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OUTPUT FEEDBACK DESIGN BY COUPLED LYAPUNOV-LIKE EQUATIONS

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Abstract: This note presents two coupled Lyapunov-like conditions under which a linear discrete-time system can be stabilized by static output feedback. The originality of these conditions is their relation to the well-known coupled Sylvester equations that describe both the (A, B) and (C, A)-invariance of a subspace. For systems verifying Kimura's condition, we show that output feedback stabilizing gain matrices can be computed through the successive resolution of two standard convex programming problems. Numerical results are provided to show the effectiveness of the proposed approach. *Copyright ©2002 IFAC*

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1. INTRODUCTION

[1] A common feature shared by different methods to treat the static output feedback stabilization problem of linear systems is that it is equivalent to obtaining solutions of coupled sets of matrix equations (Castelan et al., 2000). In particular, through the use of coupled Sylvester equations (Syrmos and Lewis, 1993) (Syrmos and Lewis, 1994), the output feedback stabilization problem can be decomposed into two stages: (i) determination of a (C, A)-outer detectable subspace, an (ii) inner stabilization of this subspace. Thus, when the number of inputs, m, plus the number of outputs, p, is greater than the number of states, n, known as Kimura's condition $(m + p > n)$ (Kimura, 1975), the Sylvester equations involved in the two stages can be solved by standard eigenstructure assignment techniques.

This paper extends to the case of discrete-time systems the results presented in (Castelan et al., 2000) for continuous-time systems. It is shown that the above mentioned geometric approach based on the solution of coupled Syslvester equations has a quadratic counterpart, so that coupled linear matrix inequalities and equalities can also be used for construction of an output stabilizable (C, A, B)-invariant subspace as an intermediate mechanism in the process of designing a static output feedback. The quadratic characterization of both stages by Lyapunov equations provides a convenient framework for the numerical resolution of the problem through standard convex programming. In particular, we show that the first stage can be accomplished through the solution of a reduced-order strict LMI. The proposed problem decomposition as two subsequent convex programming problems may be also convenient for the integration of some additional performance and robustness requirements represented in the form of LMIs .

The second section of the paper presents the key result that links the solution of the static output feedback stabilization problem to the coupled Sylvester equations and decomposes the basic solution into two steps. In the third section, the

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equivalent quadratic characterization is obtained. Then, section 4 is devoted to the presentation of the proposed algorithm, based on the use of orthogonal transformations and convex programming solutions of $LMIs$. In section 5, numerical examples are reported to illustrate the effectiveness of the proposed approach. We finally present some concluding remarks.

2. PRELIMINARIES

The considered linear discrete time-invariant systems are described by

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$
$$y_k = Cx_k \tag{2}$$

where: $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$. It is also assumed that B is full column-rank, C is full row-rank and that (C, A, B) is stabilizable and detectable. The studied problem is to find a static output feedback control law $u_k = Ky_k$, such that the closed-loop system

$$x_{k+1} = (A + BKC)x_k \tag{3}$$

is asymptotically stable, i.e. its spectrum satisfies:

$$\sigma(A + BKC) = \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \text{ with,}$$
$$\text{for } i = 1, ..., n, \quad \lambda_i \in \mathcal{C}^s = \{\lambda \in \mathcal{C}; |\lambda| < 1\}.$$

The following basic result relates the existence of a stabilizing output feedback to the solution of two coupled Sylvester equations.

Theorem 2.1. There exists a static output feedback $K : \Re^p \longrightarrow \Re^m$ such that $\sigma(A+BKC) \in \mathcal{C}^s$ if and only if the following conditions hold true for some matrices ($H_V \in \Re^{v\times v}$, $V \in \Re^{n\times v}$, $Z_v \in \Re^{m\times v}$, $H_T \in \Re^{n-v\times n-v}$, $T \in \Re^{n\times(n-v)}$, $Z_T \in \Re^{p\times n-v}$) and for some positive scalar $v \leq n$:

$$AV - VH_V = -BZ_V \;, \quad \text{with} \quad \sigma(H_V) \in \mathcal{C}^s \tag{4}$$
$$T'A - H_T T' = -Z_T' C \;, \quad \text{with} \quad \sigma(H_T) \in \mathcal{C}^s \tag{5}$$
$$T'V = 0 \tag{6}$$
$$Ker(CV) \subseteq Ker(Z_V) \tag{7}$$
$$Ker(B'T) \subseteq Ker(Z_T') \tag{8}$$

where: rank $(T) = n - v$ and rank $(V) = v$. □

The above result has been presented and exploited under different forms in the literature related to the eigenstructure assigment by output feedback (see (Syrmos et al., 1997)). The equations (4) to (5) are recognized as coupled Sylvester equations. Under the stability constraint imposed on matrix H_V, (4) means that $\mathcal{V} = Im(V)$ must be a (A, B)-inner stabilizable subspace, that is there exists F such that $(A + BF)|\mathcal{V}$ is stable. Dually, (5) means that $Ker\ T'$ has to be a (C, A)-outer detectable subspace, that is there exists L such that $(A + LC)|(\Re^n/Ker\ T')$ is stable.

Under the coupling condition (6), (4) and (5) mean that $\mathcal{V} = Ker\ T'$ is an Output Stabilizable (C, A, B) invariant subspace, as defined in (Syrmos and Lewis, 1994). Notice also that inclusions (7) and (8) can be equivalently replaced by:

$$KCV = Z_V \tag{9}$$
$$T'BK = Z_T' \tag{10}$$

From the above remarks we see that the statement of theorem 2.1 is essentially equivalent to the one of theorem 3.2 stated in (Syrmos and Lewis, 1994). For algorithmic purposes, we specialize the design to the case where $\mathcal{V} = Im\ V$ is a p-dimensional subspace. This assumption has also been adopted in many other works that, explicitly or implicitly, use the coupled-Sylvester equations for eigenstructure assignment by output feedback (Alexandridis and Parakevopoulos, 1996) (Syrmos and Lewis, 1993) (Fletcher et al., 1985). Its main advantage is to provide a straightforward solution to equation (9). Thus, based on the conditions of theorem 2.1, the following procedure generically leads to a stabilizing output feedback when the Kimura's condition $n < m + p$ is verified (Syrmos and Lewis, 1993) :

Step 1: Find a matrix $T \in \Re^{n\times n-p}$ verifying (5), and such that

$$rank \begin{bmatrix} T' \\ C \end{bmatrix} = n \Leftrightarrow Ker\ T' \cap Ker\ C = \{0\} \tag{11}$$

Step 2: Solve (4), taking into account that V must verify (6) and rank(V) must be equal to p.
Step 3: By construction (11) guarantees that rank$(CV) = p$ and K can be computed by:

$$K = Z_V(CV)^{-1} \tag{12}$$

Steps 1 and 2 can be solved by using standard eigenstructure assignment techniques. In particular, once T has been found, step 2 can be solved by using the "zero equation":

$$\begin{bmatrix} A - \lambda_i I & B \\ T' & 0 \end{bmatrix} \begin{bmatrix} v_i \\ z_{vi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13}$$

where v_i and z_{vi}, $i = 1, \ldots, p$, form the columns of V and Z_V, respectively.

Remark 2.1. To guarantee that the eigenvalues λ_i of step 2 are freely assignable, the system matrix $P(\lambda) = \begin{bmatrix} A - \lambda I & B \\ T' & 0 \end{bmatrix}$ must have full normal row rank: $rank(P(\lambda)) = 2n - p, \forall \lambda$. Otherwise the system (A, B, T') has invariant zeros, in which case they must be used to obtain the (A, B)-invariance of $Ker\, T'$. The genericity of the above procedure relies on the fact that, for $m + p > n$, $P(\lambda)$ does not lose rank for almost all triples (A, B, T') (Syrmos and Lewis, 1993).

Remark 2.2. In the case $m+p = n$, $P(\lambda)$ is square and almost all triple (A, B, T') has p finite invariant zeros (Syrmos and Lewis, 1993). Thus, the basic procedure can produce stabilizing solutions in this more difficulty case only if T found in step 1 generates p minimum-phase invariant zeros, which have to be used to solve (13) (Castelan and Hennet, 1993). However, to our knowledge, no systematic procedure exists for the search of a good T, although a "try-and-error" search may be carried-out.

3. COUPLED MATRIX INEQUALITY CONDITIONS

A quadratic characterization of Theorem 2.1 can also be obtained by replacing the stability constraints on matrices H_V and H_T by the following equivalent conditions that are obtained from Schur complement applied to the classical discrete-time Lyapunov stability conditions (Crusius and Trofino, 1999):

$$\begin{cases} H_V'\bar{\Pi}H_V - \bar{\Pi} < 0 \\ \bar{\Pi} > 0 \end{cases} \Longleftrightarrow \begin{cases} \begin{bmatrix} -\Pi & \Pi H_V' \\ H_V\Pi & -\Pi \end{bmatrix} = -Q_V \\ Q_V = Q_V' > 0 \\ \Pi = \bar{\Pi}^{-1} > 0 \end{cases} \quad (14)$$

$$\begin{cases} H_T\bar{\Gamma}H_T' - \bar{\Gamma} < 0 \\ \bar{\Gamma} > 0 \end{cases} \Longleftrightarrow \begin{cases} \begin{bmatrix} -\Gamma & \Gamma H_T \\ H_T'\Gamma & -\Gamma \end{bmatrix} = -Q_T \\ Q_T = Q_T' > 0 \\ \Gamma = \bar{\Gamma}^{-1} > 0 \end{cases} \quad (15)$$

Theorem 3.1. There exists an output feedback $K : \Re^p \longrightarrow \Re^m$ such that $\sigma(A + BKC) \in C^s$, if and only if the following conditions hold true for some full rank matrices $T \in \Re^{n\times n-v}$, $V \in \Re^{n\times v}$, with $v > 0$, such that $T'V = 0$:

(i) $\exists P = P' \geq 0, P \in \Re^{n\times n},\ W \in \Re^{m\times n}$;

$$\begin{bmatrix} -P & PA' + W'B' \\ AP + BW & -P \end{bmatrix} = \begin{bmatrix} V & 0 \\ 0 & V \end{bmatrix} Q_V \begin{bmatrix} V' & 0 \\ 0 & V' \end{bmatrix} \quad (16)$$

$$V'PV > 0\ ;\ T'PT = 0 \quad (17)$$

$$W = Z_\Pi V' \quad for \quad some\ Z_\Pi \in \Re^{m\times v} \quad (18)$$

(ii) $\exists S = S' \geq 0, S \in \Re^{n\times n},\ Y \in \Re^{n\times p}$;

$$\begin{bmatrix} -S & SA + YC \\ A'S + C'Y' & -S \end{bmatrix} = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} Q_T \begin{bmatrix} T' & 0 \\ 0 & T' \end{bmatrix} \quad (19)$$

$$T'ST > 0\ ;\ V'SV = 0 \quad (20)$$

$$Y = TZ_\Gamma' \quad for \quad some\ Z_\Gamma \in \Re^{p\times n-v} \quad (21)$$

$$(iii) \quad Ker\ CP \subseteq Ker\ W \quad (22)$$

$$Ker\ B'S \subseteq Ker\ Y' \quad (23)$$

Proof (outline):

Necessity: Consider that K is a stabilizing output feedback and, hence, that the conditions (4) to (5) of theorem 2.1 are all verified.

By recalling that the quadratic stability condition (14) holds true, we obtain:

$$\begin{bmatrix} V & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} -\Pi & \Pi H_V' \\ H_V\Pi & -\Pi \end{bmatrix} \begin{bmatrix} V' & 0 \\ 0 & V' \end{bmatrix} = \begin{bmatrix} -V\Pi V' & V\Pi H_V'V' \\ VH_V\Pi V' & -V\Pi V' \end{bmatrix} \leq 0.$$

Then, condition (16) is obtained using $AV + BZ_V = VH_V$, setting $Z_\Pi = Z_V\Pi$, $P = P' = V\Pi V'$ and $W = Z_\Pi V'$.

Now, condition (17) is obtained considering that $rank(V) = p$ and that $\Pi > 0 \Longrightarrow \begin{cases} V'V\Pi V'V > 0 \\ T'V\Pi V'T = 0 \end{cases}$.

Using similar arguments, we can show the necessity of part (ii):

$$\begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} -\Gamma & \Gamma H_T \\ H_T'\Gamma & -\Gamma \end{bmatrix} \begin{bmatrix} T' & 0 \\ 0 & T' \end{bmatrix} = \begin{bmatrix} -T\Gamma T' & T\Gamma H_T T' \\ TH_T'\Gamma T' & -T\Gamma T' \end{bmatrix} \leq 0.$$

Thus, by using $T'A + Z_T'C = H_T T'$, and setting $S = T\Gamma T'$ and $Y = T\Gamma Z_T'$, we obtain (19) and (21). Furthermore, since $rank(T) = n-v$ and $\Gamma > 0$, we also have $\begin{cases} T'T\Gamma T'T > 0 \\ V'T\Gamma T'V = 0 \end{cases}$, corresponding to condition (20).

Necessity of (22), (23) is derived from (7), (8), respectively, and from the definition of the matrices involved in these conditions.

Sufficiency: For simplicity of the presentation we show only that the verification of part(i) is equivalent to the (A, B)-inner stabilizability of the subspace $\mathcal{V} = Ker\ T'$. We first have:

$$\begin{bmatrix} V' \\ T' \end{bmatrix} P \begin{bmatrix} V & T \end{bmatrix} = \begin{bmatrix} \bar{P} & 0 \\ 0 & 0 \end{bmatrix} \geq 0;\ \bar{P} = \bar{P}' > 0 \quad (24)$$

Thus, we can decompose P as:

$$P = \begin{bmatrix} V & T \end{bmatrix} \begin{bmatrix} \Pi & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V' \\ T' \end{bmatrix} \geq 0 \quad \text{for} \quad (25)$$

$$\Pi = \Pi' = V'(V'V)^{-1}\bar{P}(V'V)^{-1}V > 0 \quad (26)$$

From (22), there exists a matrix K such that:

$$KCP = W \tag{27}$$

Thus, by substituting (27) into (16), we get

$$\begin{bmatrix} -P & P(A+BKC)' \\ (A+BKC)P & -P \end{bmatrix} = -\tilde{\mathcal{Q}}_V \tag{28}$$

where $\tilde{\mathcal{Q}}_V \in \Re^{2n\times 2n}$ is defined from $Q_V = \begin{bmatrix} Q_{V11} & Q'_{V21} \\ Q_{V21} & Q_{V22} \end{bmatrix}$ by $\tilde{\mathcal{Q}}_V =$

$$\begin{bmatrix} V \ T & \\ & V \ T \end{bmatrix} \begin{bmatrix} \begin{bmatrix} Q_{V11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} Q'_{V21} & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} Q_{V21} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} Q_{V22} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} V' & \\ T' & \\ & V' \\ & T' \end{bmatrix}.$$

Taking into account (26) and the similarity equation:

$$(A+BKC)\begin{bmatrix} V & T \end{bmatrix} = \begin{bmatrix} V & T \end{bmatrix} \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \tag{29}$$

we obtain from (28):

$$\begin{bmatrix} -\begin{bmatrix} \Pi & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \Pi\bar{A}'_{11} & \Pi\bar{A}'_{21} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} \bar{A}_{11}\Pi & 0 \\ \bar{A}_{21}\Pi & 0 \end{bmatrix} & -\begin{bmatrix} \Pi & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} =$$

$$-\begin{bmatrix} \begin{bmatrix} Q_{V11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} Q'_{V21} & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} Q_{V21} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} Q_{V22} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \tag{30}$$

By construction, $\Pi > 0$, thus from (30), $\bar{A}'_{21} = 0$, which implies that $\mathcal{V} = Im\,V$ is a $(A+BKC)$-invariant subspace. And also from (30),

$$\begin{bmatrix} -\Pi & \Pi\bar{A}'_{21} \\ \bar{A}_{21}\Pi & -\Pi \end{bmatrix} = -Q_V < 0$$

which means that the restriction of $(A+BKC)$ to $\mathcal{V} = Im\,V$ is stable. □

Remark 3.1. For any pair of matrices (P, S) solution to parts (i) and part (ii), the coupling condition between the given matrix inequality conditions (16) and (19) can be restate as $Ker\ S = Im\ \mathcal{V} = Im\ P$, or, equivalently: $SP = 0$. The coupled matrix inequality conditions (i) and (ii), together with this condition show how the geometric approach and Lyapunov conditions have been combined in our approach.

4. AN ALGORITHM FOR OUTPUT FEEDBACK STABILIZATION

Theorem 3.1 gives a necessary and sufficient condition for the existence of a stabilizing output feedback. Unfortunately, this result involves coupled matrix inequality conditions that are nonconvex in the considered decision variables T, V, P and S. However these conditions may be used to adequately construct Output Stabilizable (C, A, B)-invariant subspaces that lead to stabilizing output feedback matrices K. As in the eigenstructure assignment approach presented in section 2 (Syrmos and Lewis, 1993), this can be done by taking into account the coupling requirement and solving subsequently part (ii) and part (i) of theorem 3.1. The following procedure is proposed to compute a stabilizing output feedback when $m + p > n$ [2].

Step 1:

- Find an orthogonal decomposition $C\begin{bmatrix} M_1 & M_2 \end{bmatrix} = \begin{bmatrix} C_1 & 0 \end{bmatrix}$, where $C_1 \in \Re^{p\times p}$ is non-singular, and set $\begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} = \begin{bmatrix} M'_1 \\ M'_2 \end{bmatrix} AM_2$;
- Solve the reduced-order (strict) LMI condition below to find $S_{21} \in \Re^{n-p\times p}$ and $S_{22} \in \Re^{n-p\times n-p}$: $S_{22} > 0$ and

$$\begin{bmatrix} -S_{22} & S_{22}A_{22} + S_{21}A_{12} \\ A'_{22}S_{22} + A'_{12}S'_{21} & -S_{22} \end{bmatrix} < 0 \tag{31}$$

- Define $T' = \begin{bmatrix} S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} M'_1 \\ M'_2 \end{bmatrix}$.

Step 2:

- Compute V from (6) as an orthogonal basis of $Ker\ T'$, *i.e*: $V'V = I_p$;
- Solve conditions (16), (17) and (18) to find P, W and Z_Π.

Step 3: Compute the stabilizing output feedback matrix as the unique solution of:
$KCP = W \iff KCV\bar{P} = Z_\Pi$, since $V'V = I_p$.

Remark 4.1. In step 1, a candidate matrix T is constructed such that $Ker\,T' \cap Ker\,C = \{0\}$. In the basis formed by the columns of matriz $M = \begin{bmatrix} M_1 & M_2 \end{bmatrix}$, the open-loop system takes the form

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{32}$$

$$y = \begin{bmatrix} C_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{33}$$

where the matrices involved have the appropriate dimensions. As shown in (Chen, 84), the detectability of the pair (C, A) implies the detectability of the pair (A_{12}, A_{22}). Thus, a feasible solution to LMI (31) can always be find. Let $L_2 \in \Re^{n-p\times p}$ be such that $S_{22}L_2 = S_{21}$ and consider a Choleski decomposition of S_{22} given by: $S_{22} = T'_2T_2$. Since, by construction L_2 stabilizes

[2] If $m + p = \bar{\nu} \leq n$, a dynamic compensator of order $\nu > n - \bar{\nu}$ can be considered to recover this condition.

$(A_{22} + L_2 A_{12})$, we can define $H_T \in \mathcal{C}^s$ from the similarity relation

$$T_2 \left(A_{22} + L_2 A_{12}\right) = H_T T_2 \qquad (34)$$

Furthermore, since C_1 is invertible, a matrix $Z_T \in \Re^{p\times(n-p)}$ can always be computed from:

$$Z_T' C_1 = -\left(T_2 L_2 A_{11} + T_2 A_{21} - H_T T_1\right) \quad (35)$$

where: $T_1 = T_2 L_2$. Hence, (34) and (35) can be equivalently replaced by

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} - H_T \begin{bmatrix} T_1 & T_2 \end{bmatrix} = -Z_T' \begin{bmatrix} C_1 & 0 \end{bmatrix}, \qquad \textit{with } \sigma(H_T) \in \mathcal{C}^s. \quad (36)$$

By construction, $\begin{bmatrix} T' \\ C \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} M_1' \\ M_2' \end{bmatrix}$. Thus, $rank \begin{bmatrix} T' \\ C \end{bmatrix} = n \iff Ker\, T' \cap Ker\, C = \{0\}$.

Furthermore , let $\Gamma = \Gamma' > 0$ be a solution to (15). Thus, the following matrices Y and $S \geq 0$ verify (16) ,(17) and (18) for $Y = T Z_T' \Gamma$;

$$S = \begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} S_{11} & S_{21}' \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} M_1' \\ M_2' \end{bmatrix}, \quad \text{with}$$

$$\begin{bmatrix} S_{11} & S_{21}' \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} T_1' \\ T_2' \end{bmatrix} \Gamma \begin{bmatrix} T_1 & T_2 \end{bmatrix}.$$

Remark 4.2. Step 2 is used to solve the set of conditions for $\mathcal{V} = Ker\, T'$. Recall that these conditions are generically solvable in the sense that when $m + p > n$, $P(\lambda)$ has full row rank for almost all triples (A, B, T') (remark 2.1).

Remark 4.3. The decomposition for step 1 is not unique. The set of all decompositions can be obtained from a chosen orthogonal matrix M by

$$CMD = C \begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} D_1 & 0 \\ D_{21} & D_2 \end{bmatrix} = \begin{bmatrix} \bar{C}_1 & 0 \end{bmatrix} (37)$$

for all non-singular matrices $D_1 \in \Re^{p\times p}$, $D_2 \in \Re^{n-p\times n-p}$. For instance, in the unfrequent cases where step 2 fails, a new basis (not necessarily orthogonal) can be constructed by $M^n = MD$; the new detectable pair (A_{12}^n, A_{22}^n) satisfies:

$$\begin{bmatrix} A_{12}^n \\ A_{22}^n \end{bmatrix} = \begin{bmatrix} D_1^{-1} & 0 \\ -\bar{D}_{21} & D_2^{-1} \end{bmatrix} \begin{bmatrix} A_{21} \\ A_{22} \end{bmatrix} D_2 = \begin{bmatrix} D_1^{-1} A_{21} D_2 \\ (D_2^{-1} A_{22} - \bar{D}_{21} A_{21}) D_2 \end{bmatrix}$$

with $\bar{D}_{21} = D_2^{-1} D_{21} D_1^{-1}$. In this way, other solutions to step 1 can be found so that step 2 becomes feasible.

Finally, it is worth recalling that stabilizing solutions can also be computed from the use of the dual system (A', C', B'). In addition, standard convex programming techniques can be used to find feasible solutions for the considered Lyapunov-like conditions and also to include some additional.

5. NUMERICAL EXAMPLE

The numerical examples reported in this section were solved with Scilab (INRIA, France). Standard convex programming techniques have been applied to find feasible solutions for the coupled Lyapunov-like conditions.

The first 1 000 random triples (A, B, C) generated with unstable A verify the Kimura's condition $m + p > n$, with; $n = 5$, $m = p = 3$. The basic procedure computed stabilizing solutions in 99.6% of cases. In the other 4 examples, only one iteration was necessary to obtain a stabilizing solution; this iteration was carried out using Remark 4.3, with $D = \begin{bmatrix} I_p & 0 \\ D_{21} & I_{n-p} \end{bmatrix}$, with a random matrix D_{21}. Thus, under the Kimura's condition, the algorithm was always successful and had a performance comparable to the performance reported in (El Ghaoui et al., 1997) for the cone complementarity linearization algorithm.

Some computational experiments have also been carried out for the less restrictive case $m + p = n$, with; $n = 4$, $m = p = 2$ (see Remark 2.2). For 20 random triples (A, B, C) the basic procedure computed stabilizing solutions in 50 % of cases. Once more, only one iteration was necessary to produce a stabilizing solution, for each one of the remaining 10 examples. These results also show that the approach may be effective when Kimura's condition is not verified.

The objective of the next example is to show the numerical results obtained at each step. The following model has been obtained from a discretization of the system studied in (Fletcher et al., 1985): $A = \begin{bmatrix} 1.0502 & 0.3551 & 0 & 0 \\ 0.3551 & 1.4054 & 0 & 0 \\ -0.3049 & -0.0502 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.0502 & 0 \\ 0.3551 & 0 \\ -0.0049 & 0 \\ 0 & 0.3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, with open-loop spectrum $\sigma(A) = \{1;\ 1.6248;\ 0.8308;\ 1\}$. The corresponding triple (C, A, B) is controllable and observable and $m + p = 5 > n$. Using an orthogonal matrix M that performs step 1, a matrix $T' = \begin{bmatrix} S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} M_1' \\ M_2' \end{bmatrix}$ is readily found:

$$T' = \begin{bmatrix} 1 & 0.1868 & 0.1611 & 0.1585 \end{bmatrix} \begin{bmatrix} -0.7071 & 0 & 0 & -0.7071 \\ 0 & 0 & -1 & 0 \\ 0.7071 & 0 & 0 & -0.7071 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$= \begin{bmatrix} -0.5932 & 0.1585 & -0.1868 & -0.8210 \end{bmatrix}$.

It implies, in step 2: $V = \begin{bmatrix} -0.7879 & -0.1793 & 0.1521 \\ 0.0764 & 0.0174 & 0.9853 \\ -0.0900 & 0.9795 & 0.0174 \\ 0.6044 & -0.0900 & 0.0764 \end{bmatrix}$.

A feasible solution for the second part of step 2 is then found by minimizing the trace of matrix P:

$$P = 10^{-3} \begin{bmatrix} 0.5630 & -0.3142 & 0.3740 & -0.5525 \\ -0.3142 & 1.4728 & 0.7512 & 0.3403 \\ 0.3740 & 0.7512 & 1.8026 & -0.5354 \\ -0.5525 & 0.3403 & -0.5354 & 0.5867 \end{bmatrix},$$

$$W = 10^{-3} \begin{bmatrix} -0.5647 & -3.8352 & -1.9805 & 0.1183 \\ 0.3027 & -1.3523 & -0.5114 & -0.3634 \end{bmatrix};$$

$$Z_{\Pi} = 10^{-3} \begin{bmatrix} 0.4018 & -1.9160 & -3.8899 \\ -0.5153 & -0.5460 & -1.3230 \end{bmatrix}.$$

The corresponding stabilizing output feedback, $K = \begin{bmatrix} -21.4364 & -3.5490 & -1.7870 \\ -4.1273 & -1.0505 & -1.3373 \end{bmatrix}$, gives:

$$\sigma(A + BKC) = \{0.1355;\ 0.0098;\ 0.8056 \pm 0.1336i\}.$$

where the eigenvalue 0.1355 corresponds to step 1.

6. CONCLUDING REMARKS

Stabilization of linear systems by static output feedback is recognized a basic and still open problem in control theory. A review of existing approaches and techniques to treat different versions of this problem can be found in (Syrmos et al., 1997); other recent results not covered by this survey paper are, for instance, (Alexandridis and Parakevopoulos, 1996), (Crusius and Trofino, 1999), (Geromel et al., 1998), (El Ghaoui et al., 1997) and (Castelan et al., 2000).

The present paper has extended the results reported in (Castelan et al., 2000) to the case of discrete-time systems. The basis of the study has been the coupled Sylvester equations approach, often used in the eigenstructure assignment literature. From these equations, a quadratic approach based on the solution of two-coupled Lyapunov-like conditions has been obtained.

The effectiveness of the proposed algorithm has been shown through numerical examples for cases where the Kimura's condition are verified. Thus, stabilizing solutions can also be used in the cases where this condition is not verified by using dynamical compensators that recovers Kimura's condition for the associated augmented system.

The use of the proposed approach to integrate additional closed-loop performance requirements and also to solve less restrictive cases are the subject of underlying researches.

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