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ON MULTI-OBJECTIVE IDENTIFICATION OF TAKAGI-SUGENO FUZZY MODEL PARAMETERS

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Abstract: The problem of identifying the parameters of the constituent local linear models of Takagi-Sugeno fuzzy models is considered. In order to address the tradeoff between global model accuracy and interpretability of the local models as linearizations of a nonlinear system, two multi-objective identification algorithms are studied. Particular attention is paid to the analysis of conflicts between objectives, and we show that such information can be easily computed from the solution of the multi-objective optimization. This information is useful to diagnose the model and tune the weighting/priorities of the multi-objective optimization. Moreover, the result of the conflict analysis can be used as a constructive tool to modify the fuzzy model structure (including membership functions) in order to meet the multiple objectives. The methods are illustrated on an experimental lungs respiration application.

Keywords: Fuzzy Modelling, Identification, Multi-objective Optimization, Sensitivity Analysis, Parameter Optimization.

1. INTRODUCTION

We consider the problem of identifying the parameters of the constituent local linear models of Takagi-Sugeno fuzzy models (Takagi and Sugeno 1985). It is well known that several tradeoffs are involved in this problem. Models identified by minimizing the global prediction error need not have constituent local linear models which are interpretable as valid linearizations of the underlying nonlinear system (Murray-Smith and Johansen 1997, Shorten *et al.* 1999, Johansen *et al.* 2000, Yen *et al.* 1998, Abonyi and Babuška 2000). In the same references it is also shown that when identifying the local linear models by minimizing individual locally weighted prediction error criteria, the identified local linear models have locally valid interpretations as linearizations, under assumptions on identifiability and persistence of excitation. On the other hand, the global prediction performance is typically inferior to what can be achieved with a global performance criterion. The lack of local interpretability with global identification is also closely linked to poor identifiability and/or the choice of fuzzy membership functions. This is partly due to the interaction between the local models, or the "degree of orthogonality" among them, which is related to the degree of overlap between their associated membership functions and the degree of smoothness of the model (Murray-Smith and Johansen 1997). In other words, with a global identification approach there is a tradeoff between local interpretability and smoothness.

In this work we explicitly address the tradeoffs between local interpretability of the local models as linearizations, and the prediction performance of the global model. The objective is to identify smooth Takagi-Sugeno fuzzy models with local models that have valid interpretations as local linearizations while minimizing the model's global prediction performance. The main idea is to formulate the problem as a multi-objective optimization problem, which is a natural approach since the problem, as specified above, consists of several conflicting objectives. Minimizing a weighted sum of global and local prediction error criteria was also suggested in (Yen *et al.* 1998) for Takagi-Sugeno fuzzy models, and the present work extends this in the sense that we provide tools for analysis of the conflicts among objectives and thus methods for selecting the weights or priorities among conflicting objectives. Next, we suggest an alternative two-step multi-objective identification algorithm. The conflict between global performance and local interpretability was also discussed in (Passaquay *et al.* 2000), where a measure for consistency between the two objectives was suggested.

2. TAKAGI-SUGENO FUZZY MODEL

The framework presented here is the identification of dynamic Takagi-Sugeno fuzzy input/output models of the form (Takagi and Sugeno 1985)

$$y(t) = \sum_{i=1}^{N} \Big(-a_{1,i} y(t-1) - \cdots - a_{n_y,i} y(t-n_y) + b_{0,i} u(t) + \cdots + b_{n_u,i} u(t-n_u) + d_i \Big) w_i(z(t)) + e(t) \quad (1)$$

where $u(t) \in \mathbb{R}^r$ is the input, $y(t) \in \mathbb{R}^m$ is the output, $e(t) \in \mathbb{R}^m$ accounts for unmodelled phenomena and $z(t) \in \mathbb{R}^d$ is a vector of premise variables derived from the information vector

$$\tilde{\psi}(t) = (-y(t-1), ..., -y(t-n_y), u(t), u(t-1), ..., u(t-n_u))^T$$

Fuzzy models of the form (1) result from fuzzy inference on a set of fuzzy rules

IF $z(t) \in Z_i$ THEN

$$y(t) = -a_{1,i} y(t-1) - ... - a_{n_y,i} y(t-n_y) + b_{0,i} u(t) + ... + b_{n_u,i} u(t-n_u) + d_i$$

where the premise is defined by a fuzzy set $Z_i \subset \mathbb{R}^p$ and the consequent is a local linear dynamic model. The function $w_i : \mathbb{R}^p \rightarrow [0,1]$ is defined by the membership functions $\mu_i : \mathbb{R}^p \rightarrow [0,1]$ of Z_i

$$w_i(z) = \frac{\mu_i(z)}{\sum_{j=1}^{N} \mu_j(z)} \quad (2)$$

The only assumption we make on the set of fuzzy rules is that it is complete such that for all z, $\mu_j(z) > 0$ for some j, and (2) is well defined. Eq. (1) can be reformulated into a form that is more convenient for system identification by introducing the definitions $\psi(t) = (\tilde{\psi}^T(t), 1)^T$ and $\theta_i(t) = (a_{1,i}, ..., a_{n_y,i}, b_{0,i}, ..., b_{n_u,i}, d_i)^T$, where $\psi(t)$ is the information vector augmented with a constant element, and θ_i are the possibly unknown parameters associated with the local linear model of the i-th rule. With these definitions

$$y(t) = \sum_{i=1}^{N} \psi^T(t) \theta_i w_i(z(t)) + e(t) \quad (3)$$

Furthermore, defining

$$\varphi(t) = \begin{pmatrix} \psi(t) w_1(z(t)) \\ \vdots \\ \psi(t) w_N(z(t)) \end{pmatrix}, \; \theta(t) = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_N \end{pmatrix} \quad (4)$$

the linear regression form follows:

$$y(t) = \varphi^T(t)\theta + e(t) \quad (5)$$

2.1 *Global identification algorithm*

The objective of this algorithm is to identify the local model parameters $\theta_1, ..., \theta_N$ that give a global model with the best prediction performance (Takagi and Sugeno 1985). Consider the global least squares prediction error criterion

$$V(\theta) = \frac{1}{n} \sum_{t=1}^{n} (y(t) - \varphi^T(t)\theta)^2 \quad (6)$$

subject to equality and inequality constraints on the parameters

$$H_i(\theta_i) = 0, \; F_i(\theta_i) \leq 0, \; i = 1, 2, ..., N \quad (7)$$

where H_i and F_i are affine functions of θ_i. The usefulness of such constraints in order to improve the accuracy and robustness of the estimate is shown in (Johansen *et al.* 2000, Abonyi *et al.* 2000) and the references therein.

2.2 *Locally weighted identification algorithm*

The objective of this algorithm is to identify local model parameters $\theta_1, ..., \theta_N$ that give local models which are close local approximations to the underlying nonlinear system (Johansen and Foss 1993, Murray-Smith and Johansen 1997). Consider a locally weighted least squares prediction error criterion associated with each local model

$$V_i(\theta) = \frac{1}{n} \sum_{t=1}^{n} (y(t) - \psi^T(t)\theta_i)^2 w_i(z(t)) \quad (8)$$

subject to the constraints (7). The weighting factor $w_i(z(t))$ ensures that the parameters θ_i are influenced only by the data points within the fuzzy set Z_i that defines the region of validity of the i-th local model.

3. MULTI-OBJECTIVE IDENTIFICATION ALGORITHM I

It was suggested in (Yen *et al.* 1998) to minimize the weighted sum of the global and local identification criteria (6) and (8). Here we apply a slight extension, including the constraints (7) and individual weighting parameters for each of the individual local models. The algorithm solves the optimization problem

$$\min_{\theta} \left(V(\theta) + \sum_{i=1}^{N} \beta_i V_i(\theta_i) \right) \quad (9)$$

subject to (7). The weighting parameters $\beta_i \geq 0$ parameterize the set of Pareto-optimal solutions of the underlying multi-objective optimization problem, and essentially determine the tradeoff between the possibly conflicting objectives of global model accuracy and local model interpretability.

The selection $\beta_i = 1$ in the multi-objective criterion (9) will in general give a fairly balanced tradeoff, due to the use of w_i (which forms a partition of unity) for weighting in (8). It is still of interest to study in detail how the choice of β influences the tradeoff. This problem is not discussed in detail in (Yen *et al.* 1998). In particular, it is of interest to analyze the degree of conflict between the different objectives in (9) for a given data sequence and different values of β.

In this section, let the minimum of (9) be denoted $\hat{\theta}(\beta)$ for a given data sequence and a vector of weights β. The minimum of (9) satisfies the Karush-Kuhn-Tucker (KKT) conditions (Luenberger 1984)

$$0 = \frac{\partial V}{\partial \theta}(\hat{\theta}(\beta)) + \sum_{i=1}^{N} \beta_i \frac{\partial V_i}{\partial \theta}(\hat{\theta}(\beta)) \quad (10)$$
$$+ \sum_{i=1}^{N} \hat{\mu}_i^T(\beta) \frac{\partial F_i}{\partial \theta}(\hat{\theta}(\beta)) + \sum_{i=1}^{N} \hat{\lambda}_i^T(\beta) \frac{\partial H_i}{\partial \theta}(\hat{\theta}(\beta))$$

where $\hat{\mu}_i(\beta) \geq 0$ and $\hat{\lambda}_i(\beta)$ are the Lagrange multipliers (vectors) associated with the estimate $\hat{\theta}_i(\beta)$ of the local model parameter vector. If there are no conflicts among the objectives and constraints, i.e. $\hat{\theta}(\beta)$ minimizes all of the individual objectives simultaneously and none of the inequality constraints are active, then each of the terms in (10) will be zero. If there are conflicts, on the other hand, the directions and lengths of each of the (vector) terms of (10) will indicate the degree of conflict and which constraints and objectives are actually in conflict with each other. For the unconstrained case, eq. (10) reduces to

$$\frac{\partial V}{\partial \theta_i}(\hat{\theta}(\beta)) + \beta_i \frac{\partial V_i}{\partial \theta_i}(\hat{\theta}(\beta)) = 0 \quad (11)$$

Define the following sensitivity measures associated with each parameter $\theta_{i,j}$, which is the j-th parameter in the i-th local model:

$$\pi_{i,j}^g(\beta) = -\frac{\partial V}{\partial \theta_{i,j}}(\hat{\theta}(\beta)) \quad (12)$$

$$\pi_{i,j}^l(\beta) = -\frac{\partial V_i}{\partial \theta_{i,j}}(\hat{\theta}(\beta)) = \frac{1}{\beta_i}\frac{\partial V}{\partial \theta_{i,j}}(\hat{\theta}(\beta)) \quad (13)$$

The quantity $\pi_{i,j}^g(\beta)$ can be interpreted as the small decrease in the global identification criterion V that can be achieved by a small increase in $\hat{\theta}_{i,j}(\beta)$. Likewise, the quantity $\pi_{i,j}^l(\beta)$ can be interpreted as the small decrease in the local identification criterion V_i that can be achieved by a small increase in $\hat{\theta}_{i,j}(\beta)$. Hence, large values of $\pi_{i,j}^g(\beta)$ and $\pi_{i,j}^l(\beta)$ indicate conflicts between global and local performance. Notice that due to (11)

$$\pi_{i,j}^g(\beta) + \beta_i \pi_{i,j}^l(\beta) = 0 \quad (14)$$

An analysis of $\pi_{i,j}^g(\beta)$ and $\pi_{i,j}^l(\beta)$ can provide the user with significant information about the model and data, as illustrated in the example below.

4. MULTI-OBJECTIVE IDENTIFICATION ALGORITHM II

The idea is to first compute the locally weighted estimates $\tilde{\theta}_i$ as described in section 2.2. In the second step of the algorithm the global least squares prediction error is minimized subject to small deviations of the relevant parameters from the locally identified parameters, i.e.

$$\min_{\theta} V(\theta) \quad (15)$$

subject to (7) and

$$\tilde{\theta}_i - \Delta\theta_i \leq \theta_i \leq \tilde{\theta}_i + \Delta\theta_i \quad (16)$$

for $i = 1, 2, ..., N$. The allowed deviation $\Delta\theta_i$ from the locally identified parameters $\tilde{\theta}_i$ might be specified by the user and/or might be generated from some uncertainty estimate (e.g. standard deviation) of the locally weighted estimate $\tilde{\theta}_i$. Hence, it is guaranteed that the local model parameters retain their interpretability as local linearizations (within a user-specified tolerance) when they are tuned for global model performance in the second step of the algorithm. Notice that the interpretability of the local model parameters as linearizations may or may not involve the offset parameters d_i. Hence, depending on the application, the offset parameters d_i may in some cases be allowed to vary freely in the global optimization (15).

Also with this algorithm, a study of the KKT conditions contains information on conflicts between the objectives. In particular, it is useful to study the Lagrange multipliers associated with the deviation constraints (16). A zero Lagrange multiplier associated with some constraint $\theta_{i,j} \leq \tilde{\theta}_{i,j} + \Delta\theta_{i,j}$ (or $\tilde{\theta}_{i,j} - \Delta\theta_{i,j} \leq \theta_{i,j}$) means that the global prediction performance cannot be improved by increasing the allowed deviation $\Delta\theta_{i,j}$ alone since the Lagrange multiplier $\hat{\lambda}_{i,j}$ has the following sensitivity interpretation (Luenberger 1984):

$$\hat{\lambda}_{i,j} = -\frac{\partial V}{\partial \Delta\theta_{i,j}}(\hat{\theta}) \quad (17)$$

On the other hand, the non-negative value of the Lagrange multiplier $\hat{\lambda}_{i,j}$ tells us how much the global prediction error criterion $V(\theta)$ might be reduced by a small increase in the allowed deviation $\Delta\theta_{i,j}$. Thus, it is straightforward to determine those local model parameters that contribute to the conflict between objectives; they have non-zero Lagrange multipliers for some associated constraint.

5. EXPERIMENTAL RESULTS: ESTIMATION OF LUNGS RESPIRATION DYNAMICS

In order to further illustrate the suggested methods, we consider the problem of estimation of respiration dynamics parameters described in (Babuška *et al.* 2001). This is important for monitoring respiratory mechanics in patients on ventilatory support, for example to assess patients' pulmonary conditions (for which the interpretability of the local model parameters is of vital importance) and to automatically

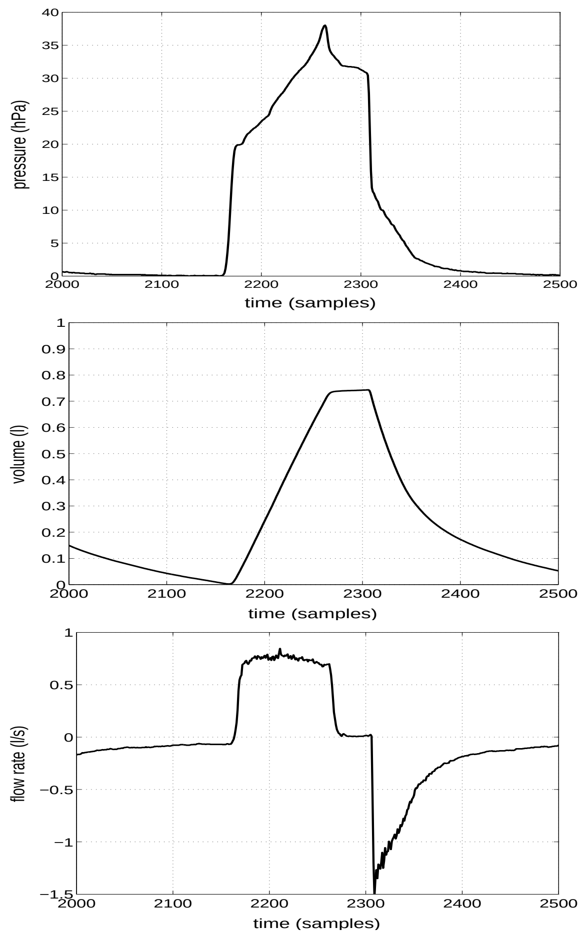

Fig. 1. Subset of a typical data sequence with respiratory data.

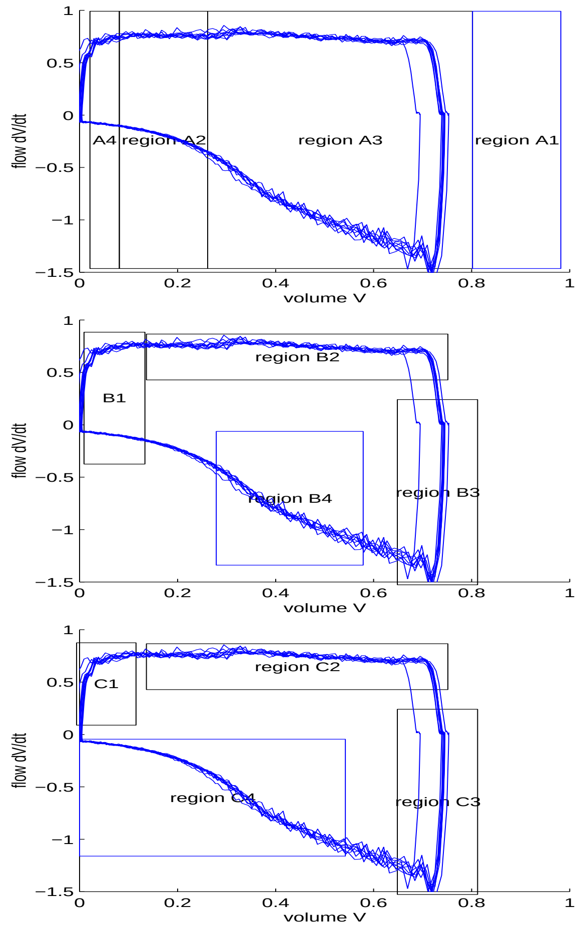

Fig. 2. Top: Partitioning of input domain for model structure A. Middle: Partitioning of input domain for model structure B. Bottom: Partitioning of input domain for model structure C. The rectangles provides a simplified illustration of the fuzzy sets.

control or optimize the ventilator settings (for which the global model accuracy may be more important). Consider the following dynamic relationship (Peslin *et al.* 1992, Lauzon and Bates 1991)

$$P = EV + R\frac{dV}{dt} + P_0 \qquad (18)$$

where V (ℓ) is the lungs volume, $\dot{V}$ (ℓ/s) is the flow rate through the ventilation tube, and P (hPa) is the pressure. We consider identification of the three parameters of this equation, namely the respiratory elestance E (hPa/ℓ), the resistance R (hPa $\cdot$ s/ℓ) and the elastic recoil pressure P_0 (hPa). This problem is viewed as a regression problem where Takagi-Sugeno fuzzy models consisting of multiple linear models of the form

$$P = E_i V + R_i \frac{dV}{dt} + P_{0,i} \qquad (19)$$

are identified. In other words, the models predict P as a function of V and $\dot{V}$. Consider three different fuzzy model structures, each with 4 local models of the form (19).

- **Model Structure A**: A fuzzy model where the membership functions are identified using the algorithm described in (Johansen and Foss 1995).
- **Model Structure B:** A fuzzy model where the membership function are selected "manually" to be consistent with the experiments using fuzzy clustering reported in (Babuška *et al.* 2001).
- **Model Structure C:** Similar to Model Structure B, with with somewhat different membership functions.

In the continuation we assume the membership function of these three model structures are fixed, and consider identification of the consequent parameters.

A single cycle from a typical data sequence is shown in Figure 1. The respiration cycle consists of three phases; i) inspiration phase (forced inlet flow), ii) respiration pause (no flow) and iii) expiration phase (free outlet flow). Figure 2 illustrates the fuzzy partition of the three model structures under consideration, with the identification data sequence projected onto the plane described by $(V, \dot{V})$. The identification data sequence consists of measurements from 10 respiration cycles for a single patient. Notice that patients may have different respiration dynamics, so it is generally desirable to identify a model for each patient (Babuška *et al.* 2001).

For each of the model structures A, B and C we identified the local model parameters using i) locally weighted least squares, ii) least squares, iii) multi-objective algorithm I with $\beta^* = 1$, and iv) multi-objective algorithm II with $\Delta\theta^* = 1$. The results are given in Tables 1, 2 and 3. The average RMS residuals for these cases are given in Table 4, and the sensitivity measures associated with the multi-objective identification algorithms are shown in Figure 3.

Table 1. Local parameters of model structure A, using different identification methods. Algorithm I is with $\beta^* = 1$ and algorithm II is with $\Delta\theta^* = 1$.

Region	Param.	LWLS	LS	Alg. I	Alg. II
A1	E	38.93	-696.5	33.93	39.88
	R	9.329	-12.84	6.469	8.329
	P_0	3.038	565.5	7.092	4.038
A2	E	28.36	-1.364	23.57	27.36
	R	20.93	26.67	22.80	21.93
	P_0	0.9699	1.429	1.371	-0.031
A3	E	42.47	21.20	42.14	41.47
	R	13.47	12.66	13.05	12.63
	P_0	-0.1901	10.42	0.075	0.665
A4	E	8.346	25.53	-2.554	7.346
	R	23.21	24.33	24.39	24.21
	P_0	1.822	-1.38	2.265	2.389

Table 2. Local parameters of model structure B, using different identification methods. Algorithm I is with $\beta^* = 1$ and algorithm II is with $\Delta\theta^* = 1$.

Region	Param.	LWLS	LS	Alg. I	Alg. II
B1	E	12.39	8.578	10.71	11.39
	R	22.89	23.18	23.35	23.89
	P_0	2.080	2.273	2.241	2.318
B2	E	29.86	30.18	29.76	30.48
	R	11.89	6.202	10.83	10.89
	P_0	7.325	12.63	8.100	7.852
B3	E	53.69	60.06	54.74	54.69
	R	10.41	9.989	10.12	9.704
	P_0	-7.809	-10.95	-8.579	-8.582
B4	E	50.85	19.10	51.31	51.36
	R	12.83	-1.049	12.42	11.83
	P_0	-5.029	-2.620	-5.485	-6.029

Table 3. Local parameters of model structure C, using different identification methods. Algorithm I is with $\beta^* = 1$ and algorithm II is with $\Delta\theta^* = 1$.

Region	Param.	LWLS	LS	Alg. I	Alg. II
C1	E	8.669	-19.49	-6.867	7.669
	R	23.15	28.94	23.75	23.50
	P_0	2.116	-0.381	3.295	3.116
C2	E	29.68	21.18	29.21	29.40
	R	11.30	4.342	9.673	10.30
	P_0	7.736	18.49	9.214	8.736
C3	E	54.37	51.76	53.26	53.92
	R	10.47	14.08	10.03	9.468
	P_0	-8.296	1.486	-7.229	-7.296
C4	E	42.07	-28.53	40.68	41.07
	R	13.63	-18.51	12.64	12.63
	P_0	-1.300	3.004	-1.547	-2.186

Table 4. Root-mean-square residuals.

Model	LWLS	LS	Alg. I	Alg. II
Structure A	1.9130	1.1067	1.6854	1.7368
Structure B	1.4631	1.3758	1.4354	1.4209
Structure C	1.9977	1.3859	1.8582	1.8565

5.1 *Discussion of the results*

There are large differences between the locally and globally identified local model parameters, especially with model structure A and C. In many cases, the globally identified local model parameter estimates of E and R are in conflict with their physical interpretation (e.g. when they are negative). Both the multi-objective identification algorithms can be used to address this tradeoff.

When comparing the sensitivities of the conflict analysis for model structures A and B we observe that the sensitivities with model structure A are up to one order of magnitude larger than the sensitivities with model structure B. This indicates that the use of model structure A leads to a significant conflict between local model interpretability and global prediction accuracy, compared to model structure B. Certainly, model structure A leads to a smaller global prediction error than model structure B when the models are identified using global least squares. However, model structure B admits a more intuitive and appealing interpretation of its membership functions as the partitioning of the input domain is closely related to the different phases of the respiration cycle; region B1 corresponds to the first part of the inspiration phase, region B2 corresponds to the final part of the inspiration phase, region B3 includes the respiration pause, while region B4 contains the expiration phase.

Perhaps more interestingly, we observe that the sensitivities with model structure C are also up to one order of magnitude larger than the sensitivities with model structure B. This large difference points out a major conflict in model structure C, especially since Figure 2 shows that the membership functions are only slightly different. The only difference is that the fuzzy sets B1 and B4 (in model structure B) are slightly shifted and resized versions of C1 and C4 (in model structure C). The other two fuzzy sets are exactly the same in both model structures. Hence, the conflict analysis for model structure C points out a structural problem, namely that there will be a conflict between global prediction performance and local interpretability due to an unfortunate interaction between the membership functions for the fuzzy sets C1 and C4. As ”proved” by model structure B, this conflict can be resolved by a small modification of these fuzzy sets.

As expected, a high level of sensitivity (or conflict) seems to be correlated with a high dependence of the residuals on which parameter identification criterion is being used, see Table 4. Especially for model structures A and C, there is a large difference between locally weighted least squares and the global least squares algorithms, while this difference is small for model structure B.

In summary, it is clear that model structure B is better than A and C, even though this is not evident from only inspecting the residuals in Table 4. The conflict analysis can be used as a constructive tool to assist modification of the model structure, including membership functions.

6. CONCLUSIONS

A multi-objective optimization formulation of the identification problem arises naturally due to the two conflicting objectives. In this paper we have studied two multi-objective formulations (one of them was previously proposed in (Yen *et al.* 1998)) and suggested algorithms for their solution and tools for analysis of the solution in terms of conflicts and sensitivity. As shown by the example, this conflict/sensitivity analysis provides useful information not only about the local model parameter estimates, but also about the adequateness of the model structure and membership functions. Hence, we believe multi-objective optimization is a useful tool for Takagi-Sugeno fuzzy identification.

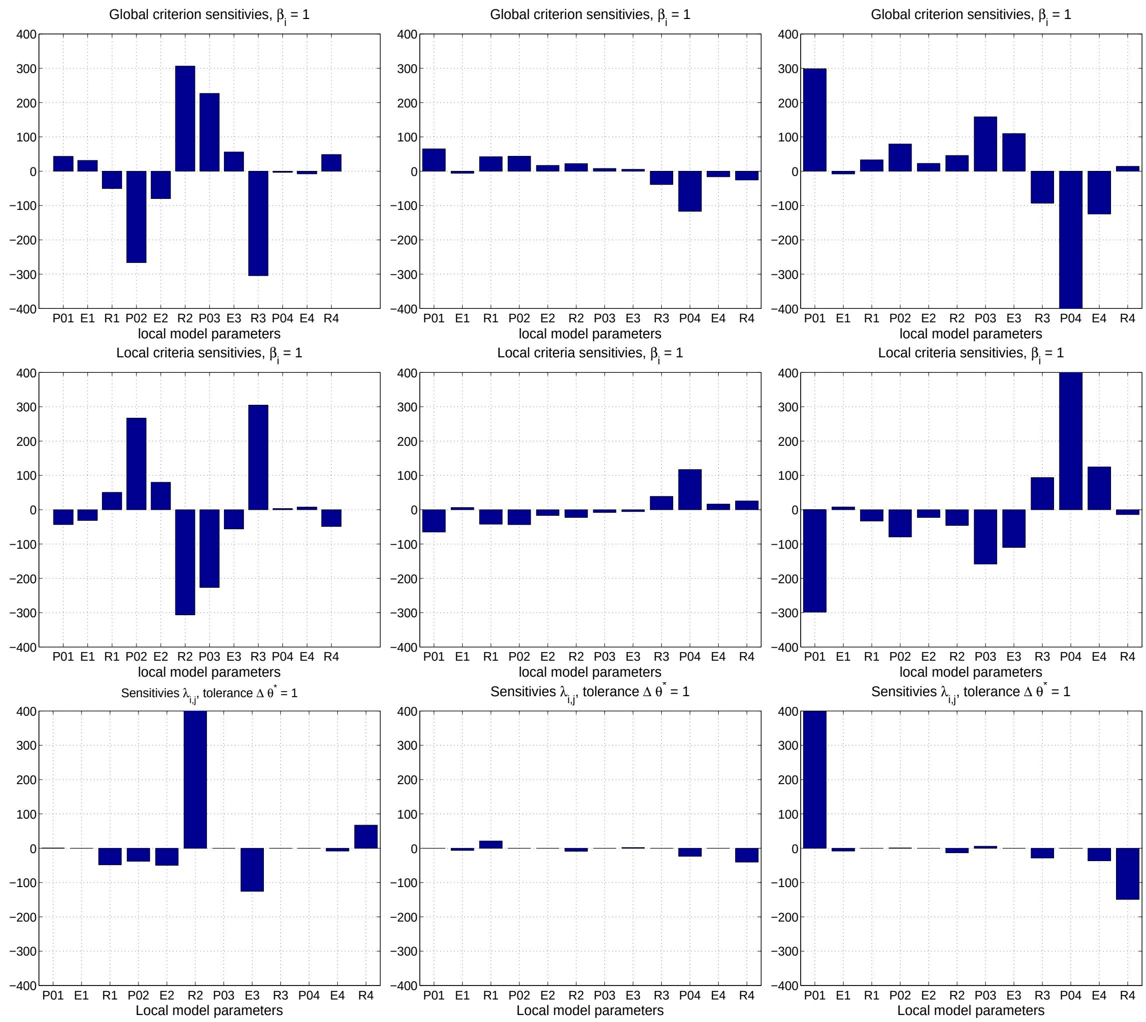

Fig. 3. Left column: Sensitivities with model structure A. Middle column: Sensitivities with model structure B. Right column: Sensitivities with model structure C.

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