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STABILITY OF FEEDBACK ERROR LEARNING METHOD FOR GENERAL PLANTS WITH TIME DELAY

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Abstract: Feedback error learning method was recently proposed by Kawato et.al. (M. Kawato and Suzuki, 1987) as a possible architecture of brain motor control which is supported by experimental results in neurophysiology. In this paper, we analyze it as a two-degree-of-freedom adaptive control for general time invariant linear plant with adaptive controller in the feedforward path. A time delay is allowed in the feedback loop as in the neuronal pathways of motor control. We derive stability condition of the feedback error learning method based on the strict positiveness of the closed loop system. The control performance of the feedback error learning method as a design strategy of adaptive control has been demonstrated by simulation results.

Keywords: time delay; motor control; adaptive control; feedforward systems; inverse system

1. INTRODUCTION

Feedback error learning method (FEL) was proposed by Kawato et. al. (M. Kawato and Suzuki, 1987) as a cerebellum model of motor control. This control scheme is considered as an adaptive version of two-degree-of-freedom control scheme with adaptive capability in the feedforward controller (Fig.1). The stability of FEL algorithm was discussed in (Miyamura and Kimura, 2000)(Miyamura, 2001) for delay free cases. In this paper, we extend the results to plants which have large time delay in feedback loop. Actually, the existence of large time delay is a salient feature of neuronal pathways and it is exactly the reason why the brain adopts feedforward controllers in addition to feedback.

In this paper, we set up the problem of FEL method with time delay in the simplest framework of linear time-invariant systems and give a stability condition of FEL algorithm with time delay based on the positive realness of the closed loop system. The main purpose is to establish control theoretical foundation of the FEL which is now a central focus of brain motor control. Another important point of this paper which was not dealt with in Kawato's work and dealt with in this paper is the problem of non-invertibility of plant, which is the big nuisance for adaptive control.

2. FEL WITH TIME DELAY

2.1 Problem Formulation

Fig.1 illustrates the feedback error learning architecture with time delay in the feedback loop. The objective of control is to minimize the error $e = r - y$ where r is the command signal and y is the plant output. The input u to the plant P is composed of the output u_{ff} of feedforward controller $Q(\theta)$ which contains tunable parameters θ, and u_{fb} of feedback controller K_1. If we disregard the learning part of the architecture, this is a typical two-degree-of-freedom control system. If P is known and P^{-1} exists and stable, choosing $Q = P^{-1}$

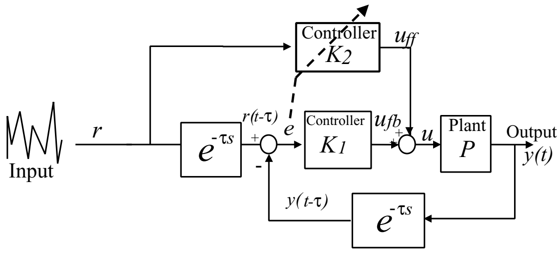

Fig. 1. Feedback Error Learning Scheme with Time Delay

makes the tracking perfect. Indeed, from the relations $u_{ff} = P^{-1}r$, $u_{fb} = K_1(r-y)$ and $y = P(u_{ff}+u_{fb})$, we easily see that $y = r$.

The basic premise of FEL is that the cerebellum cortex acquires a model of outer world through learning while it is engaged in actual motor control (M. Kawato and Suzuki, 1987). This implies that the motor control is essentially a sort of adaptive control. The crucial factor, which has been neglected in formulating motor control as an adaptive control, is the signi£cant time delay caused by neurotransmittance and visual perception.

In this section we discuss FEL from the viewpoint of adaptive control. The feedforward controller K_2 is chosen to be identical to the inverse P^{-1} of P if P is known and P^{-1} exists. Since P is unknown, we must employ some adaptive scheme for K_2 so that K_2 converges to P^{-1}. Note that the time-delay $e^{-\tau s}$, which is identical to the time delay contained in the feedback loop, is introduced to the reference signal to produce the feedback error consistent with adaptation law. We £rst make the following assumptions:

(A1) The plant P is stable and has stable inverse P^{-1}.
(A2) The upper bound of the order of P is known.
(A3) The high frequency gain $k_0 = \lim_{s\to\infty} P(s)$ is assumed to be positive.
(A4) The time delay τ is known.

The assumption (A1) is rather restrictive in the context of control system design. This may be relaxed later.

If k_0 is negative in (A3), the subsequent results are valid by taking $-P(s)$ instead of $P(s)$. Hence, (A3) is relaxed to the assumption that *the sign* of the high frequency gain is known. For the sake of the simplicity of exposition, however, we retain (A3) for simplicity of exposition.

2.2 *Parameterization of unknown systems*

Now, we describe a method of adaptive construction of a desired K_2 under the assumption that τ is known a priori. Throughout this paper, we use the following parameterization of the unknown system K_2:

$$\frac{d\xi_1(t)}{dt} = F\xi_1(t) + gr(t), \tag{1}$$

$$\frac{d\xi_2(t)}{dt} = F\xi_2(t) + gu(t), \tag{2}$$

$$u_{ff}(t) = c(t)^T\xi_1(t) + d(t)^T\xi_2(t) + k(t)r(t), \tag{3}$$

where F is any stable matrix and g is any vector with $\{F, g\}$ being controllable. In (1)-(3), $c(t)$, $d(t)$ and $k(t)$ are unknown parameters to be estimated. This is a standard parameterization of adaptive controller used in [(Narendra and Valavani, 1989)]. Assume that the *true* system is written as

$$\frac{dz_1(t)}{dt} = Fz_1(t) + gr(t), \tag{4}$$

$$\frac{dz_2(t)}{dt} = Fz_2(t) + gu_d(t), \tag{5}$$

$$u_d(t) = c_0^T z_1(t) + d_0^T z_2(t) + k_0 r(t). \tag{6}$$

It is easy to see that taking $u(t) = u_d(t)$ and appropriate selection of parameters $c(t) = c_0$, $d(t) = d_0$ and $k(t) = k_0$ can yield an arbitrary transfer function from $r(t)$ to $u_{ff}(t)$. To see this, let the matrix F and vector g in a controllable canonical form

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ -f_1 & -f_2 & -f_3 & \cdots & -f_n \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

From (1), (2), and (3), the transfer function from $r(t)$ to $u_d(t)$ is given by

$$G_{u_d r} = \frac{k_0 + c_0^T(sI-F)^{-1}g}{1 - d_0^T(sI-F)^{-1}g} = \frac{k_0 s^n + (f_n k_0 + c_n)s^{n-1} + \ldots + (f_1 k_0 + c_1)}{s^n + (f_n - d_n)s^{n-1} + \ldots + (f_1 - d_1)},$$

$$c_0 = \begin{bmatrix} c_1 & c_2 & \ldots & c_n \end{bmatrix}^T, \quad d_0 = \begin{bmatrix} d_1 & d_2 & \ldots & d_n \end{bmatrix}^T.$$

Therefore, we can construct any transfer function of degree less than or equal to n by selecting parameters c_0, d_0 and k_0 appropriately.

2.3 *Adaptation Law*

In the ideal situation, K_2 is identical to P^{-1}. In that case, $e(t) = 0$, $u(t) = u_{ff}(t) = u_d(t) = P^{-1}(s)r(t)$. The true values c_0, d_0 and k_0 of $c(t)$, $d(t)$ and $k(t)$, respectively, satisfy

$$\frac{k_0 + c_0^T(sI-F)^{-1}g}{1 - d_0^T(sI-F)^{-1}g} = P^{-1}(s). \tag{7}$$

The error signal $e(t)$ is de£ned as

$$e(t-\tau) = r(t-\tau) - y(t-\tau).$$

The cost function is de£ned as

$$J(t) = \frac{1}{2}\int_0^t e^2(\sigma)d\sigma. \qquad (8)$$

Since the unknown parameters $c(t), d(t)$ and $k(t)$ must be updated so that the error signal $e(t)$ decreases, the adaptation law is given as,

$$\begin{aligned} \theta(t) &= \left[\, c(t)^T \;\; d(t)^T \;\; k(t)^T \,\right]^T, \\ \frac{d\theta(t)}{dt} &= -\alpha\frac{\partial J(t-\tau)}{\partial\theta} \\ &= -\alpha\frac{\partial e(t-\tau)}{\partial\theta}\cdot e(t-\tau). \qquad (9) \end{aligned}$$

We choose K_1 =const. Then, it follows that

$$e(t) = \frac{u(t) - u_{ff}(t)}{K_1}.$$

Using the approximation $u(t) \sim u_d(t)$, we have

$$\frac{d\theta}{dt} = \frac{\alpha}{K_1}\frac{\partial u_{ff}(t-\tau)}{\partial\theta}e(t-\tau). \qquad (10)$$

From (3),

$$u_{ff}(t) = \theta(t)^T\xi(t),$$

where $\xi(t) := \left[\, \xi_1(t)^T \;\; \xi_2(t)^T \;\; r(t) \,\right]^T$. Then, (10) can be written as,

$$\frac{d\theta}{dt} = \frac{\alpha}{K_1}e(t-\tau)\xi(t-\tau). \qquad (11)$$

We delayed the time of adaptation in accordance with the time-delay in the feedback loop, rather than taking the real time signal.

2.4 Convergence of algorithm

Now $u(t)$ and u_d are written as

$$\begin{aligned} u(t) &= u_d(t) - P^{-1}(s)e(t-\tau), \\ u_d(t) &:= P^{-1}(s)r(t). \end{aligned}$$

Then, if we de£ne $\Delta u(t) = u_{ff}(t) - u_d(t)$, we have

$$\begin{aligned} \Delta u(t) = {} & (c(t)-c_0)^T\xi_1(t) + (d(t)-d_0)^T\xi_2(t) \\ & - d_0^T(sI-F)^{-1}gP^{-1}(s)e(t-\tau). \qquad (12) \end{aligned}$$

Since F is stable, we use the asymptotic relations

$$\begin{aligned} & \xi_1(t) \to z_1(t), \\ & \xi_2(t) \to z_2(t) - d_0^T(sI-F)^{-1}gP^{-1}(s)e(t-\tau). \end{aligned}$$

The relation (12) is written as

$$\Delta u(t) = \psi(t)^T\xi(t) - d_0^T(sI-F)^{-1}gP^{-1}(s)e(t-\tau),$$

where

$$\psi(t) := \theta(t) - \theta_0 = \begin{bmatrix} c(t)-c_0 \\ d(t)-d_0 \\ k(t)-k_0 \end{bmatrix}. \qquad (13)$$

From the relations $u(t) = u_{ff}(t) + K_1 e(t-\tau)$, we have

$$e(t) = -(G(s) + K_1 e^{-\tau s})^{-1}\psi(t)^T\xi(t), \qquad (14)$$

where $G(s)$ is given by

$$G(s) = k_0 + c_0^T(sI-F)^{-1}g. \qquad (15)$$

On the other hand, from (11), we have

$$\frac{d\psi(t)}{dt} = \frac{d\theta(t)}{dt} = \alpha e(t-\tau)K_1(s)\xi(t-\tau). \qquad (16)$$

Then, combining the above relation with (14) yields

$$\frac{d\psi(t)}{dt} = -\alpha\xi(t-\tau)L_\tau(s)\xi(t-\tau)^T\psi(t-\tau), \qquad (17)$$

where

$$L_\tau(s) = K_1(G(s) + K_1 e^{-\tau s})^{-1}. \qquad (18)$$

Theorem 1: *Consider a delay-differential equation described as*

$$\frac{dz(t)}{dt} = -\alpha\xi(t)L(s)\xi(t)^T z(t-\tau). \qquad (19)$$

Assume that the following conditions hold:
(i) The differential equation

$$\frac{dz(t)}{dt} = -\alpha\xi(t)L(s)\xi(t)^T z(t) \qquad (20)$$

where $L(s) := \{A, b, c^T, d\}$ is stable;
(ii) $\xi(t)$ is bounded, i.e., there exists $M_0 > 0$ such that

$$\|\xi(t)\| \le M_0, \quad \forall t \ge 0.$$

Then, if $\alpha > 0$ satis£es following condition

$$\alpha < \frac{\lambda}{\tau M}\left(\frac{1}{dPR + (M_\delta/\delta)SPQ}\right)$$

the delay-differential equation (19) is asymptotically stable. Here, M_δ, δ and λ are

$$\begin{aligned} \|e^{At}\| &\le M_\delta e^{-\delta t} & M_\delta > 0, \delta > 0 \\ \|U(t,s)\| &\le M e^{-\lambda(t-s)} & M > 0, \lambda > 0 \end{aligned}$$

where $U(t,s)$ is transition matrix of (20),and

$$\begin{aligned} & P = \sup_t \left\| \begin{bmatrix} -b\xi^T(t) \\ \alpha d\xi(t)\xi(t)^T \end{bmatrix} \right\| \\ & Q = \sup_t \|\xi(t)c^T\| \quad R = \sup_t \|\xi(t)\xi^T(t)\| \\ & S = \sup_t \|b\xi^T(t)\| \end{aligned}$$

The proof of Theorem 1 can be given based on the arguments in (Halanay, 1966). For the assumption (i) in Theorem 1, we introduce Lemma 1 as follows,

Lemma 1 (B.D.O. Anderson, 1986)*: Let $L(s)$ be a strongly positive real transfer function and $\xi(t)$ be an arbitrary time-varying vector. Then, the solution $z(t)$ of the differential equation*

$$\frac{dz(t)}{dt} = -\xi(t)L(s)\xi(t)^T z(t) \tag{21}$$

tends to a constant vector z_0 such that $\xi(t)z_0 \to 0$. If $\xi(t)$ satis£es the so-called persistent excitation (PE) condition (Narendra and Valavani, 1989), the above z_0 is equal to 0.

From Lemma 1, we can use Theorem 1 to establish the stability of delay-differential systems (17), provided that $L_\tau(s)$ is s.p.r.

3. THE NON-INVERTIBLE CASE WITH TIME-DELAY

3.1 Parameterization of $K_2(s)$

In the previous section, we assumed that $P^{-1}(s)$ exists and is stable. This implies that the relative degree of $P(s)$ is zero and the zeros of $P(s)$ are all stable. Now, we relax the £rst condition and introduce an approximated inverse $\hat{P}^{-1}$ as $\hat{P}^{-1}(s) = P^{-1}(s)W(s)$, where $W(s)$ is a £lter with relative degree identical to that of $P(s)$. Using this approximation, the relative degree of $P(s)$ which is the cause of non-invertibility, is compensated by the relative degree of $W(s)$.

In this section we make, instead of (A1), the assumption (A1') and an additional assumption (A4):

(A1') All the £nite zeros of $P(s)$ are stable.
(A5) The upper bound of the relative degree of P is known.

We parameterize the feedforward controller $K_2(\theta)$ in the same way as the previous section ((1)-(3)). Assume that the plant $P(s)$ has relative degree k $(k \le n)$, so we can write $P(s)$ generally as

$$P(s) = \frac{b_k s^{n-k} + b_{k+1}s^{n-k-1} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n} \tag{22}$$

We select a pre£lter $W(s)$ with relative degree k as

$$W(s) = \frac{w_{k+1}}{s^k + w_1 s^{k-1} + \ldots + w_k}, \quad (w_i,\ i = 1, 2, \ldots k+1)$$

Assume that $P^{-1}(s)W(s)$ is represented as

$$\frac{d\xi_1(t)}{dt} = F\xi_1(t) + gr(t), \tag{23}$$

$$\frac{d\xi_2(t)}{dt} = F\xi_2(t) + gu_0(t), \tag{24}$$

$$u_0(t) = c_w^T \xi_1(t) + d_w^T \xi_2(t) + k_w r(t), \tag{25}$$

where F is given as before. Then, $P^{-1}(s)W(s)$ is written as,

$$\frac{k_w s^n + (f_n k_w + c_{w,n})s^{n-1} + \ldots + (f_1 k_w + c_{w,1})}{s^n + (f_n - d_{w,n})s^{n-1} + \ldots + (f_1 - d_{w,1})}, \tag{26}$$

where

$$c_w = \left[\, c_{w,1}\ c_{w,2}\ \ldots\ c_{w,n} \,\right]^T,$$
$$d = \left[\, d_{w,1}\ d_{w,2}\ \ldots\ d_{w,n} \,\right]^T.$$

Hence, c_w, d_w and k_w must satisfy the identity

$$\begin{aligned} &\frac{k_w s^n + (f_n k_w + c_{w,n})s^{n-1} + \ldots + (f_1 k_w + c_{w,1})}{s^n + (f_n - d_{w,n})s^{n-1} + \ldots + (f_1 - d_{w,1})} \\ &= \frac{s^n + a_1 s^{n-1} + \ldots + a_n}{b_1 s^{n-k} + b_2 s^{n-k-1} + \ldots + b_{n-k+1}} \\ &\quad \cdot \frac{w_{k+1}}{s^k + w_1 s^{k-1} + \ldots + w_k}. \end{aligned}$$

Let $f := \left[\, f_1\ f_2\ \ldots\ f_n \,\right]^T$. The above identity yields the relation

$$\begin{aligned} f - d_w = &\begin{bmatrix} w_k & 0 & \ldots & 0 \\ w_{k-1} & w_k & \ldots & 0 \\ \vdots & \vdots & \ldots & 0 \\ w_1 & w_2 & \ldots & 0 \\ 1 & w_1 & \ldots & w_k \\ 0 & 1 & \ldots & w_{k-1} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_{n-k+1}/b_1 \\ b_{n-k}/b_1 \\ \vdots \\ b_2/b_1 \end{bmatrix} \\ &+ \left[\, 0\ \ldots\ 0\ w_k\ \ldots\ w_2\ w_1 \,\right]^T \end{aligned} \tag{27}$$

Let $h_i, i = 0, \ldots, n-1$ be a sequence of solutions of a difference equation

$$h_i + h_{i-1}w_1 + h_{i-2}w_2 + \ldots + h_{i-k}w_k = 0. \tag{28}$$

Using (28), we have

$$\begin{aligned} &\left[\, h_0\ h_1\ h_2\ \ldots\ h_{n-1} \,\right] \cdot [f - d_w] \\ &= w_1 h_{n-1} + w_2 h_{n-2} + \ldots + w_k h_{n-k}. \end{aligned}$$

The difference equation (28) has k independent solutions $h_i^{(j)}$, $j = 1, \ldots, k$; $i = 0, \ldots, n-k$ as

$$\begin{cases} h_i^{(j)} = -w_1 h_{i-1}^{(j)} - w_2 h_{i-2}^{(j)} - \ldots - w_k h_{i-k}^{(j)}, i \ge k \\ h_i^{(j)} = \begin{cases} 0, & i \ne j-1 \quad i \le k-1, \\ 1, & i = j-1. \end{cases} \end{cases}$$

Hence, we obtain

$$
\begin{bmatrix} 1 & 0 & \dots & 0 & h_k^{(1)} & \dots & h_{n-1}^{(1)} \\ 0 & 1 & \ddots & 0 & h_k^{(2)} & \dots & h_{n-1}^{(2)} \\ \dots & \ddots & \ddots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 1 & h_k^{(k)} & \dots & h_{n-1}^{(k)} \end{bmatrix} [f-d]
$$
$$
= \begin{bmatrix} h_{n-k+1}^{(1)} & \cdots & h_{n-1}^{(1)} \\ h_{n-k+1}^{(2)} & \cdots & h_{n-1}^{(2)} \\ \cdots & \cdots & \cdots \\ h_{n-k+1}^{(k)} & \cdots & h_{n-1}^{(k)} \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \\ \vdots \\ 1 \end{bmatrix}. \qquad (29)
$$

Using this relation, we can represent $d_1,\ d_2,\ \cdots,\ d_k$ as af£ne functions of the rest $n-k$ parameters $d_{k+1},\ \cdots,\ d_n$. More precisely, we have af£ne relations

$$
\bar{d_w} = M\hat{d_w} + m
$$

where M is a known matrix, m is a known vector and

$$
\bar{d_w} = \left[\, d_{w,1}\ \ d_{w,2}\ \ \cdots\ \ d_{w,k} \,\right]^T
$$
$$
\hat{d_w} = \left[\, d_{w,k+1}\ \ d_{w,k+2}\ \ \cdots\ \ d_{w,n} \,\right]^T.
$$

The parameters $d_{w,1},\ d_{w,2},\ \cdots,\ d_{w,k}$ are determined once $d_{w,k+1},\ \cdots,\ d_{w,n}$ are given. Hence, it is suf£cient to estimate $n-k$ unknowns $\hat{d_w}$ for estimating d_w.

3.2 Adaptation Law

Using the result of the previous section, we construct an adaptation law. From Fig.2, the error signal $e(t)$ is de£ned as

$$
e(t-\tau) = W(s)r(t-\tau) - y(t-\tau).
$$

The unknown parameters $c(t), d(t), k(t)$ must be updated so that the error signal $e(t)$ decreases. Let

$$
\bar{d}(t) = \left[\, d_1\ \ d_2\ \ \dots\ \ d_k \,\right]^T = M\hat{d}(t) + m, \qquad (30)
$$
$$
\hat{d}(t) = \left[\, d_{k+1}\ \ d_{k+2}\ \ \dots\ \ d_n \,\right]^T,
$$
$$
\xi_2(t) = \left[\, \xi_{21}\ \ \xi_{22}\ \ \dots\ \ \xi_{2n} \,\right]^T,\ \bar{\xi_2}(t) = \left[\, \xi_{21}\ \ \xi_{22}\ \ \dots\ \ \xi_{2k} \,\right]^T,
$$
$$
\hat{\xi_2}(t) = \left[\, \xi_{2(k+1)}\ \ \xi_{2(k+2)}\ \ \dots\ \ \xi_{2n} \,\right]^T,
$$
$$
\hat{\theta}(t) = \left[\, c(t)^T\ \ \hat{d}(t)^T\ \ k(t) \,\right]^T,\ \hat{\theta}_w(t) = \left[\, c_w^T\ \ \hat{d}_d^T\ \ k_w \,\right]^T.
$$

Note that the dimension of the unknown vector $\hat{\theta}(t)$ is now $2n-k$ instead of $2n$ in the previous section. The output of $K_2(\hat{\theta})$ is written as

$$
\begin{aligned}
u_{ff}(t) &= c(t)^T\xi_1(t) + \bar{d}(t)^T\bar{\xi_2}(t) + \hat{d}(t)^T\hat{\xi_2}(t) + k(t)r(t) \\
&= c(t)^T\xi_1(t) + \hat{d}(t)^T(M^T\xi_2(t) + \hat{\xi_2}(t)) \\
&\quad + m^T\bar{\xi_2}(t) + k(t)r(t).
\end{aligned}
$$

As in the invertible case, we use the same adaptation law (10), which can be written as

$$
\frac{d\hat{\theta}(t)}{dt} = \alpha \begin{bmatrix} \xi_1(t-\tau) \\ M^T\bar{\xi_2}(t-\tau) + \hat{\xi_2}(t-\tau) \\ r(t-\tau) \end{bmatrix} K_1(s)e(t-\tau) \qquad (31)
$$

3.3 Stability Proof

As in the previous case, let

$$
\hat{\psi}(t) := \hat{\theta}(t) - \hat{\theta}_w. \qquad (32)
$$

be a vector of parameter errors. Differentiation with respect to t results in

$$
\frac{d\hat{\psi}(t)}{dt} = \frac{d\hat{\theta}(t)}{dt} = -\alpha\hat{\xi}(t-\tau)K_1(s)e(t-\tau), \qquad (33)
$$

where

$$
\hat{\xi}(t-\tau) := \begin{bmatrix} \xi_1(t-\tau) \\ M^T\bar{\xi_2}(t-\tau) + \hat{\xi_2}(t-\tau) \\ r(t-\tau) \end{bmatrix}.
$$

The equation (14) is written in this case as

$$
(G_1(s) + K_1(s)e^{-\tau s})e(t) = -\hat{\xi}(t)^T\hat{\psi}(t), \qquad (34)
$$

where $G_1(s) = (1 - c_w^T(sI-F)^{-1}g_w)P^{-1}(s)$. Due to (26), we have

$$
G_1(s) = (k_w + c_w^T(sI-F)^{-1}g)W(s)^{-1}. \qquad (35)
$$

Combining (35) with (31) yields

$$
\frac{d\hat{\psi}(t)}{dt} = -\alpha\hat{\xi}(t-\tau)L_{1\tau}(s)\hat{\xi}(t-\tau)^T\hat{\psi}(t-\tau), \qquad (36)
$$

where

$$
\begin{aligned}
L_{1\tau}(s) := K_1(s)W(s)(k_w + c_w^T(sI-F)^{-1}g \\
+ K_1(s)W(s)e^{-\tau s})^{-1}
\end{aligned} \qquad (37)
$$

The equation (36) is of the same form as (16), and we can use the same reasoning as in the previous section.

Theorem 2: *Under the assumptions (A1′), (A2)-(A5), the FEL scheme (30) and (31) is stable and $e(t)$ tends to 0, if $K_1(s)$ is chosen such that $L_{1\tau}(s)$ given by (37) is strictly positive real.*

Remark: In order that $L_1(s)$ in (37) is strictly positive real, $K_1(s)$ must contain higher derivatives so that the relative degree of $K_1(s)W(s)$ is not greater than two. This seems to be a drawback of Theorem 2 which depends on the notion of the strong positive realness. However, since the error signal $e(s)$ in Figure 2 is given by $e(s) = W(s)r(s) - P(s)u(s)$, and $W(s)^{-1}P(s)$ is proper, $W(s)^{-1}e(s) = r(s) - W(s)^{-1}P(s)u(s)$ is a proper function. This implies that $W(s)^{-1}e(s)$ can be generated if the state of the plant is available for feedback. Thus, for any proper $U(s)$, $u(s) = U(s)W(s)^{-1}e(s)$ can be constructed. Then, $K_1(s) = U(s)W(s)^{-1}$ satis£es the requirement. Hence, it is not dif£cult to implement $K_1(s)$ such that the relative degree of $K_1(s)W(s) = U(s)$ is not greater than two. Moreover, the invertible and delayed-free cases have already done in which case $L_{1\tau}(s)$ can be written in more simple form.

4. SIMULATION RESULT

We show a simulation result for a plant, $P(s) = (s+1)/(s^2+7s+12)$. This plant has no stable inverse. The McMillan degree of this system is 1. By parameterizing this system as written in Section 4, the feedforward controller K_2 is written as,

$$K_2 = \frac{ks^2+(5k+c_2)s+(2k+c_1)}{s^2+(5-d_2)s+(5-d_1)}. \quad (38)$$

On the other hand we write the unknown plant $P(s)$ as,

$$P(s) = \frac{b_1 s + b_2}{a_0 s^2 + a_1 s + a_2}, \quad (39)$$

Hence when $W(s) = 10/(s+10)$, the desired object of adaptation is written as,

$$P^{-1}(s)W(s) = \frac{10a_0 s^2 + 10a_1 s + 10a_2}{b_1 s^2 + (10b_1+b_2)s + 10b_2}. \quad (40)$$

From (38) and (40) we obtain the following constraint on parameter d_1.

$$d_1(t) = 55 + 10d_2(t) \quad (41)$$

So $d_1(t)$ is tuned by (41). Figure 2 shows the result of simulation with time delay 50 steps. Adaptation starts from 700 steps and after that the error signal converges to zero and the tracking performance is obviously better than before adaptation.

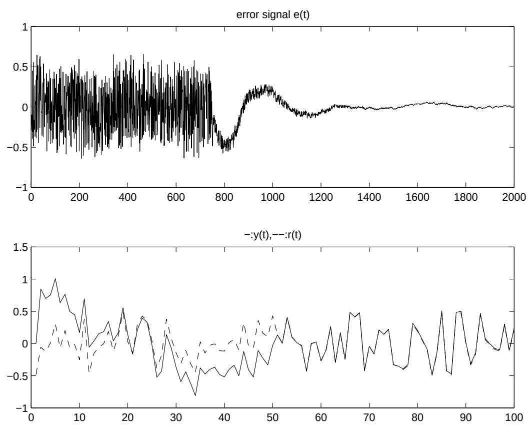

Fig. 2. A simulation result

5. CONCLUSION

In this paper FEL proposed as an architecture of brain motor control has been investigated from the viewpoint of two-degree-of-freedom adaptive control. One of the advantages of FEL is its ability to surmount the dif£culty of time delay. We have investigated its capability of compensating time delay through feedforward control. It is our intuition that adaptive control can overcome time delay by slowing down the speed of adaptation and it has been shown that the convergence of FEL for the plant with time delay by making the updating rate small. It may be interesting to exploit physiological ground of these equations, as well as to investigate mathematical properties of these equations in depth to obtain less conservative stability conditions of these equations.

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