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SOME PROPERTIES OF DIFFERENTIAL GAMES WITH NONCONVEX TARGET SETS

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Abstract: Two control systems with disturbances are considered. The first one is two-dimensional, and its target set is an arc. The second of the two conflict control systems is scalar. Its target set consists of two points. In the first system, no program control or Carathéodory strategy guarantees the evasion, but there exists a discontinuous feedback control that does. In the second system, evasion can be guaranteed by continuous strategies but not by program controls; the best possible distance for the evader is given by a discontinuous strategy and also by a multivalued upper semicontinuous strategy. In particular, this shows the difference between the case of nonconvex target set considered in this paper and the linear-convex case.

Keywords: differential games; feedback control; nonlinear control systems; disturbance; differential equations.

1. INTRODUCTION

Positional differential games can be used to model controlled systems in which the desired result should be guaranteed for all possible disturbances. The dynamics of these systems is assumed to be given by ordinary differential equations. Publications of many authors are devoted to the theory of positional differential games, cf., in particular, (Krasovskii and Krasovskii, 1995; Krasovskii and Subbotin, 1974; Subbotin and Chentsov, 1981) and references therein.

The following fact is known (Barabanova and Subbotin, 1970; Krasovskii and Subbotin, 1974, §55; Subbotin and Chentsov, 1981, §3). In a differential game with a convex target set, linear dynamics, and a fixed terminal moment, if the evasion cannot be ensured with the help of program controls, then neither can it be ensured by continuous strategies. This statement can also be formulated for a payoff functional, in which case the target set is replaced by the corresponding level set of the functional. It was shown in (Brykalov, 2001) that here the assumption of convexity of the target set (respectively, the level set) can be relaxed.

Two differential games with nonconvex (in one of the games, even disconnected) target sets are considered below. The differential equations that describe the dynamics of the games are linear in the open-loop case. The equations become nonlinear when the feedback is introduced into the systems. The evasion cannot be ensured by program controls in either of the games. Continuous strategies can guarantee the evasion in one of the games but fail to do that in the other one.

In particular, these differential games demonstrate some properties of continuous feedback in the case of nonconvex target set. Continuous strategies may turn out to be not more efficient than program controls in some games (which is typical for linear-convex case) but in others can be essentially more efficient.

At the end of the paper, the second of the considered two conflict control systems is used to show that a certain condition is essential in two statements, which deal with continuous feedback.

The following notation is used: $\mathbf{R}$ is the set of all real numbers; C^0 denotes the space of continuous functions; L_1 is the space of Lebesgue measurable integrable functions; AC is the space of absolutely continuous functions. Program controls are assumed to be Lebesgue measurable functions.

2. AN ARC AS A TARGET SET

This section deals with a planar problem of conflict control with an arc as the target set. It will be shown that program controls do not ensure the evasion in this problem. A feedback control will be constructed that guarantees the evasion and uses but one measurement of the phase vector. On the basis of Schauder fixed point theorem, it will be proved that in the considered problem, continuous in the phase vector strategies do not provide the evasion.

Problem 1. A two-dimensional vector $x = (x_1, x_2)$ is changing according to the system of differential equations

$$\dot{x}_1 = v, \quad \dot{x}_2 = 2(1-t)u \tag{1}$$

on the time interval $t \in [0,1]$. Here u, v are controls of two players and are chosen in $[0,1]$. The function $x : [0,1] \to \mathbf{R}^2$ is absolutely continuous and satisfies the differential equations almost everywhere. The initial condition has the form

$$x(0) = 0. \tag{2}$$

The target set M is given by

$$(x_1 - 1)^2 + (x_2 - 1)^2 = 1,$$

$$x_1 \leq 1, \quad x_2 \leq 1.$$

Thus, M is a quarter of a circumference of unit radius. The function $u(t)$ is Lebesgue measurable. The problem consists in choosing the control v to ensure the evasion $x(1) \notin M$ whatever the disturbance $u(t)$ might be.

To show nondegeneracy of the problem, one should, on the one hand, check that program controls $v = v(t)$ cannot guarantee the evasion, and on the other hand, should construct a feedback control that ensures the evasion. In the case of Problem 1, this feedback control will turn out to be described by a discontinuous mapping.

2.1 Program controls in Problem 1.

One can show that program controls $v = v(t)$ do not guarantee the evasion even if u is chosen as a constant.

Proposition 1. For any measurable function $v : [0,1] \to [0,1]$, there exists a constant $u \in [0,1]$ such that the corresponding solution x of the initial value problem (1),(2) satisfies the condition $x(1) \in M$.

Really, it suffices to take

$$u = 1 - \sqrt{1 - \left(1 - \int\limits_0^1 v(\tau)d\tau\right)^2}.$$

Then $u \in [0,1]$, and for the solution of problem (1),(2) one has $x_1(1) = \int_0^1 v(\tau)d\tau$, $x_2(1) = u$. Thus, $(x_1(1)-1)^2 + (x_2(1)-1)^2 = 1$, $x_1(1) \leq 1$, $x_2(1) \leq 1$, and consequently $x(1) \in M$. Proposition 1 is proved.

2.2 A method of evasion.

In Problem 1, the control v can be formed with the help of a simple feedback rule that ensures the evasion and uses only one measurement of the phase vector x at the moment $t = 1/2$.

Proposition 2. Let v be chosen in the form

$$v = \begin{cases} 0, & 0 \leq t < 1/2, \\ 0, & 1/2 \leq t \leq 1, \ x_2(1/2) \leq 1/4, \\ 1, & 1/2 \leq t \leq 1, \ x_2(1/2) > 1/4. \end{cases} \tag{3}$$

Then, for any measurable $u : [0,1] \to [0,1]$, the corresponding solution of the initial value problem (1),(2) satisfies the inequality

$$\left|1 - \sqrt{(1-x_1(1))^2 + (1-x_2(1))^2}\right| > \frac{9}{100}. \tag{4}$$

Actually, it will be shown that the control method described in Proposition 2 ensures the distance to the target set somewhat larger than $9/100$.

Proof of Proposition 2. If $x_2(1/2) \leq 1/4$, then, according to (3), $v = 0$ for all $t \in [0,1]$, and so $x_1(1) = 0$. Further,

$$x_2(1) = x_2(1/2) + 2\int\limits_{1/2}^1 (1-\tau)u(\tau)d\tau \leq$$

$$\leq 1/4 + 2\int\limits_{1/2}^1 (1-\tau)d\tau = 1/2.$$

Thus,

$$\sqrt{(1-x_1(1))^2 + (1-x_2(1))^2} \geq \sqrt{5}/2 > 1.09,$$

and inequality (4) is valid. In case $x_2(1/2) > 1/4$, formula (3) implies $v = 0$ for $t \in [0,1/2)$ and $v = 1$ for $t \in [1/2,1]$. So, $x_1(1) = 1/2$. As u is nonnegative, one has $x_2(1) \geq x_2(1/2) > 1/4$. Consequently,

$$\sqrt{(1-x_1(1))^2 + (1-x_2(1))^2} < \sqrt{13}/4 < 0.91.$$

Inequality (4) holds again. Proposition 2 is proved.

The simple feedback control method described by (3) is not optimal. For instance, by optimizing the constant 1/4 in (3), the guaranteed result can be somewhat improved.

A control law that provides the evasion can be realised in the form of positional strategy in accordance with (Krasovskii and Subbotin, 1974) by employing stepwise schemes with mesh refinement. Indeed, the feedback control method given by (3) is compatible with any positional strategy of the player u, which excludes the guaranteed reaching of the target set. Due to the theorem on alternative (Krasovskii and Subbotin, 1974, §17), there exists a positional strategy of the player v that ensures the evasion. These considerations are similar to those in (Subbotin and Chentsov, 1981, p.23; Krasovskii and Subbotin, 1974, p.243).

2.3 Carathéodory strategies.

One can show that in Problem 1 it is impossible to ensure the evasion by means of strategies $v = v(t, x_1, x_2)$ that satisfy the Carathéodory conditions, i.e., such strategies that $v(t, x_1, x_2)$ is continuous in x_1, x_2 for almost any fixed t and is Lebesgue measurable in t for any fixed x_1, x_2. In particular, this class contains strategies described by continuous functions. In the case of Carathéodory strategies, one can employ absolutely continuous solutions, which satisfy the differential equations almost everywhere. A solution for a fixed pair of functions $v(t, x_1, x_2)$, $u(t)$ is not necessarily unique. It is natural to say that a strategy $v(t, x_1, x_2)$ ensures the evasion if for any measurable disturbance $u(t) \in [0, 1]$ and any absolutely continuous solution $x(t)$ of the initial value problem (1),(2) where $v = v(t, x_1, x_2)$, $u = u(t)$, one has $x(1) \notin M_1$.

Similarly to Proposition 1, the required property here can be established even in case of constant u.

Proposition 3. Let a function $v: [0, 1] \times \mathbf{R}^2 \to [0, 1]$ satisfy the Carathéodory conditions. Then there exists a constant $u \in [0, 1]$ such that the initial value problem (1),(2) with $v = v(t, x_1, x_2)$ has at least one solution x for which $x(1) \in M$.

Proof of Proposition 3. One needs to show the existence of an absolutely continuous function $x: [0, 1] \to \mathbf{R}^2$ and a number $u \in [0, 1]$ such that

$$\dot{x}_1 = v(t, x_1, x_2),$$

$$\dot{x}_2 = 2(1-t)u,$$

$$x(0) = 0,$$

$$(1 - x_1(1))^2 + (1 - x_2(1))^2 = 1.$$

The first three equalities imply $x_1(1) \leq 1$, $x_2(1) \leq 1$, and $x_2(t) = t(2-t)u$. Substituting this expression in the last of the equalities, one can see that $u = 1 - \sqrt{1 - (1 - x_1(1))^2}$. Excluding x_2, u from the system, one obtains the following initial value problem for a scalar functional differential equation:

$$\dot{x}_1(t) =$$

$$= v\left(t, x_1(t), t(2-t)\left(1 - \sqrt{1-(1-x_1(1))^2}\right)\right),$$

$$x_1(0) = 0.$$

It suffices to show the solvability of this problem, which will imply the existence of x, u with the required properties.

Denote by S the set of all functions $z: [0, 1] \to [0, 1]$ that satisfy Lipschitz condition with the coefficient 1. According to Arzelà-Ascoli theorem, the set S is a compactum in the space of continuous functions. Besides that, S is convex. Denote

$$F(z(\cdot))(t) =$$

$$= \int_0^t v\left(\tau, z(\tau), \tau(2-\tau)\left(1 - \sqrt{1-(1-z(1))^2}\right)\right) d\tau.$$

The function $F(z(\cdot))(t)$ of the argument t satisfies Lipschitz condition with the coefficient 1 because $0 \leq v \leq 1$. Note that F maps the set S into itself and is continuous in the norm of the space of continuous functions. Due to the theorem of Schauder, the mapping F has at least one fixed point in the set S. Thus, the above-given initial value problem for functional differential equation has a solution, and Proposition 3 is proved.

Remark 1. Let the strategy $v = v(t, x_1, x_2)$ be such that for any fixed number $u \in [0, 1]$, a solution to initial value problem (1),(2) is unique. In particular, this is so if the function $v(t, x_1, x_2)$ satisfies a condition of Lipschitz. Then, in the proof of Proposition 3, one can do without the fixed point theorem of Schauder. It suffices to use the simple fact that a scalar continuous function defined on an interval vanishes provided it takes values of both signs. One can proceed further by approximating a Carathéodory strategy by Lipschitz strategies.

Remark 2. If usual restrictions are imposed on a differential game, and the initial point is fixed, then all the possible trajectories do not leave a bounded region of the phase space. One can supplement the target set by an additional point that does not belong to the above-named bounded region. If the initial target set is convex, one obtains a new target set, which is neither convex nor connected. Obviously, the differential game with the new target set inherits all the essential properties of the initial game and is in fact equivalent to it. This simple consideration can be used to construct formal examples of differential games with various properties whose target sets are not convex. However, the new target set is nonconvex just formally, as the part of the target set which is actually used in the game remains convex.

3. A TWO-POINT TARGET SET

In the conflict control problem considered in this section, the target set consists of two points. The evasion in this problem can be ensured by some continuous strategies but not by program controls. The maximal possible distance is provided by an upper semicontinuous multivalued strategy and by a discontinuous strategy, but any smaller distance can be delivered by a continuous one-valued strategy. In particular, the considered problem gives a counterexample that shows that in the conditions of some results by the author on continuous strategies, one of the main assumptions cannot be omitted.

Problem 2. Consider a control system whose motion can be described by a scalar absolutely continuous function $x(t)$ of the argument $t \in [0,1]$. Let $x(t)$ satisfy the differential equation

$$\dot{x} = u + v \tag{5}$$

for almost all $t \in [0,1]$. The initial state is zero:

$$x(0) = 0. \tag{6}$$

The disturbance u and control v are chosen in the closed intervals:

$$u \in P = [-2, 2], \quad v \in Q = [-1, 1].$$

The target set

$$M_1 = \{-2, +2\}$$

consists of two points. The controls v formed with the purpose to ensure the evasion $x(1) \notin M_1$ for any measurable disturbances $u : [0,1] \to P$.

3.1 Program controls in Problem 2.

One can prove the following

Proposition 4. Program controls $v : [0,1] \to Q$ cannot guarantee the evasion in Problem 2.

Indeed, for a fixed measurable function $v(t)$, one can find some $u \equiv \text{const} \in P$ such that the corresponding trajectory ends in the target set $x(1) \in M_1$. Denote $w = \int_0^1 v(\tau)d\tau$. Then $x(1) = w + u$. In case $w \neq 0$ take $u \equiv (2 - |w|)\text{sgn}\, w$. One has $|u| = 2 - |w| < 2$, and so $u \in P$. Besides that, $x(1) = 2\text{sgn}\, w \in M_1$. In case $w = 0$ choose an arbitrary fixed $u \in M_1 \subset P$. Then $x(1) = u \in M_1$. Thus, no program control $v(t)$ ensures the evasion, and Proposition 4 is proved.

It is interesting to note that in the proof of Proposition 4, only constant disturbances u are needed. The situation is the same as with Propositions 1 and 3.

3.2 Strategies of evasion.

In the present subsection, feedback strategies are given that guarantee evasion in Problem 2. These include a family of continuous strategies, a discontinuous strategy, and an upper semicontinuous multivalued strategy. As in the corresponding considerations in Section 2, here one can directly substitute a strategy (including a discontinuous strategy) into the right-hand side of the differential equation and consider absolutely continuous solutions that satisfy the obtained differential equation or inclusion almost everywhere (or even everywhere). This can be done here because the functions $v(x)$ employed are of sufficiently simple form.

First, consider some estimates.

Lemma. Let a number $\varepsilon \geq 0$ and a function $v(x)$, $x \in \mathbf{R}$, be fixed such that

$$v(x) = -\text{sgn}\, x \quad \text{for} \quad |x| > \varepsilon. \tag{7}$$

A measurable function $u : [0,1] \to P$ is given. Let $x : [0,1] \to \mathbf{R}$, $x(\cdot) \in AC$, satisfy the initial value problem

$$\dot{x} = u(t) + v(x), \quad x(0) = 0. \tag{8}$$

Then, for any $t \in [0,1]$, the inequality

$$|x(t)| \leq 1 + \varepsilon \tag{9}$$

holds.

In Lemma, no conditions are imposed on $v(x)$ for $|x| \leq \varepsilon$. Here the function v might be discontinuous and multivalued provided it makes sense to consider absolutely continuous solutions to the initial value problem (8).

Proof of Lemma. Fix an arbitrary point $t = t_1$ in the interval $[0,1]$. In the proof of the required inequality (9) at this point, one obviously can restrict oneself to the case $|x(t_1)| > \varepsilon$. The values of the continuous function $|x(t)|$ for $t \in [0, t_1]$ range from zero to a number larger than ε. So, there exists some $s \in [0, t_1]$ for which $|x(s)| = \varepsilon$. The set of all $s \in [0, t_1]$ with this property is nonempty, closed, and bounded. Denote the largest number in this set by s_1. Thus, $0 \leq s_1 < t_1 \leq 1$, $|x(s_1)| = \varepsilon$, and for $s_1 < t \leq t_1$ one has $|x(t)| > \varepsilon$. If the inequality $\frac{d}{dt}|x(t)| \leq 1$ holds for almost all $t \in [s_1, t_1]$, then

$$|x(t_1)| = |x(s_1)| + \int_{s_1}^{t_1} \frac{d}{d\tau}|x(\tau)|d\tau \leq$$

$$\leq \varepsilon + t_1 - s_1 \leq 1 + \varepsilon;$$

that is, (9) is true at t_1. It remains to estimate $\frac{d}{dt}|x(t)|$. Condition (7) implies that for $s_1 < t \le t_1$ one has $v(x(t)) = -\mathrm{sgn}\, x(t)$; thus, $v(x(t))\mathrm{sgn}\, x(t) = -1$. Due to (8), for almost all $t \in [s_1, t_1]$, one has

$$\frac{d}{dt}|x(t)| = \dot{x}(t)\mathrm{sgn}\, x(t) =$$

$$= (u(t) + v(x(t)))\mathrm{sgn}\, x(t) = u(t)\mathrm{sgn}\, x(t) - 1$$

The last value is not larger than 1 because $|u| \le 2$. Lemma is proved.

If $\varepsilon > 0$, then the function $v(x)$ given by (7) can be defined for $|x| \le \varepsilon$ so that one obtains a one-valued continuous strategy, for example, the following strategy:

$$v(x) = \begin{cases} -x/\varepsilon, & |x| \le \varepsilon, \\ -\mathrm{sgn}\, x, & |x| \ge \varepsilon. \end{cases} \qquad (10)$$

According to Lemma, this strategy ensures that inequality (9) holds. In case $\varepsilon < 1$, inequality (9) enables one to estimate from below the distance to the target set. Thus, one obtains

Proposition 5 For $\varepsilon \in (0, 1)$, the continuous strategy (10) ensures in Problem 2 that the distance from $x(1)$ to the target set M_1 is not smaller than $1 - \varepsilon$.

Assume now $\varepsilon = 0$. Then (7) takes the form:

$$v(x) = -\mathrm{sgn}\, x \quad \text{for} \quad x \ne 0. \qquad (11)$$

Any number from $[-1, 1]$ can be assigned to be $v(0)$. In virtue of (9), this strategy ensures that the inequality $|x(t)| \le 1$, $t \in [0, 1]$, is true for an arbitrary solution of the initial value problem (8) that corresponds to an admissible $u(t)$. Thus, the strategy (11) guarantees that the distance from the point $x(1)$ to zero is no larger than 1, and so, the distance from this point to the target set M_1 is not smaller than 1. This number is the best possible for the evader. To check that, one can take a constant control $u \equiv u_0$ with $|u_0| = 2$.

The function (11) is discontinuous whatever number is chosen as its value at zero. However, taking the set $Q = [-1, 1]$ for the value of (11) at zero, one obtains an upper semicontinuous multivalued strategy:

$$V(x) = \begin{cases} [-1, 1], & x = 0, \\ -\mathrm{sgn}\, x, & x \ne 0. \end{cases} \qquad (12)$$

So, the conclusion follows:

Proposition 6 The discontinuous one-valued strategy (11) and the upper semicontinuous multivalued strategy (12) ensure in Problem 2 that the distance between $x(1)$ and the target set M_1 is not smaller than 1. This distance is the best possible for the evader.

3.3 Two results on continuous strategies.

This subsection renders in brief two results from (Brykalov 2001) which are used in the next subsection. More details can be found in the article cited.

Fix real numbers $t_0 < \vartheta$ and integers $n, p, q \ge 1$. The functional spaces used in this subsection consist of functions that are defined on the interval $[t_0, \vartheta]$ and take values in the n-dimensional space $\mathbf{R}^n$ unless something different follows from the context. Nonempty closed $P \subset \mathbf{R}^p$, $Q \subset \mathbf{R}^q$ are given. The set P is assumed to be bounded. Let a function $f : [t_0, \vartheta] \times \mathbf{R}^n \times P \times Q \to \mathbf{R}^n$ satisfy the Carathéodory conditions. For a fixed function $\chi : [t_0, \vartheta] \to [0, \infty)$, $\chi(\cdot) \in L_1$, almost all t, and all $u \in P$, $v \in Q$, x, the estimate holds: $|f(t, x, u, v)|_n \le \chi(t)(1 + |x|_n)$. Here $|\cdot|_n$ denotes a fixed norm in $\mathbf{R}^n$. A closed set $M \subset C^0$ and a vector $x_0 \in \mathbf{R}^n$ are given. All these requirements are assumed in both propositions of this subsection.

For a multivalued mapping $[t_0, \vartheta] \ni t \mapsto W(t) \subset Q$, denote by $\Phi(W(\cdot))$ the set of all $x(\cdot) \in AC$ that satisfy the initial value problem $\dot{x}(t) \in f(t, x(t), P, W(t))$, $x(t_0) = x_0$. Here the differential inclusion is valid almost everywhere on $[t_0, \vartheta]$, and $f(t, x(t), P, W(t)) = \{f(t, x(t), u, v) : u \in P,\ v \in W(t)\}$. Sets $\Phi(w(\cdot))$ for one-valued functions $w(\cdot)$ will be used also.

Denote by Λ the set of all multivalued maps $[t_0, \vartheta] \times C^0 \ni (t, z(\cdot)) \mapsto V(t, z(\cdot)) \subset Q$ that satisfy the following properties: For almost all t and all $z(\cdot)$, the set $V(t, z(\cdot))$ is nonempty, closed, and bounded; $t \mapsto V(t, z(\cdot))$ is measurable for any fixed $z(\cdot)$; besides that, $z(\cdot) \mapsto V(t, z(\cdot))$ is upper semicontinuous for almost any fixed t.

When employing acyclic sets below, one can use Vietoris homology or Alexandroff-Čech homology.

Proposition 7 Let Ω be a family of multivalued maps of the form $[t_0, \vartheta] \ni t \to W(t) \subset Q$. For any $W(\cdot) \in \Omega$, almost all t, and any $x \in \mathbf{R}^n$, the set $f(t, x, P, W(t))$ is convex. For any $W(\cdot) \in \Omega$, the intersection $\Phi(W(\cdot)) \cap M \subset C^0$ is nonempty and acyclic. A multivalued map $(t, z(\cdot)) \mapsto V(t, z(\cdot))$ belongs to Λ. Besides that, $t \mapsto V(t, z(\cdot))$ belongs to Ω for any fixed $z(\cdot)$. Then there exists an absolutely continuous $x(\cdot) \in M$ such that $x(t_0) = x_0$ and the inclusion $\dot{x}(t) \in f(t, x(t), P, V(t, x(\cdot)))$ holds for almost all $t \in [t_0, \vartheta]$.

Consider a corollary for a one-valued strategy:

Proposition 8 Let the set $f(t, x, P, r)$ be convex for almost all $t \in [t_0, \vartheta]$ and all $x \in \mathbf{R}^n$, $r \in Q$. For any measurable function $w : [t_0, \vartheta] \to Q$, the intersection $\Phi(w(\cdot)) \cap M \subset C^0$ is nonempty and acyclic. The map $v : [t_0, \vartheta] \times C^0 \to Q$ is such that $v(t, z(\cdot))$ is measurable in t for any fixed

$z(\cdot)$ and continuous in $z(\cdot)$ for almost any fixed t. Then one can find an absolutely continuous $x(\cdot) \in M$ such that $x(t_0) = x_0$ and the inclusion $\dot{x}(t) \in f(t, x(t), P, v(t, x(\cdot)))$ is true for almost all $t \in [t_0, \vartheta]$.

3.4 Problem 2 as a counterexample.

In Propositions 7 and 8, the assumption that the intersection is acyclic cannot be omitted. The corresponding counterexample is provided by Problem 2.

It can be seen from Subsections 3.1 and 3.2 that in Problem 2, continuous strategies and upper semicontinuous multivalued strategies are more efficient then program controls. Consequently, here one cannot apply Proposition 8 and Proposition 7, which give sufficient conditions for the opposite situation. The reasons for that are discussed below.

Using the notation of Subsection 3.3, take $t_0 = 0$, $\vartheta = 1$, $x_0 = 0$, $p = q = n = 1$, $f = u + v$, $\chi \equiv 3$. Assume that M is the set of all continuous functions $x(\cdot)$ such that $x(1) \in M_1$. As M_1 is closed in $\mathbf{R}$, one can see that M is closed in the space C^0. Proposition 4 implies that $\Phi(w(\cdot)) \cap M$ is nonempty for admissible $w : [0, 1] \to Q$.

For the case of continuous strategies (10), all the conditions of Proposition 8 are valid except for the assumption that $\Phi(w(\cdot)) \cap M \subset C^0$ is acyclic for any measurable $w : [0, 1] \to Q$. One can show that this assumption is false for $w \equiv 0$. Really, the set $\Phi(0)$ consists of all solutions to the initial value problem $\dot{x}(t) \in P$, $x(0) = 0$. Thus, $\Phi(0)$ is the set of all $x\colon [0, 1] \to \mathbf{R}$ that satisfy the condition of Lipschitz with the coefficient 2 and emanate from the origin $x(0) = 0$. As M_1 consists of two points ± 2, the intersection $\Phi(0) \cap M$ is the set H that consists of two linear functions $x(t) = \pm 2t$. Indeed, for $x(\cdot) \in \Phi(0) \cap M$ one has $2 = |x(1)| = |(x(t) - x(0)) + (x(1) - x(t))| \le |x(t) - x(0)| + |x(1) - x(t)| \le 2t + 2(1 - t) = 2$, and all these inequalities turn out to be equalities. So, $|x(t)| = 2t$. Besides that, $H \subset C^0$ is homeomorphic (and even isometric) to the two-point set M_1. Thus, H is not acyclic.

For the same reason, one cannot apply Proposition 7 in the case of multivalued strategy (12). Really, by substituting $x(t) \equiv 0$ into (12), one obtains a multivalued map $W(t) \equiv Q$, which should belong to the family Ω of Proposition 7. The set $X = \Phi(Q) \cap M$ consists of all solutions to the initial value problem $\dot{x}(t) \in P + Q$, $x(0) = 0$ with an additional condition $x(1) \in M_1$. Thus, X is the family of all $x\colon [0, 1] \to \mathbf{R}$ that satisfy the condition of Lipschitz with the coefficient 3 and the relations $x(0) = 0$, $x(1) \in M_1$. One can consider $X \subset C^0$ as a metric space. The set H which was introduced above, is a subset of X. It was shown that $H \subset C^0$ is not acyclic. It suffices to check that H is a deformation retract of X. From here it follows that X is also not acyclic. Denote $\gamma(\lambda, x(\cdot))(t) = (1-\lambda)x(t)+\lambda t x(1)$. The map $\gamma\colon [0, 1] \times X \to C^0$ is continuous. Besides that, $\gamma(0, x(\cdot))(t) = x(t)$ is an identity mapping in $x(\cdot)$, and $\gamma(1, x(\cdot))(t) = tx(1)$ takes values in H and is identical on H. It only remains to show that $\gamma(\lambda, x(\cdot)) \in X$ for $x(\cdot) \in X$. Indeed, $\gamma(\lambda, x(\cdot))(0) = 0$, $\gamma(\lambda, x(\cdot))(1) = x(1) \in M_1$. The condition of Lipschitz with the coefficient 3 is also true as $|\gamma(\lambda, x(\cdot))(t_2) - \gamma(\lambda, x(\cdot))(t_1)| \le (1 - \lambda)|x(t_2) - x(t_1)| + \lambda|x(1)(t_2 - t_1)| \le (3(1 - \lambda) + 2\lambda)|t_2 - t_1| \le 3|t_2 - t_1|$. Finally, one can see that Proposition 7 cannot be applied to the case of multivalued strategy (12) because the assumption of the intersection being acyclic does not hold.

4. CONCLUSION

Properties of continuous strategies in differential games with nonconvex target sets differ essentially from those in the case of convex target sets and linear dynamics. Concrete examples can be given to demonstrate that. However, these properties can be similar to those in the linear-convex case provided some assumptions are imposed on the corresponding sets of trajectories of the considered controlled system with disturbance.

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REFERENCES

Barabanova, N.N. and A.I. Subbotin (1970). On continuous strategies of evasion in game problems on encounter of motions. *Prikl. Mat. Mekh.*, **34,** 796–803. (in Russian)

Brykalov, S.A. (2001). Continuous feedback in conflict control problems. *Dokl. Akad. Nauk,* **376,** 442–444. (in Russian)

Krasovskii, A.N. and N.N. Krasovskii (1995). *Control under lack of information,* Birkhäuser, Boston.

Krasovskii, N.N. and A.I. Subbotin (1974). *Positional differential games,* Nauka, Moscow. (in Russian)

Subbotin, A.I. and A.G. Chentsov (1981). *Optimization of guarantee in control problems,* Nauka, Moscow. (in Russian)