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A SIMPLE STABILIZATION ALGORITHM FOR THE PVTOL AIRCRAFT

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Abstract: A simple stabilizing algorithm is presented for the planar vertical takeoff and landing (PVTOL) aircraft. The controller presents no discontinuities and does not have singularities. The stability is proved by using very simple arguments and satisfactory behavior is shown in simulations.

Keywords: Aircraft control, Aerospace control, Non-linear control systems, Stabilizing controllers, Stability analysis, Lyapunov function

1. INTRODUCTION

Flight control is an essential control problem which appears in many applications, such as spacecraft, aircraft, helicopters, etc... The complete dynamics of an aircraft, taking into account aeroelastic effects, flexibility of the wings and internal dynamics of the engine is quite complex and somewhat unmanageable for the purposes of control. It is thus particularly interesting to consider a simplified aircraft which has a minimum number of states and inputs but retains the main features that must be considered when designing control laws for a real aircraft. Therefore, as considered by Hauser et al. (Hauser *et al.*, 1992), we focus our study on the Planar Vertical Take Off and Landing (PVTOL) aircraft, which is a highly maneuverable jet aircraft.

Several methodologies for controlling such a system exist in the literature. Hauser et al. (Hauser *et al.*, 1992) in 1992 developed an approximate I-O linearization procedure which results in bounded tracking and asymptotic stability for the V/STOL aircraft.

In 1996, Andrew R. Teel (Teel, 1996) illustrated his central result of nonlinear small gain theorem using the example of the PVTOL aircraft with input corruption. His theorem provided a formalism for analyzing the behavior of control systems with saturation. He established a stabilization algorithm for nonlinear systems in so-called feedforward form which includes the PVTOL aircraft.

In 1996 also, Martin et al. (Martin *et al.*, 1996) presented an extension of the result proposed by Hauser (Hauser *et al.*, 1992). Their idea was to find a flat output for the system and to split the output tracking problem in two steps. Firstly, they designed a state tracker based on exact linearization by using the flat output and secondly, they designed a trajectory generator to feed the state tracker. They thus controlled the tracking output through the flat output. In contrast to the approximate-linearization based control method proposed by Hauser, their control scheme provided output tracking of nonminimum phase flat systems. They have also taken into account in the design the coupling between the rolling moment and the lateral acceleration of the aircraft (i.e. $\varepsilon \neq 0$).

An optimal controller was applied to the PVTOL aircraft in 1999 (Lin *et al.*, 1999). Lin et al. studied robust hovering control of the PVTOL using nonlinear state feedback based on optimal control.

Reza Olfati-Saber (Olfati-Saber, 2000) proposed a global configuration stabilization for the VTOL aircraft with a strong input coupling using a smooth static state feedback.

Contrary to other approaches presented in the literature, the controller proposed in this paper is relatively simple, is continuous, has no singularities and the input is not required to be positive or different from zero. The stability analysis is carried out in a simple manner and satisfactory performance is shown in simulations.

The paper is organized as follows. In section 2, the equations of motion for the PVTOL are recalled. In section 3, we develop a stabilizing control law for the PVTOL aircraft and we present the stability analysis in section 4. Simulations are presented in section 5. Conclusions are finally given in section 6.

2. THE PVTOL AIRCRAFT MODEL

The basic equations of motion for the PVTOL aircraft are given by (see (Hauser *et al.*, 1992))

$$\begin{aligned} \ddot{x} &= -\sin(\theta)u_1 + \varepsilon\cos(\theta)u_2 \\ \ddot{y} &= \cos(\theta)u_1 + \varepsilon\sin(\theta)u_2 - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \qquad (1)$$

where x, y denote the horizontal and the vertical position of the aircraft center of mass and θ is the roll angle that the aircraft makes with the horizon. The control inputs u_1 and u_2 are the thrust (directed out the bottom of the aircraft) and the angular acceleration (rolling moment). The parameter ε is a small coefficient which characterizes the coupling between the rolling moment and the lateral acceleration of the aircraft. The coefficient "-1" is the normalized gravitational acceleration. Figure (1) provides a representation of the system.

In the present paper we will consider a simplified model of the PVTOL aircraft system, i.e. with $\varepsilon = 0$. Indeed, we propose to control the system as if there were no coupling between rolling moments and lateral acceleration. Therefore, the equations of motion of the system (1) become

$$\begin{aligned} \ddot{x} &= -\sin(\theta)u_1 \\ \ddot{y} &= \cos(\theta)u_1 - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \qquad (2)$$

This choice is due to the fact that the coefficient ε is very small $\varepsilon << 1$ and not always well-known. The controller performance should be robust to

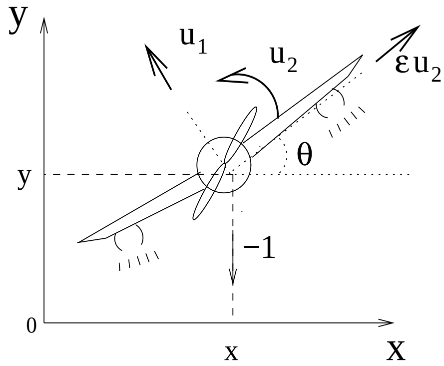

Fig. 1. The PVTOL aircraft (front view)

unmodeled dynamics, i.e. when $\varepsilon \neq 0$. Furthermore, several authors have shown that by an appropriate change of coordinates, we can obtain a representation of the system without the term due to ε. Indeed, R. Olfati-Saber (Olfati-Saber, 2000) applied the following change of coordinates

$$z = x - \varepsilon\sin(\theta) \qquad (3)$$

$$w = y + \varepsilon(\cos(\theta) - 1) \qquad (4)$$

After taking the second time derivatives of (2) and (3), the system's equations (1) in new coordinates become

$$\begin{aligned} \ddot{z} &= -\sin(\theta)\bar{u}_1 \\ \ddot{w} &= \cos(\theta)\bar{u}_1 - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \qquad (5)$$

where $\bar{u}_1 = u_1 - \varepsilon\theta^2$. The representation (**??**) has the same structure as (**??**) with the new control input $\bar{u}_1$.

3. STABILIZING CONTROL LAW

The controller is obtained by defining the following desired linear behavior for the position x and the altitude y. Let us, therefore, define the variables r_1 and r_2 as follows

$$\ddot{x} = r_1(x, \dot{x}) \triangleq -2\dot{x} - x \qquad (6)$$

$$\ddot{y} = r_2(y, \dot{y}) \triangleq -2\dot{y} - y \qquad (7)$$

Other choices are possible but the above has been chosen for simplicity. From (**??**) and (5) it follows

$$u_1 = \frac{1}{\cos\theta}(1 + r_2) \qquad (8)$$

which will not have any singularity provided $\tan\theta$ is bounded. In the stability analysis (section 4), we will prove that this is indeed the case. Introducing (**??**) into the system (**??**) gives

$$\ddot{x} = -\tan\theta(1 + r_2) \quad (9)$$
$$\ddot{y} = r_2 \quad (10)$$

From equation (**??**), it follows that $y^{(i)} \to 0$ for $i = 0, 1, \ldots$. It means that the altitude is stabilized around the origin. $y^{(i)} \in L_2$ and $r_2 \in L_2$. Independently of the value of $\cos\theta$, equation (**??**) holds. Note that if $\cos\theta \to 0$, then from (**??**), $u_1 \to \infty$. We will prove later that this is not the case.

Let us rewrite equation (**??**) as follows

$$\ddot{x} = -\tan\theta(1 + r_2) \pm r_1(1 + r_2) \quad (11)$$
$$= r_1(1 + r_2) - (\tan\theta + r_1)(1 + r_2) \quad (12)$$

Since we will prove, in the stability analysis, that r_1 will tend to zero, we also would like that $(\tan\theta + r_1)$ would converge to zero. Therefore, we introduce the error variable

$$\nu_1 \triangleq \tan\theta + r_1 \quad (13)$$

then

$$\dot{\nu}_1 = (1 + \tan^2\theta)\dot{\theta} + \dot{r}_1 \quad (14)$$
$$\ddot{\nu}_1 = (1 + \tan^2\theta)(u_2 + 2\tan\theta\dot{\theta}^2) + \ddot{r}_1 \quad (15)$$

We choose a control input u_2, so that the closed-loop system above is given by

$$\ddot{\nu}_1 = -2\dot{\nu}_1 - \nu_1 \quad (16)$$

where $s^2 + 2s + 1$ is a stable polynomial. Therefore, $\nu_1 \to 0$. The controller u_2 is then given by

$$u_2 = \frac{1}{1+\tan^2\theta}\Big(-2\dot{\theta}^2\tan\theta(1+\tan^2\theta) - \ddot{r}_1 - \tan\theta - r_1 - 2(1+\tan^2\theta)\dot{\theta} - 2\dot{r}_1\Big) \quad (17)$$

Note that u_2 is a function of $\{\theta, \dot{\theta}, r_1, \dot{r}_1, \ddot{r}_1\}$ and that all these variables can be expressed as a function of $\{x, y, \theta\}$ and their derivatives.

From (6) through (10) it follows that the closed-loop system can be rewritten as

$$\dot{\nu} = A\nu \quad (18)$$

where A is an exponentially stable matrix and $\nu^T = [\nu_1, \dot{\nu}_1]$. ν_1 converges exponentially to zero and $\nu_1 \in L_2 \cap L_\infty$. The main result is summarized in the following theorem.

Theorem 2. Consider the PVTOL aircraft model (**??**) and the control law in (**??**) and (10). Then the solution of the closed-loop system converges asymptotically to the origin, provided that $|\theta(0)| < \frac{\pi}{2}$.

We present the stability analysis of the above result in the following section.

4. STABILITY ANALYSIS

Let us rewrite the $(x, \dot{x})$ subsystem (**??**)-(6) as a linear system. Define $z^T = [x, \dot{x}]$, then

$$\dot{z} = Az + Bu \quad (19)$$
$$x = Cz \quad (20)$$

where $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$, $C = [1, 0]$, $B = [0, 1]^T$ and $u = r_1 r_2 - \nu_1(1 + r_2)$. Let us define $r_3 = \nu_1(1 + r_2)$. In view of (9), ν_1 converges exponentially to zero. Futhermore, in view of (**??**), (5) and (**??**), r_2 converges exponentially to zero.

Since ν_1 and r_2 converge both exponentially to zero, $r_3 \in L_2$, i.e. $\int_0^\infty r_3^2 dt$ is bounded. Therefore, $\int_0^t r_3^2 dt + \int_t^\infty r_3^2 dt = \int_0^\infty r_3^2 dt =$constant and $\frac{d}{dt}\int_t^\infty r_3^2 dt = -\frac{d}{dt}\int_0^t r_3^2 dt = -r_3^2$. We propose the following Lyapunov function candidate

$$V = z^T P z + 2\int_t^\infty r_3^2 dt \quad (21)$$

where P is a positive definite matrix, satisfying $A^T P + PA = -2I$. Note that $P = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$. Differentiating V, we obtain

$$\begin{aligned}
\dot{V} &= \dot{z}^T P z + z^T P \dot{z} - 2r_3^2 \\
&= \left[z^T A^T + u^T B^T\right] Pz + z^T P\left[Az + Bu\right] - 2r_3^2 \\
&= z^T\left[A^T P + PA\right] z + 2z^T PBu - 2r_3^2 \\
&= -2z^T z + 2z^T PBu - 2r_3^2 \\
&= -2(\dot{x}^2 + x^2) + 2(\dot{x} + x)\left[(-2\dot{x} - x)r_2 - r_3\right] \\
&\quad -2r_3^2 \qquad (22)
\end{aligned}$$

Since

$$\begin{aligned}
|2(\dot{x} + x)r_3| &= |2\dot{x}r_3 + 2xr_3| \le 2|\dot{x}r_3| \\
&\quad +2|xr_3| \\
&\le \dot{x}^2 + r_3^2 + x^2 + r_3^2 \qquad (23)
\end{aligned}$$

$\dot{V}$ becomes

$$\begin{aligned}
\dot{V} &\le -2(1 + 2r_2)\dot{x}^2 - 2(1 + r_2)x^2 - 6x\dot{x}r_2 \\
&\quad +x^2 + \dot{x}^2 + 2r_3^2 - 2r_3^2 \\
&\le -\dot{x}^2 - x^2 - 4r_2\dot{x}^2 - 2r_2 x^2 - 6x\dot{x}r_2 \qquad (24)
\end{aligned}$$

Since r_2 is decreasing exponentially to zero, it follows that for any $\bar{k} > 0$ there exists T large enough such that $r_2 < \bar{k}$, $\forall t > T$. In the sequel the results will hold for $t \geq T$. Since

$$|6r_2 x\dot{x}| \leq 6|\bar{k}x\dot{x}| \leq 3\bar{k}(x^2 + \dot{x}^2) \qquad (25)$$

We therefore obtain

$$\dot{V} \leq -(1 - 7\bar{k})\dot{x}^2 - (1 - 5\bar{k})x^2 \quad \forall t > T \ (26)$$

Choosing $\bar{k} = \frac{1}{10}$, it follows that $\dot{V} < 0$.

Using (4) and (6), equation (**??**) becomes

$$\ddot{x} = -\tan\theta(1 + r_2) \qquad (27)$$
$$= -(\nu_1 - r_1)(1 + r_2) \qquad (28)$$
$$= -(\nu_1 + 2\dot{x} + x)(1 + r_2) \qquad (29)$$

Since both ν_1 and r_2 are exponentially decreasing then $\ddot{x}$ is linear with respect to x and $\dot{x}$. Therefore x grows at most exponentially and does not exhibit finite escape time. From (11) and (16), we then have x and $\dot{x} \in L_2 \cap L_\infty$. Thus x and $\dot{x} \to 0$. Since

$$\tan\theta = \nu_1 - r_1 \qquad (30)$$
$$= \nu_1 + 2\dot{x} + x \qquad (31)$$

it follows that $\tan\theta \in L_2 \cap L_\infty$. We have thus proved that $\tan\theta$ is bounded and $\tan\theta \to 0$. Therefore, $\cos\theta \not\to 0$ and then the control law (**??**) is free from singularities. Finally, the solution of the closed-loop system converges asymptotically to zero for any initial condition such that $|\theta(0)| < \frac{\pi}{2}$.

5. SIMULATION RESULTS

In order to validate the results of the proposed control law, we have performed simulations. We started the PVTOL aircraft at the position $(x, y, \theta) = (2, 4, \frac{\pi}{3})$ and $(\dot{x}, \dot{y}, \dot{\theta}) = (3, 1, 1)$. We also ran simulations with the same control including in the system the term ε (see (1)). For $\varepsilon \leq 0.3$, the results were very similar as for $\varepsilon = 0$. Simulations showed that the performance of the proposed controller is satisfactory.

The simulation results for $\varepsilon = 0.3$ are shown in figures 2 and 3.

6. CONCLUSIONS

A new control strategy for the stabilization of the PVTOL aircraft is presented in this paper. The control law is simple, has no singularities and the input is not required to be positive or different from zero. Good performance of the proposed control law has been shown in simulations.

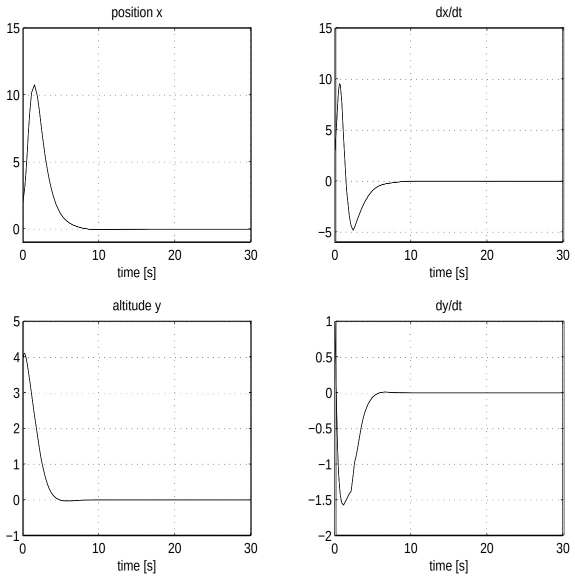

Fig. 2. States of the system

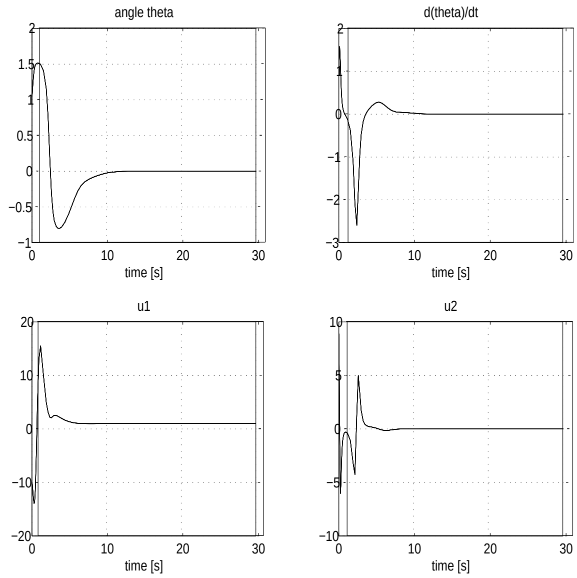

Fig. 3. States of the system and control inputs

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