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FAULT DETECTION AND ISOLATION BASED ON ADAPTIVE OBSERVERS FOR LINEAR TIME VARYING SYSTEMS

Aiping Xu * **and Qinghua Zhang** *

* IRISA-INRIA, *Campus de Beaulieu, 35042 Rennes Cedex, France*
Email: zhang@irisa.fr

Abstract: A new method for fault detection and isolation (FDI) in *stochastic* linear time *varying* (LTV) systems is proposed in this paper. It allows to completely isolate any number of faults regardless of the number of output sensors, thanks to an appropriate assumption on the fault profiles and to some persistent excitation condition. In contrast, most existing methods enabling complete fault *isolation* have been developed for linear time *invariant* (LTI) systems and require a strong condition on the number of sensors. The method proposed in this paper is based on a recent development for the design of *adaptive observers*. Its performance is illustrated by a numerical example.

Keywords: fault detection and isolation, linear time varying system, adaptive algorithms.

1. INTRODUCTION

For the purpose of fault detection and isolation (FDI), in this paper we consider *stochastic* state space systems subject to additive faults of the form

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) + w(t) \\ &\quad +\psi_1(t)\theta_1 + \psi_2(t)\theta_2 + \cdots \psi_s(t)\theta_s \quad \text{(1a)} \\ y(t) &= C(t)x(t) + v(t) \quad \text{(1b)} \end{aligned}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^l$, $y(t) \in \mathbb{R}^m$ are respectively the state, input, output of the system; $A(t), B(t), C(t)$ are known *time varying* matrices of appropriate sizes; the noises $w(t) \in \mathbb{R}^n$, $v(t) \in \mathbb{R}^m$ are bounded, centered and independent of the other signals; the additional terms $\psi_1(t)\theta_1, \dots, \psi_s(t)\theta_s$ represent the *possible faults*. The *time varying* vectors $\psi_k(t) \in \mathbb{R}^n$, $k = 1, \dots, s$, are the assumed (time varying) *fault directions* in the state space, and $\theta_1, \dots, \theta_s$ are *constant* scalar coefficients. If the k-th fault (*i.e.*, the fault in the direction $\psi_k(t)$) is absent, then $\theta_k = 0$, otherwise $\theta_k \neq 0$. The matrices $A(t), B(t), C(t)$ and the vectors $\psi_1(t) \dots \psi_s(t)$ are all assumed piecewise continuous and uniformly bounded in time.

The problem considered in this paper is to detect and to isolate the presence of any non zero θ_k, from measured input-output signals $u(t), y(t)$, the known matrices $A(t), B(t), C(t)$ and the assumed fault directions $\psi_1(t), \dots, \psi_s(t)$.

Remark 1. The assumption of constant parameters θ_k is reasonable for two practical situations: the parameters vary slowly, or the parameters are piecewise constant with rare jumps. This assumption is similar to those typically used in the FDI methods based on on-line parameter estimation (Isermann, 1993). An alternative assumption frequently used in the FDI literature is that the "fault profiles" are arbitrary functions of time. It has in principle a wider applicability, but a consequence is that the number of output sensors must be larger than or equal to the number of faults ($m \geq s$) in order to fully isolate all the faults. In contrast, as shown in this paper, with the assumption of constant parameters θ_k, it is possible to isolate any finite number of faults, regardless of the number of output sensors. □

Remark 2. Assuming constant parameters θ_k *may seem* to lead to a simple solution of the FDI problem by considering the extended system

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\theta}(t)\end{bmatrix} = \begin{bmatrix}A(t) & \Psi(t)\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \theta(t)\end{bmatrix} + \begin{bmatrix}B(t)\\ 0\end{bmatrix}u(t) + \begin{bmatrix}w(t)\\ 0\end{bmatrix} \quad (2a)$$

$$y(t) = \begin{bmatrix}C(t) & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \theta(t)\end{bmatrix} + v(t) \quad (2b)$$

where the vector θ collects all the parameters θ_k and the matrix $\Psi(t)$ is composed of the vectors $\psi_k(t)$. Indeed the extended system is linear, thus the Kalman filter can apply. However, even in the case with constant matrices A, B, C, the extended system is *time varying*. In order to guarantee the convergence of the Kalman filter for a time varying system, its uniform complete observability is usually required (Jazwinski, 1970). In practice, it is difficult to check the uniform complete observability of the above *extended* system that should take into account some persistent excitation condition. Therefore, the application of the Kalman filter to the extended system is not a trivial problem. In this paper, we typically assume the uniform complete observability of the matrix pair $(A(t), C(t))$, but nothing about the observability of the extended system.

□

Since fault detection can be handled as a particular problem of fault isolation, we concentrate our presentation on fault isolation in this paper.

Now let us formulate the problem of fault isolation in a more compact manner. Assume that we want to decide if a subset of $p \leq s$ faults is present. Group the p corresponding fault directions $\psi_k(t)$ into a matrix $\Psi_f(t) \in \mathbb{R}^n \times \mathbb{R}^p$. The remaining $q = s - p$ fault directions are grouped into a matrix $\Psi_c(t) \in \mathbb{R}^n \times \mathbb{R}^q$ (the subscript c stands for "complementary"). Accordingly, collect the scalar parameters θ_k into two column vectors $\theta_f \in \mathbb{R}^p$ and $\theta_c \in \mathbb{R}^q$. Then system (1) is rewritten as

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)u(t) + w(t)\\ &\quad + \Psi_f(t)\theta_f + \Psi_c(t)\theta_c \quad (3a)\\ y(t) &= C(t)x(t) + v(t) \quad (3b)\end{aligned}$$

With this formulation, the fault isolation problem becomes, for each considered partition between $\Psi_f(t)\theta_f$ and $\Psi_c(t)\theta_c$, to decide whether the vector θ_f is zero or not, whatever the value of the "nuisance" vector θ_c is.

Remark 3. The class of systems considered in this paper includes in fact the so-called *state-affine nonlinear* systems, in the form of

$$\begin{aligned}\dot{x}(t) &= A(t,u,y)x(t) + B(t,u,y)u(t)\\ &\quad + \Psi_f(t,u,y)\theta_f + \Psi_c(t,u,y)\theta_c\\ y(t) &= C(t,u,y)x(t) + D(t,u,y)u(t)\end{aligned}$$

where the dependence of $A, B, C, D, \Psi_f, \Psi_c$ on u, y can be *nonlinear*. Since we do not need the time derivatives of the matrices $A, B, C, D, \Psi_f, \Psi_c$, their dependence on the known signals $u(t)$ and $y(t)$ can simply be viewed as the dependence on the time t. □

The main contribution of this paper is to propose a new residual generation method for complete isolation of faults in linear *time varying* systems, whereas most residual generation methods allowing complete fault isolation are restricted to linear *time invariant* (LTI) systems. Moreover, under appropriate assumptions, our method enables *complete isolation of an arbitrary number of faults* regardless of the number of output sensors, whereas most known fault isolation methods have a strong requirement on the number of sensors.

The theoretic basis of the FDI method proposed in this paper is a recent result on adaptive observers (Zhang, 2002). Adaptive observers have been used for FDI by different authors (Ding and Frank, 1993; Yang and Saif, 1995; Wang *et al.*, 1997; Zhang, 2000). As already mentioned, the particularity of the method proposed in this paper is to consider fault isolation in LTV systems, with a minimum requirement on the number of output sensors. It turns out that our method is somewhat similar to the one presented in the recent paper (Zhang *et al.*, 2001), with the main difference in the way modeling and measurement uncertainties are handled. In (Zhang *et al.*, 2001), some projection operator is used in the algorithms to ensure the boundedness of the estimates, whereas in our paper, uncertainties are handled through a more statistical approach.

The paper is organized as follows. In Section 2 we present the proposed residual generator. Section 3 is devoted to the analysis of the behavior of the proposed residual. A simulation example is given in Section 4. Finally, some concluding remarks are drawn in Section 5.

2. RESIDUAL GENERATION

As the proposed residual generation method is based on the adaptive observer presented in (Zhang, 2002), let us shortly recall this result.

Rewrite system (3) as

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t) + \Psi(t)\theta \quad (4a)$$

$$y(t) = C(t)x(t) + v(t) \quad (4b)$$

with

$$\Psi(t) = \begin{bmatrix}\Psi_f(t) \ \Psi_c(t)\end{bmatrix} \in \mathbb{R}^n \times \mathbb{R}^s \quad \theta = \begin{bmatrix}\theta_f \\ \theta_c\end{bmatrix} \in \mathbb{R}^s$$

Let $\Gamma \in \mathbb{R}^s \times \mathbb{R}^s$ be a symmetric positive definite matrix, $\Sigma(t) \in \mathbb{R}^m \times \mathbb{R}^m$ a bounded symmetric positive definite matrix, and $K(t) \in \mathbb{R}^n \times \mathbb{R}^m$ a gain matrix. Under appropriate assumptions, the ordinary differential equation (ODE) system

$$\dot{\Upsilon}(t) = [A(t) - K(t)C(t)]\Upsilon(t) + \Psi(t) \tag{5a}$$

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t)+B(t)u(t)+\Psi(t)\hat{\theta}(t) + \left[K(t)+\Upsilon(t)\Gamma\Upsilon^T(t)C^T\Sigma(t)\right][y(t)-C(t)\hat{x}(t)] \tag{5b}$$

$$\dot{\hat{\theta}}(t) = \Gamma\Upsilon^T(t)C^T(t)\Sigma(t)\,[y(t) - C(t)\hat{x}(t)] \tag{5c}$$

is a global exponential adaptive observer for system (4) with convergence in the mean. Note that the matrix $\Upsilon(t) \in \mathbb{R}^n \times \mathbb{R}^s$ has the same size as $\Psi(t)$. See (Zhang, 2002) for the details.

Typically, $\Sigma(t)$ is chosen to be the inverse of the covariance matrix of the output noise $v(t)$, $K(t)$ is set to the Kalman gain designed for the fault-free system, Γ is used to balance the convergence speeds of state estimation and parameter estimation.

With such an adaptive observer, one may want to directly solve the FDI problem by on-line parameter estimation. Alternatively, the method proposed in this paper is based on residuals generated with the aid of the adaptive observer. One reason for this choice is its robustness against false alarms even when the system is not sufficiently excited for parameter estimation. See Theorem 1 in Section 3.

The basic idea of the residual generator presented below is to use an adaptive observer to estimate the "nuisance" parameter θ_c. The residual is then generated as the prediction error of the adaptive observer that should be insensitive to the "nuisance" parameter θ_c, but sensitive to the monitored parameter θ_f.

Residual generator Let $\Gamma \in \mathbb{R}^q \times \mathbb{R}^q$, $\Sigma(t) \in \mathbb{R}^m \times \mathbb{R}^m$, and $K(t) \in \mathbb{R}^n \times \mathbb{R}^m$ be as in the adaptive observer (5). We introduce the residual $r(t)$ as follows:

$$\dot{\Upsilon}_c(t) = [A(t) - K(t)C(t)]\Upsilon_c(t) + \Psi_c(t) \tag{6a}$$

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t)+B(t)u(t)+\Psi_c(t)\hat{\theta}_c(t) + \left[K(t)+\Upsilon_c(t)\Gamma\Upsilon_c^T(t)C^T\Sigma(t)\right] \cdot [y(t)-C(t)\hat{x}(t)] \tag{6b}$$

$$\dot{\hat{\theta}}_c(t) = \Gamma\Upsilon_c^T(t)C^T(t)\Sigma(t)\,[y(t) - C(t)\hat{x}(t)] \tag{6c}$$

$$r(t) = \Sigma^{\frac{1}{2}}(t)[C(t)\hat{x}(t) - y(t)] \tag{6d}$$

Note that the generated residual $r(t) \in \mathbb{R}^m$ has the same dimension as $y(t)$.

The residual $r(t)$ is intended to monitor the presence of the faults $\Psi_f(t)\theta_f$ but to ignore the faults $\Psi_c(t)\theta_c$. In order to isolate different faults, several residual generators should run in parallel, each with a different partition between $\Psi_f(t)\theta_f$ and $\Psi_c(t)\theta_c$.

The behavior of the proposed residual is analyzed in the following section.

3. RESIDUAL BEHAVIOR ANALYSIS

Let us investigate the behavior of the residual, first in the absence of the monitored faults $\Psi_f(t)\theta_f$, then in their presence.

3.1 *In the absence of the monitored faults*

Assumption 1. Assume that the matrix pair $(A(t), C(t))$ is such that a bounded (time-varying) matrix $K(t) \in \mathbb{R}^n \times \mathbb{R}^m$ can be designed so that the system

$$\dot{x}(t) = [A(t) - K(t)C(t)]x(t) \tag{7}$$

is exponentially stable.

This assumption means that the fault free system has an exponential observer. It is known that, if the matrix pair $(A(t), C(t))$ is uniformly completely observable, then the Kalman gain $K(t)$ can fullfil Assumption 1 (Jazwinski, 1970). Note that the observability of the extended system (2) is *never* required in this paper.

Theorem 1. Under Assumption 1, if the monitored faults are absent, *i.e.*, $\theta_f = 0$, then, whatever the value of θ_c is, the residual $r(t)$ generated by (6) tends to zero in the mean, that is, $\mathbf{E}r(t) \to 0$ when $t \to \infty$.

Note that this theorem holds regardless of any excitation condition of the system. It implies the robustness of the FDI method against false alarms when the system is not sufficiently excited for parameter estimation.

Proof of Theorem 1

For notational convenience, we do not explicitly write the dependence on t of all the variables, though the proof is valid for time varying systems.

Combine (6b) and (6c) to obtain

$$\dot{\hat{x}} = A\hat{x} + Bu + \Psi_c\hat{\theta}_c + K(y - C\hat{x}) + \Upsilon_c\dot{\hat{\theta}}_c$$

Let $\tilde{x} = \hat{x} - x$, $\tilde{\theta}_c = \hat{\theta}_c - \theta_c$ and notice that $\theta_f = 0$, $\dot{\theta}_c = 0$, then

$$\dot{\tilde{x}} = (A - KC)\tilde{x} + \Psi_c\tilde{\theta}_c + \Upsilon_c\dot{\tilde{\theta}}_c - w + Kv \tag{8}$$

The key step of the proof is to define the following linear combination of $\tilde{x}$ and $\tilde{\theta}_c$:

$$\eta(t) = \tilde{x}(t) - \Upsilon_c(t)\tilde{\theta}_c(t)$$

then we have

$$\dot{\eta} = (A - KC)(\eta + \Upsilon_c\tilde{\theta}_c) + \Psi_c\tilde{\theta}_c - \dot{\Upsilon}_c\tilde{\theta}_c - w + Kv$$
$$= (A - KC)\eta + [(A - KC)\Upsilon_c + \Psi_c - \dot{\Upsilon}_c]\tilde{\theta}_c - w + Kv$$

Because Υ_c is generated by (6a), we have

$$\dot{\eta} = (A - KC)\eta - w + Kv \tag{9}$$

Take the mathematical expectation on both sides of this equation, exchange the order between the expectation and the derivative, and denote

$$\bar{\eta}(t) = \mathbf{E}\eta(t)$$

then

$$\dot{\bar{\eta}} = (A - KC)\bar{\eta} \tag{10}$$

By Assumption 1, system (10) is exponentially stable. Choose a *constant* positive definite matrix $Q \in \mathbb{R}^n \times \mathbb{R}^n$ such that the matrix $2\Sigma(t) - \Sigma(t)C(t)Q^{-1}C^T(t)\Sigma(t)$ is positive definite for all t. According to (Brockett, 1970), for any positive definite matrix $Q \in \mathbb{R}^n \times \mathbb{R}^n$ (in particular, the above chosen one), there exists a positive definite matrix $P(t)$ such that

$$\frac{d}{dt}\left[\bar{\eta}^T(t)P(t)\bar{\eta}(t)\right] = -\bar{\eta}^T(t)Q\bar{\eta}(t)$$

Now let us study the behavior of $\tilde{\theta}_c$. As $\dot{\theta}_c = 0$, then

$$\dot{\tilde{\theta}}_c = \Gamma\Upsilon_c^T C^T\Sigma(y - C\hat{x})$$
$$= -\Gamma\Upsilon_c^T C^T\Sigma C\tilde{x} + \Gamma\Upsilon_c^T C^T\Sigma v$$
$$= -\Gamma\Upsilon_c^T C^T\Sigma C(\eta + \Upsilon_c\tilde{\theta}_c) + \Gamma\Upsilon_c^T C^T\Sigma v \tag{11}$$

Take the mathematical expectation on both sides of the last equation, exchange the order between the expectation and the derivative, and notice that v is independent of $\Gamma\Upsilon_c^T C^T$, then

$$\dot{\bar{\tilde{\theta}}}_c = -\Gamma\Upsilon_c^T C^T\Sigma C(\bar{\eta} + \Upsilon_c\bar{\tilde{\theta}}_c) \tag{12}$$

where

$$\bar{\tilde{\theta}}_c(t) = \mathbf{E}\tilde{\theta}_c(t)$$

Since $\bar{\tilde{\theta}}_c$ depends on $\bar{\eta}$, we have to study the joint behavior of $\bar{\eta}$ and $\bar{\tilde{\theta}}_c$.

Define the Lyapounov function candidate

$$V(t) = \bar{\eta}^T(t)P(t)\bar{\eta}(t) + \bar{\tilde{\theta}}_c^T(t)\Gamma^{-1}\bar{\tilde{\theta}}_c(t)$$

Then

$$\dot{V}(t) = -\bar{\eta}^T Q\bar{\eta} - 2\bar{\tilde{\theta}}_c^T\Upsilon_c^T C^T\Sigma C(\bar{\eta} + \Upsilon_c\bar{\tilde{\theta}}_c) \tag{13}$$
$$= -\begin{bmatrix}\bar{\eta}^T & \bar{\tilde{\theta}}_c^T\end{bmatrix}\begin{bmatrix}Q^{\frac{1}{2}} & 0\\ S & R\end{bmatrix}\begin{bmatrix}Q^{\frac{1}{2}} & S^T\\ 0 & R^T\end{bmatrix}\begin{bmatrix}\bar{\eta}\\ \bar{\tilde{\theta}}_c\end{bmatrix} \tag{14}$$

with

$$S = \Upsilon_c^T C^T\Sigma CQ^{-\frac{1}{2}}$$
$$R = \left[\Upsilon_c^T C^T(2\Sigma - \Sigma CQ^{-1}C^T\Sigma)C\Upsilon_c\right]^{\frac{1}{2}}$$

Note that Q has been chosen such that $2\Sigma - \Sigma CQ^{-1}C^T\Sigma$ is positive definite, so the matrix square root R exists.

From (14) we know that $\dot{V}(t) \leq 0$. It follows that $V(t)$ is non increasing. Therefore, $\bar{\eta}(t)$ and $\bar{\tilde{\theta}}_c(t)$ are both bounded.

It is easy to check that $\ddot{V}(t)$ is bounded, as $\Upsilon_c(t), \bar{\eta}(t), \dot{\Upsilon}_c(t), \dot{\bar{\eta}}(t)$ are all bounded. Then by Barbalat's lemma (Slotine and Li, 1991), $\dot{V}(t) \to 0$.

From (13), and because $\dot{V}(t) \to 0$, $\bar{\eta}(t) \to 0$, we get

$$\bar{\tilde{\theta}}_c^T\Upsilon_c^T C^T\Sigma C\Upsilon_c\bar{\tilde{\theta}}_c \to 0 \tag{15}$$

This implies

$$\Sigma^{\frac{1}{2}}C\Upsilon_c\bar{\tilde{\theta}}_c \to 0 \tag{16}$$

Therefore,

$$\mathbf{E}r(t) = \mathbf{E}\left\{\Sigma^{\frac{1}{2}}(t)[C(t)\hat{x}(t) - y(t)]\right\}$$
$$= \mathbf{E}\left\{\Sigma^{\frac{1}{2}}(t)[C(t)\tilde{x}(t) - v(t)]\right\}$$
$$= \Sigma^{\frac{1}{2}}C\mathbf{E}\tilde{x}$$
$$= \Sigma^{\frac{1}{2}}C(\bar{\eta} + \Upsilon_c\bar{\tilde{\theta}}_c)$$
$$= \Sigma^{\frac{1}{2}}C\bar{\eta} + \Sigma^{\frac{1}{2}}C\Upsilon_c\bar{\tilde{\theta}}_c$$
$$\to 0$$

□

Remark that quation (16) does not necessarily mean $\bar{\tilde{\theta}} \to 0$. The latter would be true only under some persistent excitation condition, that is not required in Theorem 1.

3.2 *In the presence of the monitored faults*

We have shown that the residual $r(t)$ converges in the mean to zero when $\theta_f = 0$, whatever the value of θ_c is, and whatever the excitation is.

The residual should also allow the detection of the monitored faults $\theta_f \neq 0$. For this purpose, some persistent excitation is required.

Assumption 2. Let $\Upsilon(t) \in \mathbb{R}^n \times \mathbb{R}^s$ be a matrix of signals generated by the ODE system

$$\dot{\Upsilon}(t) = [A(t) - K(t)C(t)]\Upsilon(t) + \Psi(t) \tag{17}$$

Assume that $\Psi(t)$ is persistently exciting, so that there exist two positive constants α, T and some bounded symmetric positive definite matrix $\Sigma(t) \in \mathbb{R}^m \times \mathbb{R}^m$ such that, for all t, the following inequality holds

$$\int_t^{t+T} \Upsilon^T(\tau)C^T(\tau)\Sigma(\tau)C(\tau)\Upsilon(\tau)d\tau \geq \alpha I \tag{18}$$

Theorem 2. Under Assumptions 1 and 2, if the monitored faults are present, *i.e.*, $\theta_f \neq 0$, then for the residual $r(t)$ generated by (6), the mean $\mathbf{E}r(t)$ cannot tend to zero when $t \to \infty$.

Proof of Theorem 2 We prove the contrapositive of the theorem instead of directly proving itself. More precisely, we show that, if $\mathbf{E}r(t) \to 0$, then $\theta_f = 0$.

As in the proof of Theorem 1, combine (6b) and (6c) to obtain

$$\dot{\hat{x}} = A\hat{x} + Bu + \Psi_c\hat{\theta}_c + K(y - C\hat{x}) + \Upsilon_c\dot{\hat{\theta}}_c$$

Let $\tilde{x} = \hat{x} - x$ and $\tilde{\theta}_c = \hat{\theta}_c - \theta_c$. Now θ_f is *not* assumed to be zero, so

$$\dot{\tilde{x}} = (A - KC)\tilde{x} - \Psi_f\theta_f + \Psi_c\tilde{\theta}_c + \Upsilon_c\dot{\tilde{\theta}}_c - w + Kv$$

Now define

$$\eta(t) = \tilde{x}(t) + \Upsilon_f(t)\theta_f(t) - \Upsilon_c(t)\tilde{\theta}_c(t) \quad (19)$$

where $\Upsilon_f(t) \in \mathbb{R}^n \times \mathbb{R}^p$ is generated by

$$\dot{\Upsilon}_f(t) = [A(t) - K(t)C(t)]\Upsilon_f(t) + \Psi_f(t) \quad (20)$$

Then we get

$$\begin{aligned}\dot{\eta} &= (A - KC)(\eta - \Upsilon_f\theta_f + \Upsilon_c\tilde{\theta}_c) - \Psi_f\theta_f + \Psi_c\tilde{\theta}_c \\ &\quad + \dot{\Upsilon}_f\theta_f - \dot{\Upsilon}_c\tilde{\theta}_c - w + Kv \\ &= (A - KC)\eta - [(A - KC)\Upsilon_f + \Psi_f - \dot{\Upsilon}_f]\theta_f \\ &\quad + [(A - KC)\Upsilon_c + \Psi_c - \dot{\Upsilon}_c]\tilde{\theta}_c - w + Kv\end{aligned}$$

Because Υ_f is generated by (20) and Υ_c by (6a), we obtain again

$$\dot{\eta} = (A - KC)\eta - w + Kv \quad (21)$$

As before, let $\bar{\eta}(t) = \mathbf{E}\eta(t)$, then

$$\dot{\bar{\eta}} = (A - KC)\bar{\eta} \quad (22)$$

By Assumption 1, system (22) is exponentially stable, therefore $\bar{\eta} \to 0$ exponentially fast.

Now let us study the behavior of $\tilde{\theta}_c$. As $\dot{\theta}_c = 0$, then

$$\begin{aligned}\dot{\tilde{\theta}}_c &= \Gamma\Upsilon_c^T C^T \Sigma(y - C\hat{x}) \quad (23) \\ &= -\Gamma\Upsilon_c^T C^T \Sigma C\tilde{x} + \Gamma\Upsilon_c^T C^T \Sigma v \\ &= -\Gamma\Upsilon_c^T C^T \Sigma C(\eta - \Upsilon_f\theta_f + \Upsilon_c\tilde{\theta}_c) \\ &\quad + \Gamma\Upsilon_c^T C^T \Sigma v \\ &= -\Gamma\Upsilon_c^T C^T \Sigma C\Upsilon_c\tilde{\theta}_c + \Gamma\Upsilon_c^T C^T \Sigma C\Upsilon_f\theta_f \\ &\quad - \Gamma\Upsilon_c^T C^T \Sigma C\eta + \Gamma\Upsilon_c^T C^T \Sigma v \quad (24)\end{aligned}$$

Let $\bar{\tilde{\theta}}_c(t) = \mathbf{E}\tilde{\theta}_c(t)$, then

$$\begin{aligned}\dot{\bar{\tilde{\theta}}}_c &= -\Gamma\Upsilon_c^T C^T \Sigma C\Upsilon_c\bar{\tilde{\theta}}_c + \Gamma\Upsilon_c^T C^T \Sigma C\Upsilon_f\theta_f \\ &\quad - \Gamma\Upsilon_c^T C^T \Sigma C\bar{\eta} \quad (25)\end{aligned}$$

Now assume that

$$\mathbf{E}r(t) = \mathbf{E}\left\{\Sigma^{\frac{1}{2}}(t)[C(t)\hat{x}(t) - y(t)]\right\} \to 0$$

It implies, through (23), that $\dot{\bar{\tilde{\theta}}}_c \to 0$, *i.e.*, $\bar{\tilde{\theta}}_c$ tends to a constant vector.

The assumption $\mathbf{E}r(t) \to 0$ implies $\Sigma^{\frac{1}{2}}C\mathbf{E}\tilde{x} \to 0$. From (19), that is, $\eta = \tilde{x} + \Upsilon_f\theta_f - \Upsilon_c\tilde{\theta}_c$, and because $\bar{\eta} \to 0$, we get

$$\Sigma^{\frac{1}{2}}C(\Upsilon_f\theta_f - \Upsilon_c\bar{\tilde{\theta}}_c) \to 0 \quad (26)$$

Notice that $\Upsilon = [\Upsilon_f, \Upsilon_c]$. Then, rewrite (26) as

$$\Sigma^{\frac{1}{2}}C\Upsilon \begin{bmatrix}\theta_f \\ -\bar{\tilde{\theta}}_c\end{bmatrix} \to 0$$

We have shown that $\bar{\tilde{\theta}}_c$ tends to a constant vector, so when $t \to \infty$,

$$\left(\int_t^{t+T} \Upsilon^T(\tau)C^T(\tau)\Sigma(\tau)C(\tau)\Upsilon(\tau)d\tau\right) \begin{bmatrix}\theta_f \\ -\bar{\tilde{\theta}}_c\end{bmatrix} \to 0$$

By Assumption 2, the integral in the last equation is positive definite and bounded from below. It follows that $\theta_f = 0$ and $\bar{\tilde{\theta}}_c \to 0$. We then have proved the contrapositive of Theorem 2, and thus the theorem itself. □

4. NUMERICAL EXAMPLE

This example, adapted from the one of (Marino and Tomei, 1995), is a single link robot arm rotating in a vertical plane. The equation of motion is

$$I\ddot{q} + \frac{1}{2}mgl\sin q = u$$

where q is the rotation angle, u the input torque, I the moment of inertia, g the gravity constant, m the mass and l the length of the arm.

Let $x_1 = q, x_2 = \dot{q}, y = q, \theta_1 = 1/I, \theta_2 = mgl/(2I)$, then the state space model is

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\theta_2\sin y + \theta_1 u \\ y &= x_1\end{aligned}$$

which fits into the form of (1) with $\psi_1(t) = [0, u(t)]^T$ and $\psi_2(t) = [0, -\sin y(t)]^T$.

The simulation parameters as follows. The nominal parameter values are: $m = 1, l = 1, I = 0.5$. The input signal is $u(t) = 5(\sin 2t + \cos 3t)$. The initial condition is $x(0) = [1, 1]^T$. A Gaussian noise with standard deviation 0.1 is added to y.

As shown in Figure 1, at the 20th second, θ_1 changes from 2 to 2.8, and at the 40th second, θ_2 changes from 9.8 to 17.64.

Two residuals as formulated in (6) are generated: $r_1(t)$ and $r_2(t)$ to monitor the changes in θ_1 and

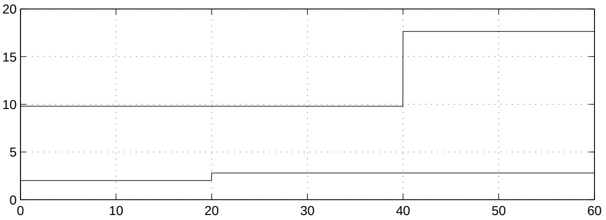

Figure 1. Single link robot arm: the parameters θ_1 (lower) and θ_2 (upper).

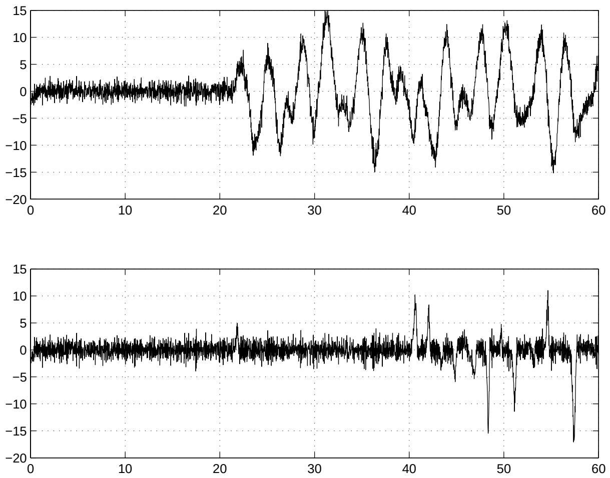

Figure 2. Single link robot arm: the residuals $r_1(t)$ monitoring θ_1 (top) and $r_2(t)$ monitoring θ_2 (bottom).

θ_2, respectively. Remind that θ_2 is estimated in the generator of $r_1(t)$, whereas θ_1 is estimated in the generator of $r_2(t)$. The parameters of the residual generator $r_1(t)$ are $K = [2, 2]^T, \Sigma = 10, \Gamma = 6, \Sigma = 10$. For $r_2(t)$ the parameters are $K = [2, 2]^T, \Sigma = 10, \Gamma = 5, \Sigma = 10$.

The two residuals are plotted in Figure 2. At the beginning, the behaviors of the two residuals are essentially due to the simulated measurement noise. At the 20th second, both $r_1(t)$ and $r_2(t)$ react to the change of θ_1. The residual $r_2(t)$ quickly reestablishes its nominal behavior after a short transient, whereas $r_1(t)$ is persistently affected by the change. Starting from the 40th second, the behavior of $r_2(t)$ is affected by the change of θ_2.

Because of the presence of the noise, a statistical method should be used for the evaluation of the residuals. A related result can be found in (Zhang, 2000). This issue is not further discussed here due to the limitation of space.

5. CONCLUSION

We have proposed a new *residual generation* method for detection and isolation of faults in linear time *varying* (LTV) systems. It allows to completely isolate any number of faults regardless of the number of output sensors, thanks to an appropriate assumption on the fault profiles and to some persistent excitation condition.

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