatrix $\psi[A\tilde{V} \otimes \tilde{U} - \tilde{V} \otimes A^T \tilde{U}]$ of less dimension $r_A \times (n-k)(n-m)$ has full column rank. Taking into account that $rank(\Gamma X) = rank(\Gamma X(V \otimes U))$, we can formulate the following proposition.

Proposition 7. (rank condition for s.l.i.).
Let in (1) $D = 0$, B and C be arbitrary numerical matrices, vector of system parameters $s = \bar{A}$, r_A independent constraints

$$\psi \bar{A} = \psi_0, \tag{15}$$

be imposed on A. Necessary and sufficient condition for such model to be s.l.i. under assumption of controllability and observability is that for almost all θ excluding sets of measure zero

$$rank\ \psi \tilde{X}(\theta) \quad = \quad (n-k)(n-m), \tag{16}$$

where $\tilde{X}(\theta) = A(\theta)\tilde{V} \otimes \tilde{U} - \tilde{V} \otimes A(\theta)^T \tilde{U}$.

Corollary 8. (order condition).
Necessary condition for model structure defined in the proposition 7 to be s.l.i. is that

$$r_A \quad \geq \quad (n-k)(n-m).$$

Note, that result of corollary 8 was first derived in paper (Delforge, 1980) in a more intricate way with the help of modal matrix approach.

If the model is s.n.i. we can investigate identifiability of individual parameters analyzing matrix $\Lambda = -\tilde{X}(\theta)\hat{L}$, where $\hat{L}$ is determined from $\psi \tilde{X}(\theta)\hat{L} = 0$, see also (Avdeenko and Je, 2000).

3.2 *Second class of model structures*

Let each column of B and each row of C contain only one nonzero element. Renumbering components of the state vector, such matrices can be presented in the form:

$$B^T = \left[\, B_1 \mid 0 \,\right], \quad C = \begin{bmatrix} C_1 & 0 & | & 0 & 0 \\ 0 & 0 & | & C_2 & 0 \end{bmatrix}, \tag{17}$$

where $B_1 = diag\{b_1, \cdots, b_k\}, C_1 = diag\{c_1, \cdots, c_q\}$, $C_2 = diag\{c_{q+1}, \cdots, c_m\}, q <= k$. Denote $\tilde{J}_1 = \{k+1, \cdots, n\}$ and $\tilde{J}_2 = \{q+1, \cdots, k, k+m-q+1, \cdots, n\}$ – sets consisting of numbers of zero columns of matrices B^T and C.

Let $b = (b_1, \cdots, b_k)^T$, $c = (c_1, \cdots, c_m)^T$ be vectors consisting of nonzero elements of B and C. Let constraints on A be presented in form (15). By analogy constraints on the nonzero elements of B and C are presented as follows:

$$\phi b = \phi_0, \quad \xi c = \xi_0. \tag{18}$$

Renumbering elements of system vector such that nonzero elements of matrices B and C stand before their zero elements, matrix of constraints Γ can be transformed to the form:

$$\begin{bmatrix} \psi & 0 & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & I_{(n-1)k} & 0 \\ 0 & 0 & 0 & 0 & I_{(n-1)m} \end{bmatrix}.$$

It is not difficult to check that matrix ΓX for that class of models has full column rank if and only if its submatrix of less dimension $r_{Abc} \times [(n-k)(n-m) + (k+m-q)]$ ($r_{Abc} = r_A + r_b + r_c$ is total number of constraints on A,b and c). Form of the latter matrix is given in the following proposition.

Proposition 9. (rank condition for s.l.i.).
Let in (1) $D = 0$, B and C be of the form (17), constraints (15), (18) be imposed on A and nonzero elements of B and C, vector of system parameters $s^T = [\bar{A}^T, b^T, c^T]$. Necessary and sufficient condition for such model to be s.l.i. under assumption of controllability and observability is that for almost all θ excluding sets of measure zero

$$rank\ \gamma\chi(\theta) = (n-k)(n-m) + (k+m-q),$$

where $\gamma = \begin{bmatrix} \psi & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & \xi \end{bmatrix}$, $\chi(\theta) = \begin{bmatrix} \alpha(\tilde{J}_1, \tilde{J}_2) & \breve{\alpha} \\ 0 & -\breve{B}^T \\ 0 & \breve{C} \end{bmatrix}$,
$\alpha(\tilde{J}_1, \tilde{J}_2) = A(\theta)I_n(\tilde{J}_1) \otimes I_n(\tilde{J}_2) - I_n(\tilde{J}_1) \otimes A^T I_n(J_2)$ ($I_n(\tilde{J}_1)$ and $I_n(\tilde{J}_2)$ are submatrices of the identity matrix I_n containing columns with numbers from $\tilde{J}_1$ and $\tilde{J}_2$), $\breve{\alpha} = [\breve{\alpha}_1 | \cdots | \breve{\alpha}_{k+m-q}]$, $\breve{\alpha}_i = A_i(\theta) \otimes I_i - I_i \otimes A_i^T$ (A_i, I_i and A_i^T are the i-th columns of matrices A, I_n and A^T), $\breve{B}^T$ and $\breve{C}$ are submatrices of matrices B^T and C consisting of the first $(k+m-q)$ columns, θ is vector of independent system parameters of the dimension $(n^2 + k + m - r_{Abc})$

Corollary 10. (order condition).
Necessary condition for the model structure defined in the proposition 9 to be s.l.i. is

$$r_{Abc} \geq (n-k)(n-m) + (k+m-q).$$

If the model is s.n.i. we can investigate identifiability of individual parameters analyzing matrix $\Lambda = -\chi(\theta)\check{L}$, $\check{L}$ is determined from the equation $\gamma\chi(\theta)\check{L} = 0$.

Sufficient conditions for testing global identifiability for two considered classes of model structures are obtained from condition (12) perfectly in the same way as conditions for local identifiability.

4. EXAMPLE

To illustrate advantages of proposed conditions we consider an example being slight modification of case study presented in (Audoly *et al.*, 1998). Note that our approach permits to test models of much larger dimensions, but we can not present them here because of lack of space.

Let model structure be determined by the following numerical control and observation matrices and six constraints on matrix A:

$$B^T = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$a_{13} = a_{31} = a_{24} = a_{42} = \sum_{i=1}^{4} a_{i1} = \sum_{i=1}^{4} a_{i2} = 0.$$

Thus, we have 10 independent system parameters $\theta = (a_{12}, a_{14}, a_{21}, a_{23}, a_{32}, a_{33}, a_{34}, a_{41}, a_{43}, a_{44})^T$.

To test identifiability of the model structure we compute the following matrix $\psi\tilde{X}^*(\theta^*, \theta) =$

$$\begin{pmatrix} 0 & a_{14}^* & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{44}^* + a_{12} + a_{32} & -a_{32} & 0 \\ -a_{21} & 0 & -a_{41} \\ a_{14}^* + a_{34}^* + a_{44}^* + a_{12} + a_{32} & -a_{32} & 0 \end{pmatrix}.$$

To test s.l.i. we use proposition 7 and check condition $rank\psi\tilde{X}(\theta) = (n-k)(n-m) = 3$, where $\tilde{X}(\theta) = \tilde{X}^*(\theta, \theta)$. It is clear from column echelon form of this matrix that rank condition for s.l.i. is valid. To test s.g.i. we have to determine sets in the parametric space for which condition $rank\psi\tilde{X}^*(\theta^*, \theta) = (n-k)(n-m) = 3$ is not valid. One can see that it is not valid only if $a_{41} = 0$ or if $a_{14}^* = a_{34}^* = 0$, i.e. on sets of measure zero in the parametric space. Therefore we conclude that the model structure is s.g.i.

It is interesting that with use of our approach we can foresee consequences of imposing some additional constraints on model structure. For our example it is seen from the form of matrix $\psi\tilde{X}(\theta)$ that adding constraint $a_{41} = 0$ may lead to non-identifiable model structure. Indeed, one can check that matrix $\psi\tilde{X}(\theta)$ in that case (of dimension 7×3) has zero column. Thus, resulting model structure is s.n.i.

5. CONCLUSION

In this paper we have presented conditions for testing local and global identifiability of system parameters of linear state space models. The conditions proved to be much simpler for symbolic computation than those generated by other methods. To test rank condition for local identifiability one can apply built-in procedures of rank computation accessible in most computer algebra packages. As for testing global identifiability we can not simply apply built-in procedure. On the basis of sufficient condition for s.g.i. we develop an algorithm being combination of Gaussian method of exclusion with Buchberger algorithm for evaluation of Grobner basis. The method is implemented in MAPLE V and ensures fulfillment of both necessary and sufficient conditions for global identifiability.

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