

## TOWARD THE DESIGN AND IMPLEMENTATION OF A GLOBAL SLIDING-MODE CONTROL SCHEME WITH INPUT CONSTRAINT

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**Abstract:** A global sliding-mode control (GSMC) scheme with input constraint is presented in this article. The scheme offers a measure to weigh the contribution between GSMC and the nominal dynamics so that the chattering level and the maximal control effort can be reduced based on an on-line tuning of the weight on GSMC. A moving sliding function is adopted on the control design, the input can thus be confined within a predefined boundary during transient period, while a robust performance can be guaranteed in the steady state. The efficacy of this scheme is further validated via implementation on a linear variable reluctance motor (LVRM) servo system. Both simulation and experimental studies further demonstrate its feasibility and effectiveness.

**Keywords:** sliding-mode control, linear motor, servo control.

### 1. INTRODUCTION

Sliding-mode control (SMC) schemes have been successfully applied to various systems (Baja, 1993; Chiang *et al.* 1998; Cho *et al.* 1993; Lu and Chen 1995; Utkin, 1993) for the last decades. Sliding-mode control originating from a variable structure system is designed so that all trajectories in the state space are directed toward a set of predefined switching planes (Itkis, 1976). Once the system states are trapped on the switching planes (or manifolds), the system response is solely determined by the switching planes and is insensitive to bounded parameter uncertainties and bounded external disturbances. Theoretically, the switching planes can be chosen to achieve arbitrary rapid response, the physical bounds on control effort limit the bandwidth of the closed-loop systems, however. If the desired system response is excessive, the sliding-mode may fail and the reaching phase may be induced, then sliding behaviour cannot be ensured throughout the entire response (Hung *et al.* 1993). Moreover, the complete robustness of sliding-mode is achieved by a high-gain switching action; it will inevitably induce undesirable chatters and cause unfavourable effects, which would limit the bandwidth of closed-loop system response further. The global sliding-mode control scheme (Lu and Chen, 1995) and a scheme where adjustable robustness is considered (GSMCAR), previously presented in Lin *et al.* (2002). The scheme considered a linear time-invariant uncertain system and has been successfully applied to linear variable reluctance motor (LVRM) positioning system. The scheme offers sliding function together with the nominal system dynamics to weigh the contribution of GSMC so that the performance of the robustness in the sliding-mode can be adjusted while the chatter level and the control effort can be decreased

accordingly. By using this scheme, we can limit the control effort within a predefined region if certain tolerance and/or design windows are permitted. In this article, we extend the GSMCAR scheme to a tracking control system with hard bounds on the control action. The controller design ensures sliding-mode to exist throughout an entire response and the sliding behaviour can be assigned according to system specifications and bounds. It should be noted that the maximum control effort and the maximum chattering level usually occurs in the transient period. In order to consider the problems of input constraint and performance robustness simultaneously, a moving sliding function is developed. The basic ingredient of the moving sliding surface design is that the weight on GSMC is initially chosen to constrain the control input within certain physical limits and will be subsequently updated toward a designated sliding-mode regime if the input bounds are satisfied. By this scheme, an adjusted robustness can be achieved during transient period and the robust performance can also be guaranteed in the steady state.

### 2. PROBLEM STATEMENT

Consider the following linear time-invariant (LTI) uncertain system

$$x^{(n)} = \sum_{i=0}^{n-1} a_i x^{(i)} + bu + d \quad (1)$$

where  $x^{(i)}$  for  $i=0,1,2, \dots, n-1$  are the state variables and  $u$  is the control input. The coefficients  $b$  and  $a_i$  are the uncertain parameters of the system (1). Without the loss of generality, assuming that  $b \neq 0$  then the range of variation of each coefficient is listed as follows

$$\hat{b} - \Delta b < b < \hat{b} + \Delta b \quad (2)$$

$$\hat{a}_i - \Delta a_i < a_i < \hat{a}_i + \Delta a_i \quad \text{for } i=0,1,2,\dots,n-1$$

in which  $\hat{b}$ ,  $\hat{a}_i$  are nominal values, and  $\mathbf{D}b$ ,  $\mathbf{D}a_i$  are bounds of uncertainties of  $b$  and  $a_i$ , respectively. The term  $d$  represents external disturbance, of which the bounds is also assumed to be known as,

$$|d| \leq D \quad (3)$$

Furthermore, it is assumed that the following constraint of the control input must be satisfied

$$0 < |u(t)| \leq U_{\max} \quad (4)$$

Obviously, the nominal model of the system (1) can be represented as,

$$x_f = \sum_{i=0}^{n-1} \hat{a}_i x^{(i)} + \hat{b}u \quad (5)$$

Given a desired trajectory, the tracking error vector of the system (1) is defined as,

$$\mathbf{e} = [e, \dot{e}, \dots, e^{(n-1)}]^T \quad (6)$$

$$= [x - x_d, \dot{x} - \dot{x}_d, \dots, x^{(n-1)} - x_d^{(n-1)}]^T$$

in which  $x_d^{(i)}$  for  $i=1,2,\dots,n-1$  are reference signals.

Then we have

$$e^{(n)} = \sum_{i=0}^{n-1} a_i e^{(i)} + bu + d - x_d^{(n)} \quad (7)$$

In the global sliding-mode control (GSMC) design (Lu and Chen, 1995), if a desired characteristic equation of the closed-loop system is defined as

$$s^n + \sum_{i=0}^{n-1} c_i s^i = 0$$

$$\text{or } (s - I_1)(s - I_2)\dots(s - I_n) = 0 \quad (8)$$

where  $s$  is the Laplacian operator and  $I_i$  for  $i=1,2,\dots,n$  are the desired characteristic roots with negative real part, thus  $c_i$  can be determined accordingly. In other words, (8) leads to a desired asymptotically stable dynamics described by

$$e^{(n)} + \sum_{i=0}^{n-1} c_i e^{(i)} = 0 \quad (9)$$

In general, a switching surface can be chosen as

$$\mathbf{s} = e^{(n-1)} + \sum_{i=1}^{n-1} c_i e^{(i-1)} + c_0 \int_0^t e(\mathbf{t}) dt \quad (10)$$

Based on (1), if  $b$  does not change sign for  $t > 0$ , the sliding-mode control law can be derived from the sliding condition  $\mathbf{s}\dot{\mathbf{s}} < 0$  and expressed as

$$u_s = u_{eq} + \tilde{u} \quad (11)$$

$$u_{eq} = -\frac{1}{b} \left( \sum_{i=0}^{n-1} [\hat{a}_i x^{(i)} + c_i e^{(i)}] - x_d^{(n)} \right)$$

$$\tilde{u} = -\left[ \sum_{i=0}^{n-1} k_{ai} |x^{(i)}| + k_b \left( \sum_{i=0}^{n-1} c_i e^{(i)} \right) + |x_d^{(n)}| + k_d \right] \text{sgn}(b\mathbf{s})$$

in which

$$\begin{aligned} k_{ai} &\geq \frac{|\hat{a}_i \Delta b| + |\hat{b} \Delta a_i|}{\hat{b}[\hat{b} - \Delta b \text{sgn}(\hat{b})]}, \\ k_b &\geq \frac{\mathbf{D}b}{\hat{b}[\hat{b} - \mathbf{D}b \text{sgn}(\hat{b})]}, \\ k_d &\geq \frac{1}{\hat{b} - \Delta b \text{sgn}(\hat{b})} D \end{aligned} \quad (12)$$

And the function  $\text{sgn}(\cdot)$  is defined as,

$$\text{sgn}(\mathbf{s}) = \begin{cases} 1 & \text{for } \mathbf{s} > 0 \\ -1 & \text{for } \mathbf{s} < 0 \end{cases} \quad (13)$$

It is evident that  $u_{eq}$  is the so-called equivalent control effort which is defined as the solution of the problem  $\dot{\mathbf{s}} = 0$  under the nominal model (5) and the system dynamics in sliding-mode can thus be determined solely. On the other hand, switching term  $\tilde{u}$  is adopted to suppress the adverse effect from parameter uncertainties and external disturbance so as to guarantee the existence of sliding-mode. Once the sliding-mode exists and holds, then the system performance can be guaranteed and the system response will be completely insensitive to those perturbations. Nevertheless, drawbacks on GSMC design are encountered, and they are listed as follows.

- (1) The invariant properties of sliding-mode are achieved by a high-gain switching action, it will require a large control effort and undesired chatter is inevitable. Namely, the final design may be too conservative.
- (2) Since the input constraint is not taken into account, the control effort may be saturated during transient period and the desired sliding dynamics cannot be ensured at all time.

Here, we will extend the GSMCAR scheme to ensure that the control input can be constrained on certain physical limitations, while an adjustable robust performance can be guaranteed throughout an entire response.

### 3. GLOBAL SLIDING-MODE CONTROL WITH INPUT CONSTRAINT

Motivated by the GSMCAR scheme, if a sliding function together with nominal system dynamics is used to decrease the switching control term, the control effort would be reduced and can be constrained on some predefined bounds. Furthermore, if the system states are trapped on a sliding surface initially and stay thereafter, sliding dynamics would be ensured during the entire response.

#### 3.1 Choice of a sliding surface

Consider a sliding function as follows

$$\mathbf{s}_n = [k e^{(n-1)} + (1-k)v] + \sum_{i=1}^{n-1} c_i e^{(i-1)} + c_0 \int_0^t e(\mathbf{t}) dt - \mathbf{s}_0 \quad (14)$$

where  $k$  is a weighting factor,  $0 \leq k \leq 1$ , representing the combining effect from the realistic system dynamics and the nominal system dynamics. The auxiliary variable  $v$  is defined as

$$\dot{v} = x_f - x_d^{(n)} = \sum_{i=0}^{n-1} \hat{a}_i x^{(i)} + \hat{b}u - x_d^{(n)} \quad (15)$$

If the state error trajectory of (7) is trapped on the switching surface (14), namely,  $\mathbf{s}_n = \dot{\mathbf{s}}_n = 0$ , the sliding dynamics of the closed-loop system would be governed by,

$$[\hat{b}b^{-1} + k(1 - \hat{b}b^{-1})]e^{(n)} + \sum_{i=0}^{n-1} [c_i + (1-k)\hat{d}a_i]e^{(i)} = (1-k)\hat{b}b^{-1}d \quad (16)$$

in which  $\hat{d}a_i = \hat{a}_i - \hat{b}b^{-1}a_i$ . Accordingly, the characteristic equation of the closed-loop system can be rewritten as,

$$[\hat{b}b^{-1} + k(1 - \hat{b}b^{-1})]s^n + \sum_{i=0}^{n-1} [c_i + (1-k)\Delta a_i]s^i = 0 \quad (17)$$

It is obvious that, with given bounds of parameter uncertainties, the weighting factor  $k$  would determine the pole-location of the closed-loop system on the complex plane. Therefore, choosing  $k$  properly, the closed-loop poles can be constrained within a predefined region on the complex plane and robust performance can thus be adjusted accordingly. Moreover, the closed-loop system would be asymptotically stable if the predefined region is designed on the left-half plane. It is noted that the system states of a global sliding-mode control scheme are trapped on the switching surface starting from the initial time [9,12]. For the switching function to be equal to zero initially, the initial condition of (14) would be chosen as,

$$\mathbf{s}_0 = -ke_0^{(n-1)} + \sum_{i=1}^{n-1} c_i e_0^{(i-1)} \quad (18)$$

where  $e_0^{(i-1)}$  for  $i=1,2,\dots,n-1$  represent the initial values of corresponding error state variables. Hence, the sliding-mode would be ensured all the time and the sliding dynamics would represent the overall system performance.

### 3.2 Design of sliding-mode controller

In the sliding-mode control, the sliding condition that ensures the convergence of a sliding function is derived from Lyapunov stability criterion by selecting a functional  $V$  in which  $V = 0.5\mathbf{s}^T \mathbf{s}$ , and the feedback gains are chosen so that,

$$\dot{V} = 0.5 \frac{d\mathbf{s}^T}{dt} \mathbf{s} = \mathbf{s}^T \dot{\mathbf{s}} < 0 \quad (19)$$

is satisfied.

In this way, the system states are directed toward sliding regime and restricted on it thereafter. Based on the sliding surface (14), the following sliding-mode control law is derived

$$\begin{aligned} u_n &= u_1 + u_2 \\ u_1 &= -\frac{1}{\hat{b}} \sum_{i=0}^{n-1} [\hat{a}_i x^{(i)} + c_i e^{(i)} - x_d^{(n)}] \\ u_2 &= -k_r \left[ \sum_{i=0}^{n-1} k_{ai} |x^{(i)}| + k_b \left( \sum_{i=0}^{n-1} c_i e^{(i)} \right) + |x_d^{(n)}| \right] \text{sgn}(b\mathbf{s}_n) \end{aligned} \quad (20)$$

where  $k_{ai}, k_b, k_d$  are defined by (12), and the fraction number  $k_r$ ,  $0 \leq k_r \leq 1$ , can be expressed in the term of  $k$

$$k_r = \frac{k[\hat{b} - \Delta b \text{sgn}(b)]}{\hat{b} - k\Delta b \text{sgn}(b)} \quad (21)$$

To verify satisfaction of sliding condition (19), taking time derivative of (14) and substituting (7) and (15) into the resulting equation to yield

$$\begin{aligned} \dot{\mathbf{s}}_n &= k \left[ \sum_{i=0}^{n-1} a_i x^{(i)} + bu + d - x_d^{(n)} \right] + \\ &\quad (1-k) \left[ \sum_{i=0}^{n-1} \hat{a}_i x^{(i)} + \hat{b}u - x_d^{(n)} \right] + \sum_{i=0}^{n-1} c_i e^{(i)} \\ &= \sum_{i=0}^{n-1} [ka_i + (1-k)\hat{a}_i] x^{(i)} + \sum_{i=0}^{n-1} c_i e^{(i)} + [kb + (1-k)\hat{b}]u + kd - x_d^{(n)} \end{aligned} \quad (22)$$

Dividing both sides of (22) by  $kb(t) + (1-k)\hat{b}$  to yield

$$\begin{aligned} \frac{1}{kb + (1-k)\hat{b}} \dot{\mathbf{s}}_n &= \sum_{i=0}^{n-1} \left[ \frac{ka_i + (1-k)\hat{a}_i}{kb + (1-k)\hat{b}} \right] x^{(i)} + \frac{1}{kb + (1-k)\hat{b}} \sum_{i=0}^{n-1} c_i e^{(i)} \\ &\quad + u + \frac{k}{kb + (1-k)\hat{b}} d - \frac{1}{kb + (1-k)\hat{b}} x_d^{(n)} \end{aligned} \quad (23)$$

Define  $\mathbf{a}_i = \frac{ka_i + (1-k)\hat{a}_i}{kb + (1-k)\hat{b}}$ ,  $\mathbf{b} = \frac{1}{kb + (1-k)\hat{b}}$  and

$\mathbf{g} = \frac{kd}{kb + (1-k)\hat{b}}$ . Hence, (23) can be rewritten as

$$\mathbf{b} \dot{\mathbf{s}}_n = \sum_{i=0}^{n-1} \mathbf{a}_i x^{(i)} + \mathbf{b} \left( \sum_{i=0}^{n-1} c_i e^{(i)} - x_d^{(n)} \right) + u + \mathbf{g} \quad (24)$$

If  $\mathbf{b}$  does not change sign, both  $\mathbf{b}$  and  $b$  should have the same sign, then we have,

$$\begin{aligned} |\mathbf{b}| \dot{\mathbf{s}}_n \text{sgn}(b) &= \\ &\quad \sum_{i=0}^{n-1} \mathbf{a}_i x^{(i)} + \mathbf{b} \left( \sum_{i=0}^{n-1} c_i e^{(i)} - x_d^{(n)} \right) + u + \mathbf{g} \end{aligned} \quad (25)$$

Multiplying (25) by  $\mathbf{s}_n \text{sgn}(b)$  and substituting (20) into the resulting equation to yield

$$\begin{aligned} |\mathbf{b}| \mathbf{s}_n \dot{\mathbf{s}}_n &= \sum_{i=0}^{n-1} [(\mathbf{a}_i - \hat{\mathbf{a}}_i) \text{sgn}(b\mathbf{s}_n x^{(i)}) - k_r k_{ai}] |\mathbf{s}_n x^{(i)}| \\ &\quad + [(\mathbf{b} - \hat{\mathbf{b}}) \text{sgn}(b\mathbf{s}_n x_d^{(n)}) - k_r k_b] |\mathbf{s}_n x_d^{(n)}| + \\ &\quad + [(\mathbf{b} - \hat{\mathbf{b}}) \text{sgn}(b\mathbf{s}_n \sum_{i=0}^{n-1} c_i e^{(i)}) - k_r k_b] \left| \mathbf{s}_n \sum_{i=0}^{n-1} c_i e^{(i)} \right| \\ &\quad + [\mathbf{g} \text{sgn}(b\mathbf{s}_n) - k_r k_d] |\mathbf{s}_n| \end{aligned} \quad (26)$$

where  $\hat{\mathbf{a}}_i = \frac{\hat{a}_i}{\hat{b}}$  and  $\hat{\mathbf{b}} = \frac{1}{\hat{b}}$ .

Because  $|\mathbf{a}_i - \hat{\mathbf{a}}_i| \leq k_r k_{ai}$ ,  $|\mathbf{b} - \hat{\mathbf{b}}| \leq k_r k_b$  and  $|\mathbf{g}| \leq k_r k_d$ , therefore,  $\mathbf{s}_n \dot{\mathbf{s}}_n < 0$  for  $\mathbf{s}_n \neq 0$  and then the sliding condition (19) is satisfied.

It is noted that the difference between the proposed control law (20) and the GSMC control law (11) is the coefficient,  $k_r$ , in the switching term, that is  $u = k \tilde{u}$ . Therefore, the proposed scheme should have similar property of GSMC, yet, its effect will depend on the choice of  $k_r$ , and the proposed

scheme should be designed under the framework of GSMC scheme.

**Remark 1:** If the factor  $k$  equals zero, (i.e.  $k_r = 0$ ), then (20) can be rewritten as,

$$u_n = -\frac{1}{b} \left( \sum_{i=0}^{n-1} \hat{a}_i x^{(i)} - \sum_{i=0}^{n-1} c_i e^{(i)} - x_d^{(n)} \right) \quad (27)$$

It is clear that the control law (27) can be viewed as a linear feedback with pole-assignment design, the switching term will be reduced and the control effort can thus be decreased. However, some poles may have exceeded the predefined regime under perturbation and the system performance cannot be guaranteed. On the other hand, as  $k$  approaches one, (i.e.  $k_r = 1$ ), the sliding dynamics (16) would be simplified as (9) of a GSMC design. All the closed-loop poles are placed exactly at the predefined locations on the complex plane and thus insensitive to parameter uncertainties. However, this design is usually conservative and would induce high frequency chattering during implementation.

### 3.3 Determination of $k$ based on input constraint

As previously described, the switching gain of (20) is a function of the weighting factor  $k$ ; decreasing  $k$  will suppress the chattering level. Moreover, the maximal control effort can be reduced and a physical limit of control input can thus be achieved. However, the parameter uncertainties and external disturbances would have certain influence on the sliding dynamics. On the other hand, increment of  $k$  will definitely suppress the effect caused by the system uncertainties and disturbances. Alternatively, the weighting factor  $k$  will be determined by a trade-off between input constraint and robust performance. A design procedure of factor  $k$  such that the problems of bounded input and robust performance can be considered concurrently is described in the following.

Assume that the equivalent control effort  $u_{eq}$  of (11), (and  $u_1$  of (20)), is determined by a pole-placement scheme according to the design specifications first, the switching gain  $\tilde{u}$  of (11) is also determined. For a bounded input control, if the equivalent control effort is always less than a predefined input constraint, i.e.

$$|u_1| \leq U_{\max} \quad (28)$$

then the ideal value of  $k_r$  of (20) can be chosen as,

$$k_r = \frac{(U_{\max} - |u_1|)}{|\tilde{u}|} > 0 \quad (29)$$

and the factor  $k$  is also to be designed according to (21). Since the factor  $k_r$  (and  $k$ ) may be a function of time, the sliding function (14) will be viewed as a moving sliding function (Choi et al. 1993; Rajiv and Netjat, 1997). Therefore, the sliding function (14) cannot be maintained to zero continually if the factor  $k$  is with a step change or with a fast variation. How to modify the factor  $k$  (and  $k_r$ ) to

guarantee the existence of a sliding-mode or a quasi sliding-mode at all time is the next problem.

Now, we will tune the factor  $k_r$  on-line by an appropriate small change,  $\Delta k_r$ , without violating the sliding condition (19). The tuning algorithm of  $k_r$  in this study may be outlined as follows.

Step 1. Calculate the initial value  $k_{r0}$  to satisfy (29)

according to the predefined bounds  $U_{\max}$ , and also determine the factor  $k_0$  of (14).

Step 2. Predefine an appropriate small constant  $\Delta k_r$  and  $k_{r(i)} = k_{r(i-1)} + \Delta k_r$ , where  $k_{r(i)}$  denotes the value of  $k_r$  of the  $i$ -th step.

Step 3. Estimate  $k_{r(i)}$  by (29) and compare with the previous  $k_{r(i-1)}$ , determine the sign of  $\Delta k_r$ .

Step 4. Update  $k_r$  by  $k_{r(i)} = k_{r(i-1)} + \Delta k_r$ , modify the sliding function with a new value of  $k$  obtained by (21) and check the existence of sliding behaviour.

**Remark 2:** During the factor  $k$  modification, it is recommended to modify the control law (20) as follows.

$$u_n = -\frac{1}{b} \sum_{i=0}^{n-1} [\hat{a}_i x^{(i)} + c_i e^{(i)} - x_d^{(n)}] - k_r \left[ \sum_{i=0}^{n-1} k_{ai} |x^{(i)}| \right] \quad (30)$$

$$+ k_b \left( \sum_{i=0}^{n-1} c_i e^{(i)} \right) + |x_d^{(n)}| + k_d ] \text{sgn}(b \mathbf{s}_n) - K_p \mathbf{s}_n$$

The parameter  $K_p$  is a positive constant to reduce the hitting time. As the sliding-mode has been reached, the added term  $-K_p \sigma_n$  has no effect on the control effort.

## 4. EXPERIMENTAL STUDIES

In general, a linear variable reluctance motor (LVRM) offers the advantages of low cost and simplex mechanical construction, wide operating range of thrust force, and direct-drive. However, due to the fact that the phase inductance of slider is a function of both the slider position and the phase current, the thrust force usually has highly nonlinear properties. Moreover, the dynamics of uncontrolled system is highly dependent on the external load variation. These features make most classical control methods insufficient to achieve a satisfactory dynamic performance. Therefore, a robust controller is usually necessary for a high performance motor-drive design. Here, an LVRM drive system (Lin et al. 2002) is adopted to show the effectiveness of the proposed scheme. The photograph of this LVRM drive system under study is shown in Figure 1, while the schematic diagram of this system is shown in Figure 2.

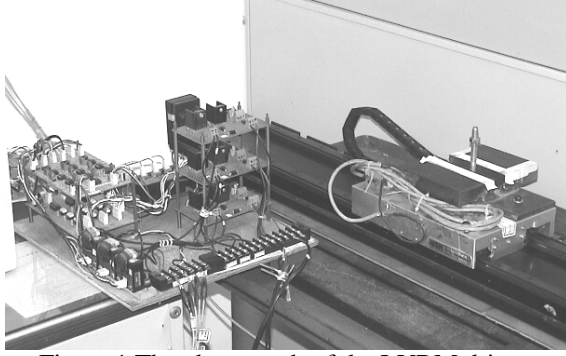


Figure 1 The photograph of the LVRM drives system under study.

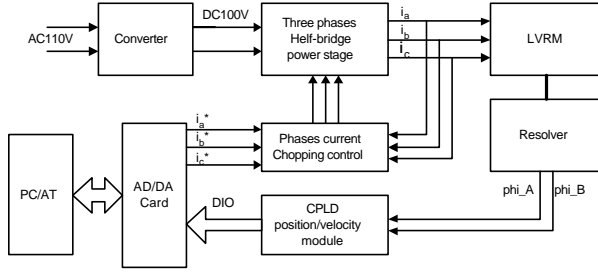


Figure 2. Schematic diagram of the LVRM driver system under study.

The experimental dynamic equation of the LVRM system under study is expressed as,

$$\ddot{x} = a_1 \dot{x} + bF + d \quad (31)$$

where  $-5 \leq a_1 \leq -3$ ,  $16 \leq b \leq 48$ , which is caused by the external load variation (from 0 kg to 6.0 kg) and the nonlinear effects including friction and dead-zone. This experimental model is determined through a Dynamic Signal Analyzer and by means of a curve-fitting scheme (Lin *et al.* 2002).

Assuming the system tracking error is defined as  $e = x - x_d$  and  $x_d$  is the desired position trajectory, if the tracking controller is designed to have closed-loop poles equal to  $(-40, 0)$  with a multiplicity of two, then the proposed sliding function can be chosen as

$$\mathbf{s}_n = [k\dot{e} + (1-k)e_v] + 80e + 1600 \int_0^t e(t) dt + \mathbf{s}_0 \quad (32)$$

where  $e_v$  is defined as,

$$\dot{e}_v = -4\dot{x} + 32u - \ddot{x}_d \quad (33)$$

The initial  $\mathbf{s}_0$  is chosen as  $\mathbf{s}_0 = -k\dot{e}_0 - 80e_0$  so that the switching function is equal to zero initially.  $k$  will be adjusted on-line according to the control law. To guarantee the existence of sliding-mode, the proposed control law is designed as,

$$u_n = -[-0.125\dot{x} + 0.03125e - \ddot{x}_d] - k_r [0.1875|\dot{x}| + 0.03125(|80\dot{x} + 1600x| + |\ddot{x}_d|)] \text{sgn}(\mathbf{s}) - K_p \mathbf{s}_n \quad (34)$$

where  $K_p \mathbf{s}_n = 1.5 \mathbf{s}_n$  is used to improve the reaching time of the sliding function.  $k_r$  is

determined by the input constraint and can be obtained as described in section 3.3. Once the  $k_r$  is determined,  $k$  can be calculated by

$$k = 2k_r / (1 + k_r) \quad (35)$$

The simulation result for regulation control is shown in Figures 3. The dynamic response of the system using the proposed scheme with worst system parameters and a disturbance of 10N is added at  $t=0.4$  sec is shown in this Figure. It reveals that even the control effort is always less than the input constraint (i.e.60N), the existence of sliding-mode can always be ensured all the time under input constraint, while the initial value of factor  $k$  is determined by the input constraint and updated online to maintain sliding during transient period, while robust performance can be achieved in the steady state.

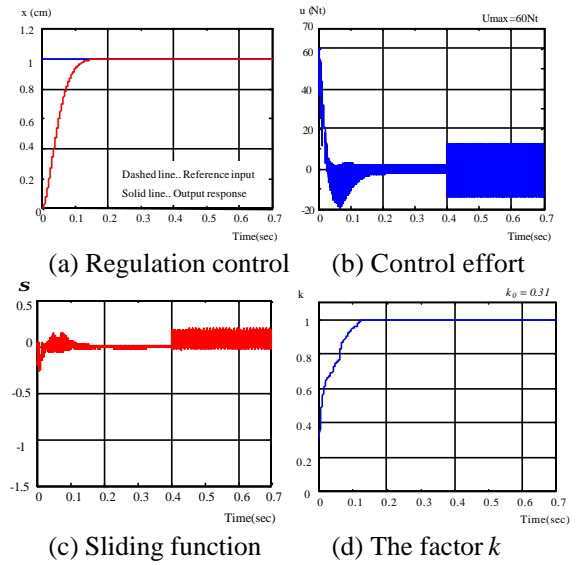
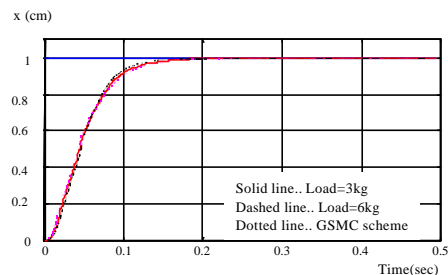
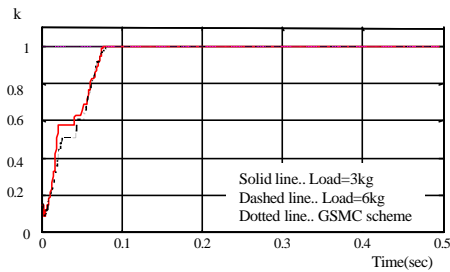


Figure 3. A simulated regulation control with an 60N input constraint and a disturbance of 10N added at  $t=0.4$  sec.

Applying the design from simulation studies, both the GSMC scheme and the proposed scheme are investigated experimentally on the LVRM servo system. The experimental results of the regulation control under different loadings and the sinusoidal tracking control under different input constraints are shown in Figure 4 and Figure 5, respectively. Figure 4 showed that the proposed scheme could provide excellent robustness as well as the GSMC scheme under different loadings.



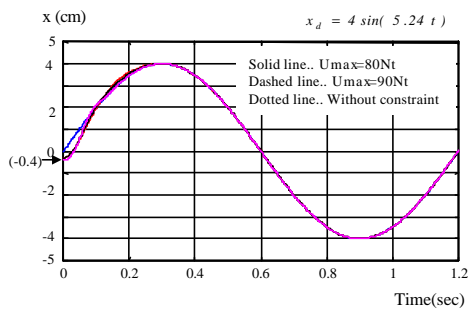
(a) Regulation control



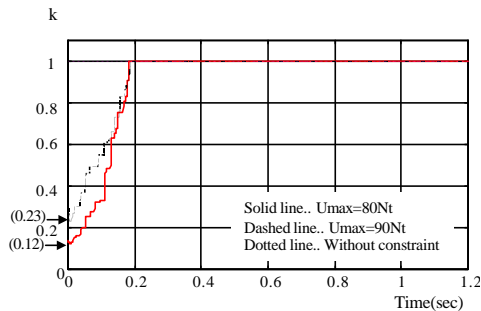
(b) The updating of factor K

Figure 4. Experimental results for regulation control with an 60N input constraint under various loadings.

Figure 5 showed that the proposed scheme can achieve satisfactory tracking performance under different input constraint even the system states are not trapped on the reference command initially.



(a) The sinusoidal tracking



(b) The updating of factor K

Figure 5. Experimental results on sinusoidal tracking with 6kg loading under different input constraints for the proposed scheme.

It is seen from Figure 4(b) and Figure 5(b) that the initial value of  $k$  is determined by the input constraint solely and can be updated to one automatically under different load conditions or different input constraints. From these results, the proposed scheme has demonstrated its effectiveness in a high performance design of the LVRM servo system.

## 5. CONCLUSION

In this article, a global sliding-mode control scheme with input constraint has been proposed to ensure the existence of sliding-mode throughout an entire response. A switching function together with nominal system dynamics is designed to decrease the switching term of control effort. In order to

fulfil the bounded input constraint while achieving robustness simultaneously, a design procedure of on-line updating the weight of GSMC based on the Lyapunov criterion is also presented. By this scheme, an adjusted robustness can be ensured during the transient period while a robust performance can be guaranteed in the steady state. Experiments were conducted on an LVRM servo system, in which both regulation control and sinusoidal tracking control were performed. The results have demonstrated that high performance could be achieved by applying the proposed scheme if certain tolerance of transient response is permitted.

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