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MULTI-AGENT CONTROL OF QUEUING PROCESSES [1]

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Abstract: Multi-agent control is an efficient method to optimize local systems in decentralized environments. Market-based algorithms are multi agent scenarios in which producer and consumer agents compete and cooperate on a market at the same time. The method is applied to a set of local Markov/queuing processes, or birth and death processes, respectively, where the local death rates can be influenced by some external control strategy but the local birth rates cannot. The goal is to change the individual death rates so that, after some finite time, the distributions of all queuing processes are adjusted. The death rates of the local queuing processes are changed on the basis of the probabilities of the appearance of a selected event in each of the local queuing processes.

Keywords: Agents, market-based control, decentralized systems, queues, Markov processes

1. INTRODUCTION

Centralized control of complex systems is known to be difficult in the presence of a large number of local systems. A decentralized option is multi agent control which is able to cope better with the problems arising with centralized control and optimization methods. Multi agent control methods are especially applied in congestion control of traffic networks (Altman,1999) and manufacturing systems (Bussmann, 1998, Wallace 1998, Zaremba, 1999). Another important and growing field of application is logistics. In (Moore, 1997) a multi-agent framework is presented that deals with logistic operations in a distributed network environment. This approach is used both for analysis and for the design of a distributed intelligent agent architecture. In (Falk, 1993) a multi-level warehouse hierarchy with its market mechanisms is applied to real-time transportation, dynamic freight allotment, depot agents, scheduling problems, and production planning.
One of the most promising approaches to decentralized systems is market-based control where the behavior of economic systems is imitated. In this framework producer and consumer agents both compete and cooperate on a market of certain commodities. Several market-based control and optimization strategies are presented in (Clearwater, 1996, Guenther, 1997, Berlin, 1998). In (Voos (1), 1999) and (Voos (2), 1999) market-based control algorithms are presented in more detail. On the basis of cost functions used by producer and consumer agents the optimization of distributed coupled linear systems is shown. The present paper adopts mainly ideas from (Guenther, 1997) and (Voos (1), 1999). The method is applied to a set of local Markov processes , e.g. queuing processes, respectively. In the case of several queues of customers it might be of interest to keep all queue lengths approximately the same in order to occupy the whole system equally. It is assumed that the local death rates can be influenced by some external control strategy but the local birth rates cannot. The main goal is to change the individual death rates so that, after some finite time, the distributions of all queuing processes are adjusted. The death rates of the local queuing processes are changed on the basis of the probabilities of the appearance of a selected event in each of the local queuing processes. The paper is organized as follows. Section 2 gives a short introduction into the problem of decentral queuing (birth and death) processes acting in parallel. In Section 3 the market-based algorithm is applied to a set of birth and death processes. In Section 4 simulation results are presented. Section 5 concludes with a short summary.

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2. DECENTRAL QUEUING PROCESSES

The problem of decentral queuing processes acting in parallel will be explained by a simple application example. Given a company producing n types of commodities (e.g. computers). Let further be given an unknown number of customers who order different numbers of each type of commodity. Let, for example, some customers order a number of computers of type 1, and let other customers order another number of computers of type 2 etc. Each computer needs for its production different materials (elements) some of them may be used for both types of computers. Let both the stream of demands by the customers and the times for handling the commissions by the company be exponentially distributed (see Fig 1). Furthermore, for simplicity we assume that a customer orders only one piece of a special type of commodity. We also assume that at a time instant t the change in the number of demands is 0 or 1.

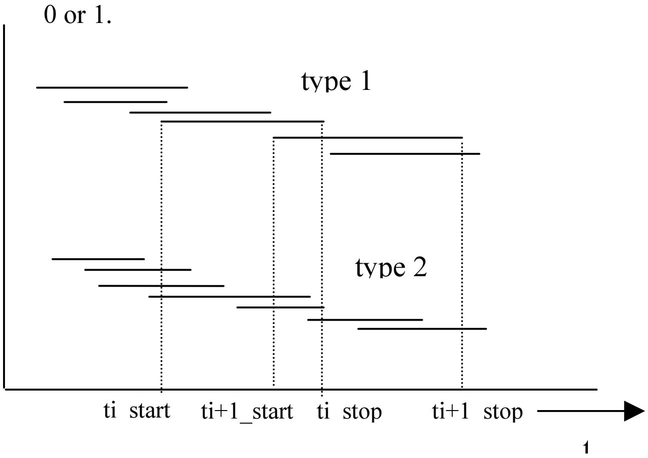

Fig. 1. Occurence of demands and handling times of the commissions

Let the time difference between two demands be $\Delta t_{i\,start} = t_{i+1\,start} - t_{i\,start}$. Let further the difference between the stops of orders be $\Delta t_{i\,stop} = t_{i+1\,stop} - t_{i\,stop}$. The corresponding probability functions for $t, \tau > 0$ are

$$P(\Delta t_{i\,start} < t) = 1 - e^{-\lambda_t t} \qquad t > 0 \quad (1)$$

$$P(\Delta t_{i\,stop} < \tau) = 1 - e^{-\lambda_\tau \tau} \qquad \tau > 0 \quad (2)$$

respectively. $\lambda_t, \lambda_\tau > 0$ are the parameters of the exponential processes (1) and (2). According to (Wolff, 1989) this type of stochastic process can be modeled by a homogeneous Markov process. Since demands are stochastically "born" and the corresponding handlings of commissions "die out" the process can be modeled by a so-called birth and death process. Let j be the state denoting "j active commissions" of an arbitrary commodity. The differential equations for the probability of the occurence of the states j=1,2,....*n* are

$$\begin{aligned} \dot{P}_0(t) &= -\lambda_0 P_0(t) + \mu_1 P_1(t) \\ \dot{P}_j(t) &= \lambda_{j-1} P_{j-1}(t) - (\lambda_j + \mu_j) P_j(t) \quad (3) \\ &\quad + \mu_{j+1} P_{j+1}(t) \end{aligned}$$

where λ_j are the birth rates and μ_j the death rates. $P_j(t)$ is the probability to reach the state j at time t when having started with some state *i* at time t_0. In (3) the birth and death rates are different for every state j. When assuming constant rates for all states one obtains instead

$$\begin{aligned} \dot{P}_0(t) &= -\lambda P_0(t) + \mu P_1(t) \\ \dot{P}_j(t) &= \lambda P_{j-1}(t) - (\lambda + \mu) P_j(t) + \mu P_{j+1}(t) \end{aligned} \quad (4)$$

We call a process *ergodic* or stable, respectively, if

$$0 < \lambda / \mu < 1.$$

In this case the process converges. That is, for a specific state $\tilde{j}$ the probability to reach a higher state becomes increasingly unlikely

$$P_j < P_{j-1} \quad \text{for} \quad j > \tilde{j}.$$

For our example the states j are the number of handled commissions of a commodity type *k=1,2,...,M* at time t. Then, $P_j^{\,k}(t)$ is the probabi-lity to have *j* commissions of a commodity type *k* in the queue at time t. $\dot{P}_j^{\,k}(t)$ is the change in the probability $P_j^{\,k}(t)$ at time t. Hence, equation (4) can be rewritten

$$\begin{aligned} \dot{P}_0^{\,k}(t) &= -\lambda^k P_0^{\,k}(t) + \mu^k P_1^{\,k}(t) \\ \dot{P}_j^{\,k}(t) &= \lambda^k P_{j-1}^{\,k}(t) - (\lambda^k + \mu^k) P_j^{\,k}(t) \quad (5) \\ &\quad + \mu^k P_{j+1}^{\,k}(t) \end{aligned}$$

where for each type *k* the λ^k and μ^k, respectively, are in general different.

Equations (4) can be considered as a system of equations concerning all types of commodities. This corresponds to the total Markov process of all events with n different states (see graph in Fig. 2). That is, in (4) and Fig. 2 we do not distinguish between different types *k* of commodities.

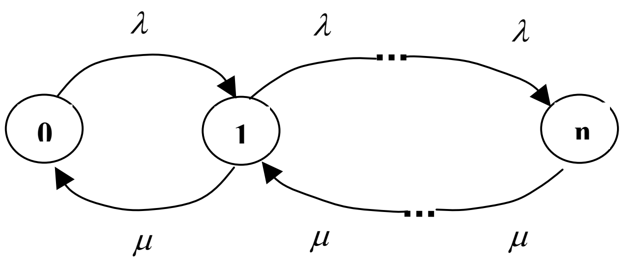

Fig. 2 Graph of the centralized Markov process

When splitting the process into subprocesses each of which belonging to a special type of commodity one obtains M different Markov processes (5) with the corresponding graphs (see Fig 3) each of which having n^k states.

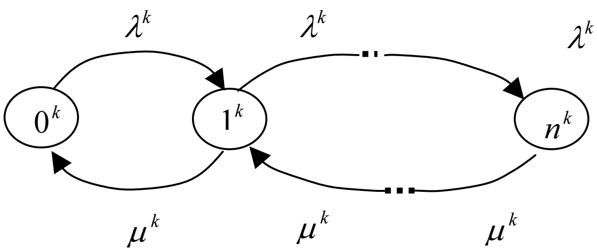

Fig. 3 Graph of the decentralized Markov processes

State $j^k = 1,2,...,n^k$ denotes "j^k active commissions" of commodity type *k*. The relation between the λ and μ, respectively, of the centralized process and the λ^k and μ^k, respectively, of the decentralized processes is

$$\lambda = \sum_{k=1}^{M} \lambda^k; \qquad \mu = \sum_{k=1}^{M} \mu^k \qquad (6)$$

because of the OR-combinations of independent probabilities. That is, a connection between the decentralized processes (5) and their individual graphs are connected via (6). In addition, the total number *j* of active commissions in the centralized model and the numbers j^k of individual type related active commissions are connected by

$$j = \sum_{k=1}^{M} j^k \qquad (7)$$

However, since *j* can be obtained in many ways the number of possible combinations of j^k to get the same total number *j* can be very high .

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3. PROCESS CONTROL BY MARKET-BASED ALGORITHMS

In the following the way of how to influence the whole process and the subprocesses, respectively, is investigated. The birth and death processes (5) are completely determined by their initial conditions and their parameters λ^k and μ^k. The birth rates λ^k cannot be influenced since they depend completely on the customer. The only parameters that can be influenced are μ^k. Increasing μ^k means that a commodity can be delivered faster (earlier) than before. One way of increasing μ^k is the exchange or loaning of materials or elements , respectively, from other types of commodities. This, however, leads to a decrease of the μ^k for these other types.

To reach a compromise for all subsystems a market-based algorithm is applied to find a so-called Pareto-optimum for the whole set of subprocesses . One can argue that it may be sufficient to make the queues as fast as possible. But under the condition of restricted recources of the suppliers this control policy may lead to longer queues, larger buffer capacities, and lower quality of service. Instead, the control policy applied here is to change the death rates of the individual queues in a way that their dynamical behaviors become equal. This is identical with the requirement for using the capacity all subprocesses in a well-balanced way which in turn leads to a synchronization of the queues. This can be reached by minimizing a cost /energy function of the total process via the local cost/energy functions.
An appropriate calculation of the death rates is done by a market-based approach that is presented in the following. In order to avoid misinterpretations one has to note that the below-mentioned terms "producer" and "consumer" should not be confused with real customers/consumers and suppliers/producers acting in the commisioning process described in the previous section. Subsequently, producer and consumer agents are virtual actors trading with virtual commodities like death rates.

Now, we define a producer agent Pag_j^k and a consumer agent Cag_j^k, respectively, for each state j^k belonging to a commodity of type *k*. Pag_j^k "produces" a certain death rate μ_p^k , and tries to maximize a local profit function $\rho_j^k > 0$. Cag_j^k "demands" for a certain death rate μ_c^k , and tries to maximize a local utility function $U_j^k > 0$. Both ρ_j^k and U_j^k will be specified later. The trade between producer and consumers agents takes place on the basis the local profit and utility functions, and common prices p_j. The latter can be calculated from the equlibrium of the whole "economy" in every time step. An equilibrium is reached as the sum over all supplied death rates μ_p^k is equal to the sum over all utilized death rates μ_c^k.

$$\sum_{l=1}^{M} \mu_p^l = \sum_{l=1}^{M} \mu_c^l \qquad (8)$$

from which the common prices p_j can be calculated. The utility function for the consumer agent Cag_j^k is now defined by

Utility = benefit - expenditure

$$U_j^k = \tilde{b}_j^k \mu_c^k - \tilde{c}_j^k p_j (\mu_c^k)^2 \qquad (9)$$

where $\tilde{b}_j^k, \tilde{c}_j^k > 0$ will be determined later in connection with the queue dynamics (5).
The profit function for the producer Pag_j^k is defined by

profit = income - costs

$$\rho_j^k = g_j^k p_j \mu_p^k - e_j^k (\mu_p^k)^2 \qquad (10)$$

where $g_j^k, e_j^k > 0$ are free parameters that determine the average price level. p_j is the actual price that has to be payed for μ_p^k by each consumer Cag_j^k. With regard to the choice of the individual terms in U_j^k and ρ_j^k see Guenther, 1997. The parameters $\tilde{b}_j^k, \tilde{c}_j^k$ in (9) can be determined by using local energy functions based on the dynamics (5) where, for simplicity, the argument *t* is eliminated

$$\dot{P}_j^k = \lambda^k P_{j-1}^k - (\lambda^k + \mu_c^k) P_j^k + \mu_c^k P_{j+1}^k \qquad (11)$$

From (11) a local energy function for each subprocess is defined as

$$J_{jc}^k = \left(\dot{P}_j^k\right)^2 = \alpha_j^k + \beta_j^k \mu_c^k + \gamma_j^k (\mu_c^k)^2 \qquad (12)$$

where

$$\begin{aligned} \alpha_j^k &= (\lambda^k)^2 \cdot (P_{j-1}^k - P_j^k)^2 \geq 0 \\ \beta_j^k &= -2\lambda^k \cdot (P_{j-1}^k - P_{j+1}^k)(P_j^k - P_{j+1}^k) \\ &\leq 0 \quad \text{for small } |\dot{P}_j^k| \\ \gamma_j^k &= (P_j^k - P_{j+1}^k)^2 \geq 0 \end{aligned} \qquad (13)$$

It can be shown that for the stationary case $\dot{P}_j^k = 0$, and for $p_j = 1$, the energy function (12) reaches its minimum at the maximum of the utility function (9), independently of the parameter α_j^k. Therefore, a comparison of (9) and (12) leads to the intuitive choice

$$\tilde{b}_j^k = |\beta_j^k|, \qquad \tilde{c}_j^k = \gamma_j^k \qquad (14)$$

in order to guarantee $\mu_c^k \geq 0$. The optimization of the whole process takes place by individual maximization of the local utility and profit functions, respectively. Maximization of the utility function (9) yields

$$\frac{\partial U_j^k}{\partial \mu_c^k} = \tilde{b}_j^k - 2\tilde{c}_j^k p_j \mu_c^k = 0 \qquad (15)$$

from which optimum "demanded" μ_c^k's are obtained

$$\mu_c^k = \frac{\tilde{b}_j^k}{2\tilde{c}_j^k} \cdot \frac{1}{p_j} \qquad (16)$$

Maximization of the profit function (10) yields

$$\frac{\partial \rho_j^k}{\partial \mu_p^k} = g_j^k p_j - 2e_j^k \mu_p^k = 0 \qquad (17)$$

from which optimum "produced" μ_p^k's are obtained

$$\mu_p^k = \frac{p_j}{2\eta_j^k}; \qquad \eta_j^k = \frac{e_j^k}{g_j^k} \qquad (18)$$

The requirement for an equilibrium between the sums of the "produced" μ_p^k's and the "demanded" μ_c^k's led to the balance equation (8).
Substituting (16) and (18) into (8) gives the price p_j for μ_p^k and μ_c^k, respectively.

$$p_j = \sqrt{\sum_{l=1}^{M} \tilde{b}_j^l / \tilde{c}_j^l \Big/ \sum_{l=1}^{M} 1/\eta_j^l} \qquad (19)$$

Substituting (19) into (16) yields the final equation for the death rate to be implemented in each subsystem. η_j^k can be chosen as a constant which gives reasonable results.

A better choice, however, is $\eta_j^k = \dfrac{\mu^k}{\lambda^k}$ to let η_j^k be dependent on μ^k and λ^k.

For the *stationary case* we obtain

$$P_j^k = \frac{\lambda^k}{\mu^k} P_{j-1}^k \qquad (20)$$

For β_j^k, and γ_j^k we obtain with (13) and (20)

$$\begin{aligned} \beta_j^k &= -2\mu^k \cdot \left(1 + \frac{\lambda^k}{\mu^k}\right)\left(1 - \frac{\lambda^k}{\mu^k}\right)^2 (P_j^k)^2 \leq 0 \\ \gamma_j^k &= \left(1 - \frac{\lambda^k}{\mu^k}\right)^2 (P_j^k)^2 \geq 0 \end{aligned} \qquad (21)$$

The determination of β_j^k and γ_j^k for the *nonstationary case*, however, requires the computation/measurement of the probabilities $P_j^k(t)$ that can be done by constructing histograms for every point in time. The probability of a state $P_j^k(t)$ is approximated by the number of events j^k divided by the total number of events

$$P_j^k(t) = n(j^k) \Big/ \sum_{i=1}^{N} n(i^k) \qquad (22)$$

4. SIMULATION EXAMPLES

Example 1

The first example deals with 3 different Markov processes with 11 states for each process.

The continuous processes and the optimization strategy are implemented as discrete models. The simulations have been done on a small time scale which can be changed for a real process accordingly. The initial values are

Process P1:

P0_1=1; P1_1=0; P2_1=0; P3_1=0; P4_1=0; P5_1=0; P6_1=0; P7_1=0; P8_1=0; P9_1=0; P10_1=0;

Process P2:

P0_2=1; P1_2=0; P2_2=0; P3_2=0; P4_2=0; P5_2=0; P6_2=0; P7_2=0; P8_2=0; P9_2=0; P10_2=0;

Process P3:

P0_3=1; P1_3=0; P2_3=0; P3_3=0; P4_3=0; P5_3=0; P6_3=0; P7_3=0; P8_3=0; P9_3=0; P10_3=0;

The corresponding parameters are

lambda_1=1; lambda_2=2.5; lambda_3=3.8; mu_1=1.4; mu_2=1.7; mu_3=1.9;

Process P1 is ergodic since lambda_1< mu_1. The processes P2 and P3 are non-ergodic since lambda_2 > mu_2 and lambda_3 > mu_3. Figure 4 shows the evolution of P1-P3 without any cooperation or interconnection between the processes. After T=10s the processes are almost stationary. For this case P1 shows the characteristic ergodic feature with $P_k > P_{k+1}$ where, on the other hand, for the processes P2 and P3 we have the non-ergodic feature $P_k < P_{k+1}$. Despite of this, P1 and P2 are not unstable but come to rest at a stationary distribution depicted in Fig. 4. The reason is that the number of states is restricted. Figure 5 presents the case in which the processes cooperate (compete) with each other. The corres-ponding parameters for the market-based algorithm are

e_1=2; g_1=1; eta_1=e_1/g_1;
e_2=2; g_2=1; eta_2=e_2/g_2;
e_3=2; g_3=1; eta_3=e_3/g_3;

The initial distribution is the same as in the previous case. The μ's are changed at each time step according to (16) or (18), respectively.

Already after T=1s it can be observed that the process distributions approach to each other. After T=10s all processes exhibit ergodic features which means that the corresponding μ's take finally larger values than the corresponding λ's

lambda_1 = 1;lambda_2 = 2.5; lambda_3 = 3.8; mu_1 = 1.3799; mu_2 = 3.5065 ; mu_3 = 5.3328.

Example 2

The distributions in the previous were generated directly by the differential equations (5) and (13), respectively. Instead, in the following example the distributions are the result of stochastic processes generated by noise generators that produce both the birth processes of demands and the death processes of handling the commissions It is assumed that only one time series is available. All birth and death processes are assumed to be stochastically independent, stationary, and ergodic.

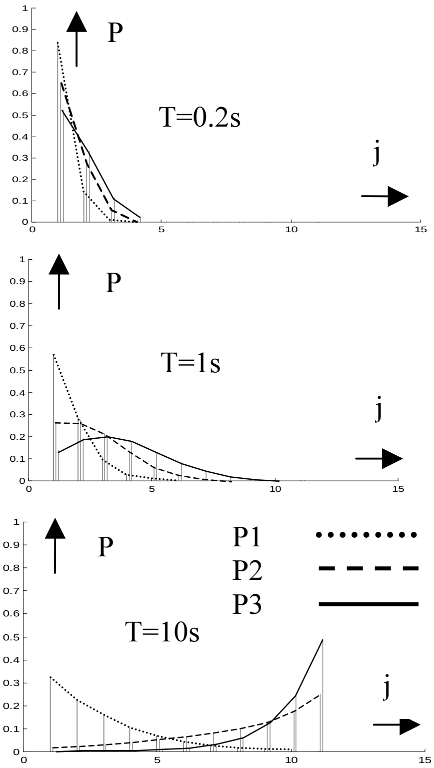

Fig 4. Evolution of P1- P3 (no optimization)

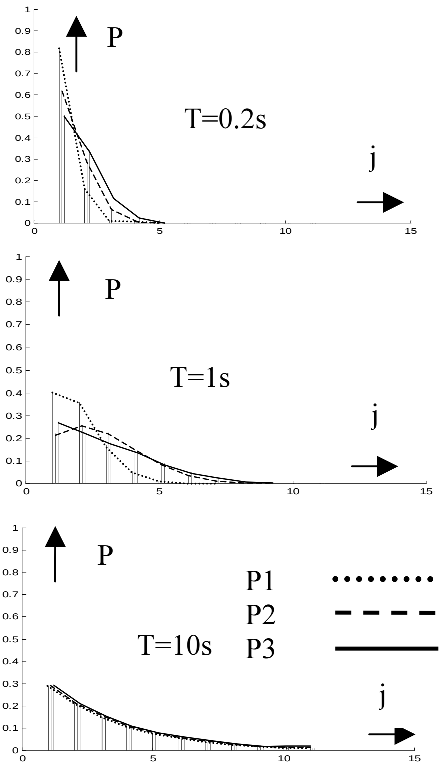

Fig 5. Evolution of P1-P3 (with optimization)

The corresponding distributions for the non-optimized and the optimized case after T=10s, respectively, are depicted in Figs. 6a,b. The birth and death rates λ (ld) and μ (mu) in Fig. 6 are the final rates after the experiment. The market-based optimization leads, as in the previous example, to a stable behavior and a synchronization of the three different queues.

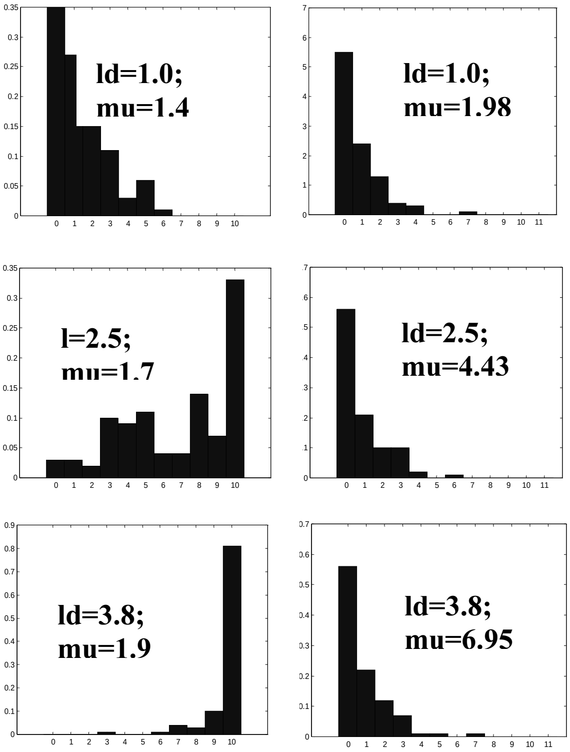

Fig 6 Processing of the death rates
a) no optimization b) with optimization

5. CONCLUSIONS

In the paper a commisioning process is modeled by set of local birth and death processes. Although the local birth rates are fixed, the corresponding death rates can be influenced by some specific control algorithms. The control goal is to change the local death rates so that the distributions of all queuing processes are adjusted. This is done by a market-based optimization method. In the paper an introduction to distributed birth and death processes is given, and the problem of a centralized or decentral representation of the problem is discussed. For the market-based algorithm applied here, local utility and profit functions, respectively, are defined on the basis of which the particular queuing processes are optimized. The result of the optimization is a so-called Pareto-optimum that represents a compromise between the competing processes. The simulation experiments show that the algorithm leads to excellent results for the adjustment of local Markov/queuing processes.

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