15th Triennial World Congress, Barcelona, Spain

A NEURAL NETWORK-BASED ADAPTIVE SLIDING MODE CONTROL FOR ROBOT MANIPULATORS

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Abstract An adaptive sliding mode tracking controller using neural network is proposed for robot manipulators with uncertainties. In this new control scheme, a RBF neural network is used to adaptively learn system uncertainties bounds, and then the outputs of neural network is used to adjust the switching gain. This new controller can guarantee both strong robustness with respect to system nonlinearities and uncertainties and the asymptotic convergence of the tracking error to zero.

Keywords Manipulators, neural networks, adaptive control, sliding-mode control, robustness

Notation $\lambda_{\min}(A)$ denotes the minimum eigenvalue of matrix *A*. For $a \in R^n$, $|a|$ denotes the absolute value vector, i.e. $|a|=(|a_1|,|a_2|,\ldots,|a_n|)^T$. $\|a\|$ denotes the Euclidean norm, i.e. $\|a\|=\sqrt{a^T a}$. $sign(a)=(sign(a_1), sign(a_2),\ldots,sign(a_n))^T$.

Ⅰ. INTRODUCTION

The popularly known computed-torque method (Craig, J. J., 1989) as been shown to be effective for robot manipulator control. However, the control scheme relies on an exact robot model which is practically unavailable in many cased due to inevitable uncertainties. Hence, the approach of solving model uncertain problems has attracted considerable attentions in the past decade.

So far, in order to compensate for such uncertainties in the robot manipulator dynamic equation, many control strategies have been proposed. There are basically three approaches to the control of such uncertain systems: 1) adaptive control, 2) sliding mode control, 3) neural network control. Adaptive control methods are applied to systems mainly with parametrized uncertainties. In adaptive control design, the linear-in-parameter assumption is usually used to formulate error equations which relate measurable signals to parameter errors whereby a parameter adaptive updating law can be formed (Sun D. *et. al.*, 1992, Battilotti, S., *et.al.*,1997, and Slotine, J-J.E., *et.al.* 1989). Sliding-mode control exploits the variable structure concept. It first defines the sliding surface in the error state space, and then employs a discontinuous control law to drive the error state to slide along the surface until it converges. The major advantages of sliding mode control are robustness to parameter uncertainty and invariance to unknown disturbances. However, in sliding-mode control, to calculating the switching gains, it is usually assumed that the bounds on the uncertainties are known (Yeung, K. S. *et. al.*, 1988, Fu, L. C. *et. al.*, 1992). This may lead to a conservative design. In general, precise bounds on the uncertainty are difficult to compute.

In recent years, neural networks have been extensively studied for use in the field of control engineering, especially robot manipulator control, primarily because of their excellent features such as learning ability, nonlinear mapping, and parallel processing. Many neural network-based controller, as in Nam, B. H., *et. al.*, 1997, Kwan, C. M., *et. al.*, 1995, Carelli, R., *et. al*, 1995 and Man Z., *et. al.*, 1995) have been proposed for the compensation for the effects of nonlinearities and uncertainties so that system performances can be improved.

In this paper, a new neural network-based adaptive robust tracking control scheme is proposed for robot manipulators with uncertainties. In this scheme, a neural network is used to adaptively learn the unknown bound of system uncertainties, and then the output of the neural network is used to adaptively adjust the switching gain. Through integrating the sliding mode control technique and the excellent learning ability of neural network, the proposed controller has strong robustness with respect to system uncertainties and nonlinearities, and can guarantee the asymptotic convergence of joint position and velocity tracking errors to zero.

Ⅱ. STATEMENT OF PROBLEM

Consider an n-degree-freedom robot manipulator system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_d \qquad (1)$$

Where $q, \dot{q}, \ddot{q} \in R^n$ are the link position, velocity, and acceleration vectors, respectively, $M(q) \in R^{n\times n}$ the inertia matrix, $C(q, \dot{q}) \in R^{n\times n}$ the centripetal-Coriolis matrix, $G(q) \in R^n$ the gravity forces acting on the links, $\tau_d \in R^n$ is a disturbance such as static friction, and $\tau \in R^n$ the control input torque vector. The rigid dynamics (1) has the following properties (. Lewis , F. L, *et. al.*, 1993):

P1. The inertia and centripetal-Coriolis matrices have the following property

$$y^T\left(\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right)y = 0 \qquad \forall y \in R^n \quad (2)$$

Where $\dot{M}(q)$ is the time derivative of matrix *M*.

P2. The inertia matrix $M(q)$ is symmetric, positive definite, and bounded.

Generally, in practical robot systems, the perturbations in system parameters are inevitable. Hence, this paper considers the case that the parameter matrices in model (1) can be divided as following

$$M(q) = M_0(q) + \delta M(q),$$
$$C(q,\dot{q}) = C_0(q,\dot{q}) + \delta C(q,\dot{q}),$$
$$G(q) = G_0(q) + \delta G(q) \qquad (3)$$

where $M_0(q)$, $C_0(q,\dot{q})$, and $G_0(q)$ are the nominal parts and are assumed to be known exactly, whereas $\delta M(q)$, $\delta C(q,\dot{q})$, and $\delta G(q)$ are the estimating errors, and denote parametric uncertainties.

Substituting expressions (3) into (1), the dynamic model of robot system can be rewritten as

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) + F(q,\dot{q},\ddot{q}) = \tau + \tau_d \qquad (4)$$

with $F(q,\dot{q},\ddot{q}) = \delta M(q)\ddot{q} + \delta C(q,\dot{q})\dot{q} + \delta G(q)$ denoting system uncertainty.

It is assumed that system uncertainty $F(q,\dot{q},\ddot{q})$ is upper bounded by a positive function $F_0(t)$, i.e.

$$|F(q,\dot{q},\ddot{q})| < F_0(t).$$

Further, we assume the following known bound for disturbance τ_d

$$|\tau_d| < T_0 \qquad (5)$$

where T_0 is known positive constant.

Let $q_d, \dot{q}_d \in R^n$ denote the desired joint position and velocity, respectively, and define the tracking error as $e = q - q_d$, $\dot{e} = \dot{q} - \dot{q}_d$.

For robot system (4), we choose the following switch planes

$$s = \dot{e} + \Lambda e \qquad (6)$$

with $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\lambda_i > 0$.

Using (6), the model (4) can be rewritten as

$$M\dot{s} = -M_0\ddot{q}_r - C_0\dot{q}_r - G_0 - Cs + \tau - F + \tau_d \quad (7)$$

where $\dot{q}_r = \dot{q}_d - \Lambda e$, $\ddot{q}_r = \ddot{q}_d - \Lambda\dot{e}$.

Our objective is to design a control law such that the joint position and velocity errors go to zero. In other words, the control law should be able to force the system to move along the sliding surface s=0.

If the uncertain bound F_0 is known, based on the sliding mode control technique, we can design the following law that can guarantee the desired dynamic performance.

Theorem 1. Consider the system (4) with the sliding surface s=0 defined by (6). If the control law is designed as

$$\tau = M_0\ddot{q}_r + C_0\dot{q}_r + G_0 - Ks - (F_0(t) + T_0)sign(s) \quad (8)$$

where $K \in R^{n\times n}$ is positive definite matrix.

Then, the tracking errors of joint position and velocity asymptotically converge to zero.

Proof : Choose the Lyapunov function candidate as

$$V = \frac{1}{2}s^T Ms \quad (9)$$

Differentiating V with respect to time, and using expression (7), we get

$$\dot{V} = \frac{1}{2}s^T\dot{M}s + s^T M\dot{s}$$

$$= s^T(\frac{1}{2}\dot{M} - C)s + s^T(-M_0\ddot{q}_r - C_0\dot{q}_r -$$

$$G_0 + \tau - F + \tau_d) \quad (10)$$

Substituting (8) into (10), and using property 1, we have

$$\dot{V} = -s^T Ks + s^T(-(F_0 + T_0)sign(s) - F + \tau_d)$$

Noting that

$$s^T(-(F_0 + T_0)sign(s) - F + \tau_d)$$

$$\leq -|s|^T(F_0 + T_0 - |F| - |\tau_d|) < 0 \quad \text{for } |s| \neq 0$$

Therefore, we get

$$\dot{V} < -s^T Ks \leq -\lambda_{\min}(K)\|s\|^2 < 0$$

Thus, by means of Lyapunov theory, s reaches the sliding mode s=0 in a finite time. This, in turn, implies that $e \to 0$, $\dot{e} \to 0$, as $t \to \infty$.

Remark 1. In sliding mode control law (8), the only knowledge required is the upper bound of system uncertainty. However, in general, the precise bound on the uncertainty is rarely available in practice. Therefore, in the following, we consider the case that the positive nonlinear function F_0 is unknown.

Ⅲ. NEURAL NETWORK

In this paper, we use a RBF neural network to adaptively learn the bound F_0 of system uncertainty, i.e.

$$\hat{F}_0(x, \hat{\theta}) = \hat{\theta}^T\phi(x) \quad (11)$$

Where $\hat{\theta}$ is the estimation of network weight vector θ , and the vector $\phi(x)$ is Gaussian type of function whose ith element is defined as

$$\phi_i(x) = \exp\left(-\| x - c_i\|^2/\sigma^2\right) \quad (12)$$

Where c_i represents the center of the ith basis function, σ represents the spread of the basis function.

Further, the following assumption are made

A1) The network approximation error $\varepsilon(x) \in R^n$ is bounded by known positive constant, i.e.

$$|\varepsilon(x)| = |\theta^{*T}\phi(x) - F_0(t)| < w \quad (13)$$

where θ^* is the optimal weight.

Remark 2. Assumption A1 is quite common in the neural networks literature, and has been proved by many researchers(Sadegh, N., 1991 , Sanner, R . M., *et. al.* , 1992).

A2) For the bound $F_0(t)$ of system uncertainty and the vector w in (13), the following inequality is satisfied

$$F_0(t) - |F(q,\dot{q},\ddot{q})| > w \qquad (14)$$

In the following, we will use the output of neural estimator (11) to adjust the switching gain so that switching planes s=0 are ensured asymptotically stable, and then the tracking errors asymptotically converge to zero.

Ⅳ. NEURAL SLIDING-MODE CONTROL

Considering the following control law

$$\tau = M_0\ddot{q}_r + C_0\dot{q}_r + G_0 - Ks - (\hat{\theta}^T\phi(x) + T_0)sign(s) \quad (15)$$

with the weight adaptive updating rule

$$\dot{\hat{\theta}} = \eta\phi(x)\,|s|^T \qquad (16)$$

where $\eta > 0$, is the adaptive rate.

Theorem 2. For the robot system (4) with the sliding surface (6) and assumptions A1 and A2, the control law (15) with (16) can guarantee that the tracking errors of joint position and velocity asymptotically converge to zero.

Proof. Define the following Lyapunov function

$$V = \frac{1}{2}s^T Ms + \frac{1}{2}tr(\tilde{\theta}^T\eta^{-1}\tilde{\theta}) \qquad (17)$$

where $\tilde{\theta} = \hat{\theta} - \theta^*$, $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$.

Differentiating V with respect to time and considering expression (7), we have

$$\dot{V} = \frac{1}{2}s^T\dot{M}s + s^T M\dot{s} + tr(\tilde{\theta}^T\eta^{-1}\dot{\tilde{\theta}})$$

$$= s^T(\frac{1}{2}\dot{M} - C)s + s^T(-M_0\ddot{q}_r - C_0\dot{q}_r - G_0 + \tau - F + \tau_d) + tr(\tilde{\theta}^T\eta^{-1}\dot{\tilde{\theta}}) \qquad (18)$$

Using control law (15), (16), and property 1, we have

$$\dot{V} = -s^T Ks + s^T(-(\hat{\theta}^T\phi + T_0)sign(s) - F + \tau_d) + tr(\tilde{\theta}^T\eta^{-1}\dot{\tilde{\theta}})$$

$$= -s^T Ks + s^T(-(\hat{\theta}^T\phi sign(s)) + s^T(-T_0 sign(s) - F + \tau_d) + + tr((\hat{\theta} - \theta^*)^T\eta^{-1}\dot{\tilde{\theta}})$$

$$= -s^T Ks - |s|^T\hat{\theta}^T\phi + tr(\hat{\theta}^T\phi|s|^T) + s^T(-T_0 sign(s) - F + \tau_d) - tr(\theta^{*T}\phi|s|^T)$$

Considering the following equalities

$$tr(\hat{\theta}^T\phi|s|^T) = tr(|s|^T\hat{\theta}^T\phi) = |s|^T\hat{\theta}^T\phi$$

$$tr(\theta^{*T}\phi|s|^T) = tr(|s|^T\theta^{*T}\phi) = |s|^T\theta^{*T}\phi$$

we have

$$\dot{V} = -s^T Ks + s^T(-T_0 sign(s) - F + \tau_d) - |s|^T\theta^{*T}\phi$$

Further, Noting that

$$s^T(-F + \tau_d) \le |s|^T|F - \tau_d| \le |s|^T(|F| + |\tau_d|)$$

we can get

$$\dot{V} \le -s^T Ks - |s|^T(\theta^{*T}\phi - |F|) - |s|^T(T_0 - |\tau_d|) \quad (19)$$

Finally, considering Assumption A1, A2, and expression (5), the second term and the third term in (19) satisfy the following inequalities, respectively,

$$-|s|^T(\theta^{*T}\phi - |F|) = -|s|^T(\theta^*\phi - F_0 + F_0 - |F|)$$

$$= -|s|^T(\varepsilon(X) + F_0 - |F|)$$

$$\le -|s|^T(\varepsilon(X) + w) < 0 \qquad \text{for } |s| \ne 0$$

and $-|s|^T(T_0 - |\tau_d|) < 0$.

Hence, we get

$$\dot{V} < -s^T Ks \le -\lambda_{\min}(K)\|s\|^2 < 0$$

Then, it following from the Lyapunov theory that s approaches zero in finite time, i.e. the sliding mode occurs in finite time. That implies $e \to 0$, $\dot{e} \to 0$, as $t \to \infty$.

Remark 3. In this theorem, the uncertain bound needed for calculating the switching gain is estimated by RBF neural network. Hence, this proposed control scheme removes the limitation that *a priori* knowledge about the bound of unknown parameters is required in conventional sliding mode control.

Remark4 .Since the control law (15) is discontinuous along the sliding surface, it can lead to control chattering. Chattering is undesirable in practice because it involves high control activity and may excite high frequency unmodelled dynamics. The problem of chattering can be solved by smoothing out the discontinuous control inside a boundary layer neighboring the sliding surface (Slotine, J.J. E. , 1987, T. P. Leung, *et. al*., 1991).

Ⅴ. CONCLUSION

A neural network-based adaptive sliding-mode control design methodology is proposed by using the theory of VSS and the nonlinear mapping of neural network. The major contribution of this scheme lies in removing the requirement of *a priori* knowledge of the uncertain bound by adaptive of the switching gain using neural network. This control technique can guarantee the asymptotic convergence of joint position and velocity tracking error to zero, and has strong robustness with respect to system uncertainties and external disturbance.

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