15th Triennial World Congress, Barcelona, Spain

A SLIDING MODE CONTROL APPROACH TO ROBOTIC TRACKING PROBLEM

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Abstract: In the conventional sliding mode control, a discontinuous switching control signal is applied. It makes the system invariant to parametric uncertainty and external disturbance, but also causes actuator chattering. This paper considers a tracking control of robot manipulators using a dynamic sliding mode control, where a global invariance is achieved and the resulting control signal is chattering free. The resulting performances are illustrated by its application to the tracking problem of robot manipulators. *Copyright © 2002 IFAC*

Keywords: Sliding mode control, chattering, robot manipulator, invariance, trajectory tracking problem

1. INTRODUCTION

To achieve good tracking under uncertainties, one usually requires to combine some mechanisms in the control design, such as adaptation, feedforward, and high-gain. Sliding mode control has been developed for theoretical and practical studies of control engineering since the 1970's (Utkin, 1978). One of the advantages of sliding mode control is its invariance against parametric uncertainties and external disturbances. However, it also introduces actuator chattering phenomenon that should be avoided in many physical systems, such as servo control systems, structure vibration control systems and robotic systems. In 1996, Chen and Lin proposed a chattering-free sliding mode control law using Fliess's generalized controller canonical form (Fliess, 1990) of the nonlinear system. This approach can be applied to a nonlinear dynamical system control provided that the internal dynamics of the system are stable.

Basically, the sliding mode control motion has two phases, i.e., the reaching phase and the sliding phase. The invariance of the sliding mode control only applies to the sliding phase. Therefore, the system is sensitive to parameter uncertainty and disturbance in the reaching phase. An approach (Lee and Xu, 1994) to reduce the effect of reaching phase dynamics on control performance is to make the sliding mode occur while the control is applied. However, for some applications, the above approach may fail to eliminate chattering with an auxiliary control input associated with the plant uncertainty. Further comment on this approach can be seen in (Kwan, 1995).

In this paper, the computed-torque technique (An et al., 1987) is used to propose a systematic design procedure for a dynamic sliding mode control, resulting in continuous control signal with global invariance. Based on the proposed method, a dynamic sliding surface across the initial states can be designed for robotic tracking control problems. Without using approximate methods, the proposed sliding mode control law not only eliminates chattering but also preserves its invariance to parametric uncertainties and external disturbances..

2. CONVENTIONAL SLIDING MODE CONTROL DESIGN

The dynamic equations of a n-link robot are usually obtained by the Euler-Lagrange equations as the following.

$$H(q)\ddot{q} + M(q,\dot{q})\dot{q} + G(q) = \tau + d\ , \qquad (1)$$

where $q, \dot{q}, \ddot{q} \in R^n$ are the joint position, velocity and acceleration vectors; $H(q)\in R^{n\times n}$ is a positive-definite inertia matrix; $M(q,\dot{q})\in R^{n\times n}$ is a matrix containing the Coriolis and centrifugal terms; $G(q)\in R^{n}$ is a vector of gravitational terms; $\tau\in R^{n}$ is the actuator torque vector acting on the joints of the robot, and $d\in R^{n}$ is the vector of disturbances referred to the actuator input.

If the desired system states are available, the desired angle, angular velocity, and angular acceleration of the *i*th manipulator joint are, respectively, denoted by $q_d, \dot{q}_d, \ddot{q}_d$, and the corresponding error equations can be written as follows:

$$e = q - q_d,\ \dot{e} = \dot{q} - \dot{q}_d,\ \ddot{e} = \ddot{q} - \ddot{q}_d \qquad (2)$$

The sliding surface of the sliding mode control design should satisfy two requirements, i.e., the closed-loop stability and performance specifications. A conventional sliding surface corresponding to the error state equation can be represented as

$$S = \dot{e} + ce = 0, \qquad (3)$$

where c can be chosen according to the desired system specifications. From Eq. (3), since the order of the sliding surface is one order less than the system of Eq.(1), the initial error states are not on the sliding surface of (3).

According to the sliding control method, the equivalent control of the sliding mode control law can be chosen as:

$$\tau_{eq} = H(q)(\ddot{q}_d - c\dot{e}) + M(q,\dot{q})\dot{q} + G(q). \qquad (4)$$

Without external disturbances and modelling uncertainties, the equivalent control (4) makes $H(q)(\ddot{e}+c\dot{e})=0$, and $\dot{S}=0$ is achieved.

In practice, the external disturbances and the modelling uncertainties always exist, and only the nominal model of the robots can be obtained. It is supposed that the nominal model of the robots is denoted by $H_0(q), M_0(q,\dot{q})$, and $G_0(q)$. To compensate the effect of uncertainties and disturbances, the switching control law is applied. In general, the sliding mode control law can be represented as:

$$\tau = \tau_{eq} + \tau_{sw} = H_o(q)(\ddot{q}_d - c\dot{e}) + M_o(q,\dot{q})\dot{q} + G_o(q) + \tau_{sw} \qquad (5)$$

where τ_{eq} is the equivalent control law for sliding phase motion and τ_{sw} is the switching control for the reaching phase motion. In the conventional sliding control, it can be chosen as:

$$\tau_{sw} = -\text{sgn}(S) H_0(q) W \qquad (6)$$

where W is a $n\times 1$ weighting vector and sgn(S) = $diag\{\text{sgn}(s_i)\}, i = 1\cdots n$ with

$$\text{sgn}(s_i) = \begin{cases} 1, if\, s_i > 0 \\ -1, if\, s_i < 0 \\ 0, otherwise. \end{cases} \qquad (7)$$

Since the discontinuous switching control signal is applied, it makes the system invariant to parametric uncertainty and external disturbance, but also causes chattering. To reduce chattering, the switching function sgn(S) can be replaced by continuous approximations in the neighbourhood of the sliding surface, such as the sigmoid-like function.

3. MAIN RESULTS

For the global invariance of the sliding mode control, the following sliding surface is proposed.

$$S = \ddot{e} + K_v\dot{e} + K_p e = 0 \qquad (8)$$

where $S = [s_1 \quad s_2 \quad \cdots \quad s_n]^T$, $e = [e_1 \quad e_2 \quad \cdots \quad e_n]^T$, $\dot{e} = [\dot{e}_1 \quad \dot{e}_2 \quad \cdots \quad \dot{e}_n]^T$ and $\ddot{e} = [\ddot{e}_1 \quad \ddot{e}_2 \quad \cdots \quad \ddot{e}_n]^T$. The design parameters K_v and K_p can be chosen according to the desired stable switching surfaces. These sliding surfaces represent the error dynamics of the resulting control system.

Remark 1:

If the conventional sliding surface $S = \dot{e} + ce = 0$ is applied, the initial error state $(e(0), \dot{e}(0))$ is not on the sliding surface, so there exists a reaching phase motion. The proposed sliding surface (9) is determined by the initial error state $(e(0), \dot{e}(0))$ and the specified parameters K_P, K_v , and can be described by the state space equation as follows.

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -K_p & -K_v \end{bmatrix} X, \qquad (9)$$

where $X = [e, \dot{e}]^T$.

If the equivalent control law of the sliding-mode controller is chosen according to the computed-torque method (Wijesoma, and Richards, 1990).

$$\tau_{eq} = H(q)(\ddot{q}_d - K_v\dot{e} - K_p e) + M(q,\dot{q})\dot{q} + G(q). \qquad (10)$$

where $e = q - q_d, \dot{e} = \dot{q} - \dot{q}_d$.

K_p and K_v are constant diagonal $n\times n$ gain matrices with k_{p_i} and k_{v_i} on the diagonals. Without the external disturbances and the modelling uncertainties, the equivalent control (10) makes

$$H(q)(\ddot{e} + K_v\dot{e} + K_p e) = 0 \qquad (11)$$

where $H(q) \neq 0$, so that

$$\ddot{e} + K_v\dot{e} + K_p e = S = 0 \qquad (12)$$

is achieved by control law (11). □

From Eq. (12), the proposed sliding surface across the initial error state. On the other hand, since the sliding surface is stable, the system steady state $(e(\infty), \dot{e}(\infty)) \to (0,0)$. From this point of view, it also indicates that there is no reaching phase motion, if the equivalent control (10) is applied.

Suppose that the nominal model matrices of the robotic system are denoted by $H_0(q)$, $M_0(q,\dot{q})$ and $G_0(q)$, respectively. Therefore, the equivalent control law according to the nominal model can be described by

$$\tau_{eq} = H_0(q)(\ddot{q}_d - K_v\dot{e} - K_p e) + M_0(q,\dot{q})\dot{q} + G_0(q). \qquad (13)$$

Since the external disturbances and the model uncertainties always exist, the switching control law is needed to compensate these uncertainties and to keep the error states staying on the sliding surface. Without introducing actuator chattering, the modified switching control law is applied.

$$\tau_{sw} = -\int_0^t \mathrm{sgn}(S) H_0(q) W dt, \quad (14)$$

where W is a $n \times 1$ weighting vector and sgn(S) = $diag\{\mathrm{sgn}(s_i)\}, i = 1 \cdots n$.

Because the integral of sgn(S) is applied, which is like a low-pass filter, the proposed switching controller can avoid chattering efficiently.

Then, the sliding mode control law is defined as:

$$\begin{aligned}\tau &= \tau_{eq} + \tau_{sw} \\ &= H_0(q)(\ddot{q}_d - K_v \dot{e} - K_p e) + M_0(q,\dot{q})\dot{q} + G_0(q) \quad (15) \\ &\quad - \int_0^t \mathrm{sgn}(S) H_0(q) W dt\end{aligned}$$

By Eq. (15) and (1), it results that

$$\begin{aligned}&H(q)\ddot{q} + M(q,\dot{q})\dot{q} + G(q) \\ &= H_0(q)(\ddot{q}_d - K_v \dot{e} - K_p e) + M_0(q,\dot{q})\dot{q} + G_0(q) \quad (16) \\ &\quad - \int_0^t \mathrm{sgn}(S) H_0(q) W dt + d\end{aligned}$$

Define the modelling uncertainties as $\Delta H = H_0 - H$, $\Delta M = M_0 - M$, $\Delta G = G_0 - G$ then,

$$\begin{aligned}&\Delta H(q)\ddot{q} + \Delta M(q,\dot{q})\dot{q} + \Delta G(q) + d - \int_0^t \mathrm{sgn}(S) H_0(q) W dt \\ &= H_0(q)(\ddot{e} - K_v \dot{e} - K_p e)\end{aligned} \quad (17)$$

From (18), if the switching control is designed to make the error state $(e, \dot{e})$ stay on the sliding surface $S = 0$, as equation (8), then

$$\begin{aligned}&H_0 S \\ &= H_0(q)\{\ddot{e} + k_v \dot{e} + k_p e\} \\ &= \Delta H(q)\ddot{q} + \Delta M(q,\dot{q})\dot{q} + \Delta G(q) + d - \int_0^t \mathrm{sgn}(S) H_0(q) W dt \\ &= 0\end{aligned} \quad (18)$$

i.e.

$$\begin{aligned}S &= H_0^{-1}\{\Delta H(q)\ddot{q} + \Delta M(q,\dot{q})\dot{q} + \Delta G(q) + d - H_0(q)\int_0^t \mathrm{sgn}(S) W dt\} \\ &= H_0^{-1}(\Delta H(q)\ddot{q} + \Delta M(q,\dot{q})\dot{q} + \Delta G(q) + d) - \int_0^t \mathrm{sgn}(S) W dt\end{aligned} \quad (19)$$

Then, the reaching condition for S = 0 can be derived by defining the Lyapunov function as

$$V = \frac{1}{2} S^T S \ . \quad (20)$$

Differentiating *V(t)* with respect to time yields

$$\begin{aligned}\dot{V} &= S^T \dot{S} = S^T \left\{ H_0^{-1}\left[-\frac{d}{dt}(\Delta H\ddot{q} + \Delta M\dot{q} + \Delta G + d) \right] - \mathrm{sgn}(S) W \right\} \\ &= -S^T \left[H_0^{-1} \frac{d}{dt}(\Delta H\ddot{q} + \Delta M\dot{q} + \Delta G + d) \right] - |S|^T \cdot W \\ &\le |S|^T \cdot \left\{ \left| H_0^{-1} \frac{d}{dt}(\Delta H\ddot{q} + \Delta M\dot{q} + \Delta G + d) \right| - W \right\}\end{aligned} \quad (21)$$

where $|S| = [|s_1| \quad |s_2| \quad \cdots \quad |s_n|]^T$.

Now, if $\dot{V}$ is negative semi-definite with respect to S (i.e., $\dot{V}$ <0 for $S \neq 0$, and $\dot{V}$ =0 for S = 0) with a large enough W, the approaching condition will be satisfied. An immediate choice for W to make $\dot{V}$ a negative semi-definite function of S would be

$$W \ge \left| H_0^{-1} \frac{d}{dt}(\Delta H\ddot{q} + \Delta M\dot{q} + \Delta G + d) \right| \ . \quad (22)$$

In practice, it may be argued that $\left| \frac{d}{dt}\left[H_0^{-1}(\Delta H\ddot{q} + \Delta M\dot{q} + \Delta G + d) \right] \right|$ is difficult to estimate. However, the condition can reasonably be achieved with a large enough *W*. If the disturbance can be estimated more accurate, then the resulting control input to keep the state trajectory on sliding surface will be more appropriate. This situation is similar to the conventional sliding mode control law.

4. PERFORMANCE ANALYSIS

The use of the proposed sliding mode control law results only the sliding phase dynamics. Therefore, the behaviour of the resulting control system can be determined as long as the initial error state is available. The system time response can be adjusted more explicitly by assigning the sliding surface dynamics.

Based on the above analysis, a systematic design procedure for achieving required output performance with a given input space could be described as follows. It will be applied to robot manipulator control in the next section.

Based on the above analysis, the design procedure for tracking control problem is described as follows.

(1). Determine w and η with respect to system uncertainties and disturbances.

$$W \ge \left| \frac{d}{dt}\left[H_0^{-1}(\Delta H\ddot{q} + \Delta M\dot{q} + \Delta G + d) \right] \right|$$

(2). Determine the coefficients K_p , K_v of the sliding surface by selecting the corresponding eigenvalues λ, such that the required tracking performance is obtained.

(3). The resulting control law can be described as

$$\begin{aligned}\tau &= H_0(q)(\ddot{q}_d - K_v \dot{e} - K_p e) + M_0(q,\dot{q})\dot{q} \\ &\quad + G_0(q) - \int_0^t H_0(q) W \,\mathrm{sgn}(S) dt\end{aligned} \ .$$

5. CASE STUDY

In this section, a 2 D.O.F. manipulator as shown in Fig.1 will be studied. The dynamic model is described in the Appendix. In the simulation studies, the desired joint trajectory is

$$\begin{bmatrix} q_{d1} \\ q_{d2} \end{bmatrix} = \begin{bmatrix} 1 + 0.2\sin \pi t \\ 1 - 0.2\sin \pi t \end{bmatrix}$$

and the matched disturbance added to the joints is described as

$$d(t) = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0.5\sin(\pi t) \\ 0.5\sin(\pi t) \end{bmatrix}$$

The performance specifications and conditions for numerical studies are stated as follows.

(a). The initial angular position and velocity of the joints are $[0,0]^T$ (rad) and $[0,0]^T$ (rad/sec), respectively. And the desired angular position and velocity at the first step is $[1.0063, 0.9937]^T$ (rad) and $[0.6280, -0.6280]^T$ (rad/sec), respectively. That is, the initial value of error states is $Z(0) = [1.0063 \quad 0.9937 \quad 0.628 \quad -0.628]^T$.

(b). The settling time of the tracking control system is required to less than 3 seconds.

(c). The steady-state tracking error of the angular position and angular velocity should be less than 0.005 rad, and 0.1 rad/sec, respectively.

For a critically damped control system design, the gains K_p and K_v can be chosen as:

$$K_v = 2\alpha I$$
$$K_p = \alpha^2 I$$

In this study, the parameters $\alpha = [2,2]^T$, $W = [1,3]^T$ are used. From Fig.2, the trajectory tracking error of the resulting system satisfies the system requirement . Fig.3 and 4 show that the control torques generated by the proposed sign-function-based control does not result in chattering phenomena. The modified switching control torque obtained by (14) is in a similar form as the matched disturbance with opposite sign. It reveals that the effect of matched disturbance can be dispelled by the proposed method. Fig.5 illustrates that the proposed method makes the error state $(e, \dot{e})$ stay on the sliding surface, and the global invariance is achieved without chattering.

6. CONCLUSIONS

In this paper, an alternative approach to the design of sliding mode control is proposed. A systematic design procedure based on the concept of a dynamic sliding surface is presented. The proposed scheme results in a continuous control signal such that chattering is eliminated. The simulation results show that the proposed approach achieves design requirements without chattering effect.

APPENDIX

The dynamic equations of the planar robot with 2 DOF, shown in Fig. 1, are described as follows.

$$H(q)\ddot{q} + M(q,\dot{q})\dot{q} = \tau \ ,$$

where

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix},$$

$$M = \begin{bmatrix} -2m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2 & -m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2 \\ m_2 l_1 l_{c2}\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$\tau = \begin{bmatrix}\tau_1 & \tau_2\end{bmatrix}^T.$$

$$q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T,$$

$$H_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)) + I_1 + I_2,$$

$$H_{12} = H_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2,$$

$$H_{22} = m_2 l_{c2}^2 + I_2,$$

and $l_{c1} = \frac{1}{2} l_1$, $l_{c2} = \frac{1}{2} l_2$.

The nominal and actual robot parameters used in the following table.

Table A. The parameters of the robot

Parameters	Nominal Value	Actual Value
l_1	*1m*	*1m*
l_2	*1m*	*1m*
m_1	*1kg*	*1kg*
m_2	*1kg*	*1.5kg*
l_{c1}	*0.5m*	*0.7m* (37)
l_{c1}	*0.5m*	*0.7m*
I_1	$0.0833\ kgm^2$	$0.0833\ kgm^2$
I_2	$0.0833\ kgm^2$	$0.1513\ kgm^2$

ACKNOWLEDGEMENT

The work was supported by the National Science Council, Taiwan, under the grant No. NSC 89-2212-E006-064.

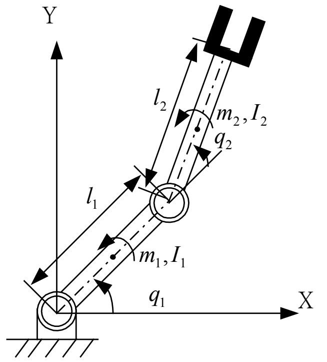

Fig. 1. The schematic diagram of a 2 D.O.F. robot.

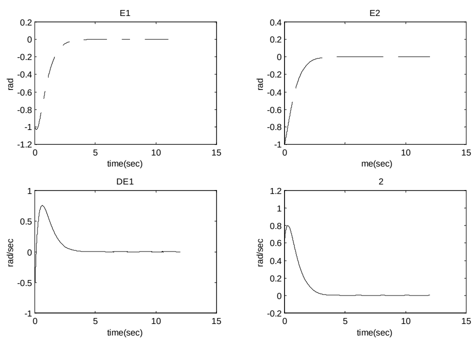

Fig. 2. The trajectory tracking errors of the resulting system.

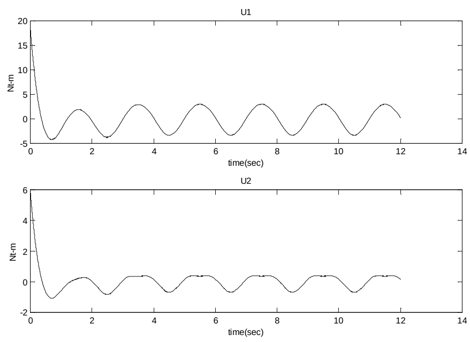

Fig. 3. The torque outputs obtained by the proposed method.

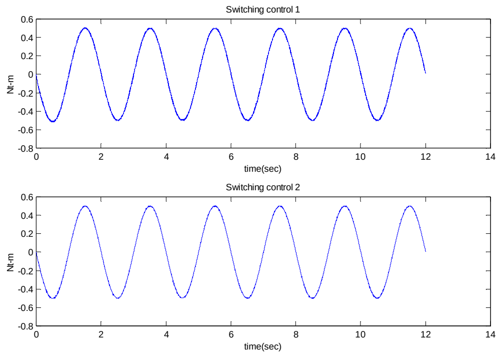

Fig.4. The sign-function-based control τ_{sw} obtained by the proposed method.

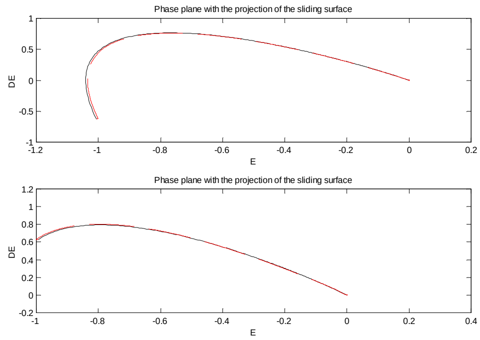

Fig. 5. The phase plane trajectory and the sliding surface for arms 1 and 2.

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