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H_2 OPTIMAL DECOUPLING FIRS

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Abstract: FIR compensator design for H_2-optimal decoupling of measurable or previewed signals in discrete-time linear time-invariant systems is considered. The algorithm for the FIR system weighting matrices computation is based on pseudoinversion techniques aiming to minimize the l_2 norm of the overall system impulse responses. The inherent dimensionality constraint of these techniques is overcome by welding problems referring to subsequent time subintervals of the FIR system window. The solution of the H_2-optimal state observation problem with unknown inputs straightforwardly comes by duality.

Keywords: H_2 optimal decoupling, preview control, FIR systems.

1. INTRODUCTION

It is well known that in the discrete-time case the use of FIR systems is particularly convenient for the solution of the decoupling problem of measurable or previewed signals as well as the perfect tracking problem (that can be considered as a particular case of the former). The dual problem i.e., the possibly delayed unknown-input observation of a linear function of the state as well as left inversion as a particular case is also conveniently solved with FIR systems.

The necessity of using FIR systems is due to the nature of the modes which the optimal finite-time state trajectory arcs consist of. In fact, both stable and anti-stable modes (corresponding to eigenvalues reciprocal to each other) are to be reproduced in the control function.

If a set of geometric-type conditions are met, H_2 optimal decoupling or tracking problems and their duals can be solved cost-free (or almost cost-free). This aspect has recently been investigated in (Marro *et al.*, 2000*c*), where a compensator with a peculiar structure, a parallel of a FIR system and a dynamic unit, has been proposed.

As far as previewed signal decoupling and tracking is concerned, it is well-known that perfect or almost perfect tracking can be achieved also in the nonminimum phase case if the reference signal is known in advance. See, for instance, (Devasia *et al.*, 1996) and (Hunt *et al.*, 1996) for the infinite horizon nonlinear and linear cases, respectively. Instead, refer to (Gross and Tomizuka, 1994) and (Marro and Fantoni, 1996) for two different approaches to the receding horizon SISO case and also to (Marro *et al.*, 2000*a*) for the introduction of FIR systems in obtaining the noncausal inversion of MIMO discrete-time linear systems.

Regarding the dual problems, FIR filter and smoother design has been extensively investigated and its use is now well established, mainly for the estimation of some state variables of stochastic systems ((Park *et al.*, 2000), (Kwon *et al.*, 1999), (Kwon *et al.*, 1994), (Kwon and Byun, 1989), (Kwon and Kwon, 1987), (Kwon *et al.*, 1983), (Kwon and Pearson, 1978)), although some attempts have also been made to introduce the receding horizon technique for the design of observers in a deterministic environment ((Ling and Lim, 1996)).

The novelty of this contribution consists in presenting a solution of the decoupling problem with preview (hence of its dual) by means of a FIR compensator achieving the minimum H_2 norm of the transfer function matrix from the input to be decoupled to the controlled output (or, in the dual case, of the transfer function from the unknown input to the estimation error).

Throughout this paper, $\mathbb{R}$ stands for the field of real numbers, sets, vector spaces and subspaces are denoted by script capitals like $\mathcal{V}$, matrices and linear maps by slanted capitals like A, the image and the null space of A by $\mathrm{im}A$ and $\ker A$ respectively, the trace by $\mathrm{tr}A$, the transpose by A', the pseudo-inverse by $A^{\#}$.

2. STATEMENT OF THE PROBLEM

Consider the linear discrete time-invariant system Σ described by

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k)+Hh(k)\,, \\ y(k) &= Cx(k)+Du(k)+Gh(k)\,, \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^p$, previewed or measured input $h \in \mathbb{R}^s$ and controlled output $y \in \mathbb{R}^q$. Assume that matrix A is stable, pair (A,B) controllable and matrices $[B'\ D']'$ and $[H'\ G']'$ full column rank. Refer to the block diagram in Fig. 1, where the N_p-

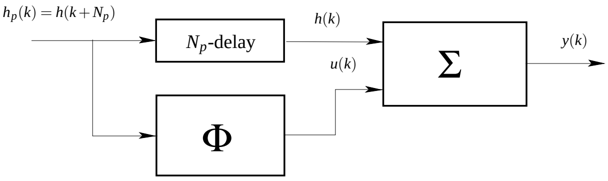

Fig. 1. Decoupling a measurable or previewed signal.

step preview interval of signal $h(k)$ is accounted for by the N_p-delay unit, so that the overall system having $h_p(k)=h(k+N_p)$ as input and $y(k)$ as output is causal. The particular case $N_p=0$ corresponds to the decoupling of a signal which is measurable but not known in advance. The block Φ denotes a FIR system described by

$$u(k) = \sum_{\ell=0}^{N-1} \Phi(\ell) h_p(k-\ell)\,, \tag{2}$$

with window $N > N_p$. Referring to Fig. 1, denote by $W(z)$ the transfer function matrix from $h_p(k)$ to $y(k)$. The H_2 optimal decoupling problem is stated as follows.

Problem 1. (H_2 optimal decoupling problem with preview) Refer to systems (1) and (2) connected as shown in Fig. 1. Given the window N and the preview time N_p, find the FIR weighting matrices $\Phi(\ell)$ $(\ell=1,\ldots,N-1)$ minimizing $\|W\|_2$.

It is worth noticing that solution of Problem 1 also applies to the dual problem, as briefly shown in the sequel. Refer to the linear discrete-time system Σ_{d} described by

$$\begin{aligned} x(k+1) &= A_d\, x(k)+B_d\, u(k)\,, \\ y(k) &= C_d\, x(k)+D_d\, u(t)\,, \\ e(k) &= H_d\, x(k)+G_d\, u(t)\,, \end{aligned} \tag{3}$$

with state $x \in \mathbb{R}^n$, inaccessible input $u \in \mathbb{R}^q$, informative output $y \in \mathbb{R}^p$, output to be estimate $e \in \mathbb{R}^s$. Matrix A_d is assumed to be stable, pair (A_d, C_d) observable and matrices $[C_d\ D_d]$, $[H_d\ G_d]$ full row rank.

Let us consider Fig. 2, where Φ_{d} is a FIR system with weighting matrices $\Phi_d(\ell)$, $(\ell=1,\ldots,N-1)$. Denote by $W_d(z)$ the transfer function matrix from $u(k)$ to $\eta(k-N_p)$.

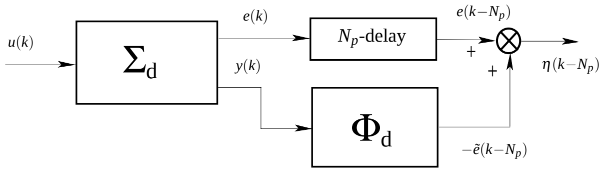

Fig. 2. Unknown-input current or delayed observation of a linear function of the state.

Problem 2. (H_2 optimal unknown-input estimation of a linear function of the state with delay) Refer to system (3) connected as shown in Fig. 2. Given the window N and the delay time N_p, find the FIR weighting matrices $\Phi_d(\ell)$ $(\ell=1,\ldots,N-1)$ minimizing $\|W_d\|_2$.

Solution of Problem 2 can be derived from that of Problem 1 as $\Phi_d(\ell)=\Phi'(\ell)$ $(\ell=1,\ldots,N-1)$, provided the following correspondences are set: $A=A_d'$, $B=C_d'$, $C=B_d'$, $D=D_d'$, $H=H_d'$, $G=G_d'$.

2.1 *Geometric conditions for perfect decoupling*

In this section some geometric conditions guaranteeing perfect or almost perfect decoupling are briefly recalled. They have been proven and applied in (Barbagli *et al.*, 2000) and (Marro *et al.*, 2000*c*). Let us denote by $\hat{\mathcal{V}}^*$ the maximum $(\hat{A}, \mathrm{im}\,\hat{B})$-controlled invariant contained in $\ker\hat{C}$ and $\hat{\mathcal{S}}^*$ the minimum $(\hat{A}, \ker\hat{C})$-conditioned invariant containing $\mathrm{im}\,\hat{B}$, with $(\hat{A},\hat{B},\hat{C})=(A,B,C)$ if both D and G are null matrices and

$$\hat{A} := \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \quad \hat{B} := \begin{bmatrix} B \\ D \end{bmatrix}, \quad \hat{C} := \begin{bmatrix} 0 & I_q \end{bmatrix}, \tag{4}$$

if not. Also, define $\hat{\mathcal{H}} := \mathrm{im}H$ if both D and G are null matrices, $\hat{\mathcal{H}} := \mathrm{im}([H'G']')$ if not.

If system (1) is minimum phase with respect to u, the condition

$$\hat{\mathcal{H}} \subseteq \hat{\mathcal{V}}^* \tag{5}$$

guarantees that perfect decoupling is achievable with a stable feedforward dynamic unit without any preaction. The condition

$$\hat{\mathcal{H}} \subseteq \hat{\mathcal{V}}^* + \hat{\mathcal{S}}^* \tag{6}$$

guarantees that perfect decoupling is achievable with a stable feedforward dynamic unit with only a relative-degree preaction.

On the other hand, if the system (1) is nonminimum phase, condition (6) enables perfect decoupling only as the preaction time N_p approaches infinity. However, almost perfect decoupling is achievable if N_p is large enough with respect to the time constant of the unstable zero closest to the unit circle. In the above mentioned cases $\|W\|_2$ is zero or can made arbitrarily small.

If condition (6) is not satisfied or system (1) is non-minimum phase and the available preaction time is not large enough, the H_2 optimal design object of this paper is a convenient resort and the use of FIR compensators instead of dynamic systems unifies and greatly simplifies the synthesis procedures.

3. THE MODIFIED FINITE HORIZON LQ PROBLEM

Solution of Problem 1 can easily be obtained by slightly extending an efficient algorithm for solving the finite-horizon linear quadratic optimal control problem, possibly cheap or singular. This extension and the corresponding algorithmic solution are presented below as Problem 3 and Theorem 1.

Problem 3. (Finite-horizon LQ problem with a previewed impulse input and constrained final state) Consider system (1) with given initial state $x(0)=x_0$ and final state constrained as

$$\Gamma x(N) = y_f\,, \tag{7}$$

where matrix Γ and vector y_f are given. The final time N is assumed to be greater than the controllability index of (A,B). Let $h(k)=\overline{h}\delta(k-N_p)$, where both $\overline{h}$ and $N_p<N$ are given. Find a control sequence $u(k)$ $(k=0,\dots,N-1)$ minimizing the cost

$$J := \sum_{k=0}^{N-1} y'(k)y(k) + x'(N)Z'Zx(N). \tag{8}$$

where Z, a penalty matrix on the final state, is given.

Let us introduce the following compact notation for the control sequence:

$$u_N := \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}. \tag{9}$$

A solution of Problem 3 is provided by the following theorem.

Theorem 1. A solution of Problem 3 is given as

$$u_N^o = T_N x_0 + V_N y_f + W_N \overline{h}\,, \tag{10}$$

where the matrices T_N, V_N and W_N are defined by

$$T_N := -P_{B_N}(\Gamma L_N)^{\#}\Gamma A^N - K(B_N K)^{\#} A_N,$$
$$V_N := P_{B_N}(\Gamma L_N)^{\#},$$
$$W_N := -P_{B_N}(\Gamma L_N)^{\#}\Gamma A^{N-N_p-1}H - K(B_N K)^{\#} H_N,$$

being $P_{B_N} = \left(I - K(B_N K)^{\#} B_N\right)$ and

$$A_N := \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \\ ZA^N \end{bmatrix},\; B_N := \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-2}B & CA^{N-3}B & \cdots & D \\ ZA^{N-1}B & CA^{N-2}B & \cdots & ZB \end{bmatrix}, \tag{11}$$

$$H_N := \left[\, 0 \,\cdots\, G' \;\, (CH)' \,\dots\, (CA^{N-N_p-2}H)' \;\, (ZA^{N-N_p-1}H)' \,\right]',$$

$$L_N := \left[\, A^{N-1}B \; A^{N-2}B \,\cdots\, B \,\right].$$

and K is a basis matrix for $\ker(\Gamma L_N)$. The optimal cost is

$$J^o = \begin{bmatrix} x_0 \\ y_f \\ \overline{h} \end{bmatrix}' \begin{bmatrix} C_N' C_N & C_N' D_N & C_N' E_N \\ D_N' C_N & D_N' D_N & D_N' E_N \\ E_N' C_N & E_N' D_N & E_N' E_N \end{bmatrix} \begin{bmatrix} x_0 \\ y_f \\ \overline{h} \end{bmatrix}, \tag{12}$$

where matrices C_N, D_N and E_N are defined by

$$C_N = P_{B_N}\left(A_N - B_N(\Gamma L_N)^{\#}\Gamma A^N\right), \tag{13}$$
$$D_N = P_{B_N} B_N (\Gamma L_N)^{\#}, \tag{14}$$
$$E_N = P_{B_N}\left(H_N - B_N(\Gamma L_N)^{\#}\Gamma A^{N-N_p-1}H\right). \tag{15}$$

The proof of Theorem 1 has been omitted. A complete proof of the theorem is given in (Marro *et al.*, 2000*b*).

Theorem 1 provides an algorithm framework to deal with the following finite horizon LQ optimization problems.

1. Standard LQ problem with both the initial and the final state completely assigned. In this case matrices H and G and, consequently, W_N, H_N and E_N are not defined. The problem is solved by assuming $\Gamma=I_n$, $y_f=x_f$, and $Z=0_{1,n}$.
2. Standard LQ problem with the initial state assigned and the final state simply weighted. In this case matrices H, G, W_N, H_N and E_N are not defined like in the previous case. The problem is solved by assuming $\Gamma=0_n$, $y_f=0_{n,1}$, while Z is given to define the cost of the final state. The solution does not depend explicitly on the final state (matrices V_N and D_N are null).
3. LQ problem with both the initial and the final state completely assigned and an impulsive input $h(k)=\overline{h}\delta(k-N_p)$. The problem is solved like in case 1 above, but with all the matrices defined.
4. LQ problem with both the initial state assigned, the final state simply weighted, and an impulsive $h(k)=\overline{h}\delta(k-N_p)$. The problem is solved like in case 2 above, with all the matrices defined.

Theorem 1 describes a pseudoinversion procedure to solve the optimal control Problem 3 with a N_p–previewed impulse disturbance input and weighted final state. Since the dimensions of the matrices to be pseudo-inverted are proportional to the number of steps N of the control time interval, this technique is subject to a dimensionality constraint depending on the computational capability available. However, this drawback can be overcome by means of the additive procedure described in Section 5.

4. DESIGN OF THE H_2 OPTIMAL FIR COMPENSATOR

The solution of the H_2 optimal control problem stated in Problem 1 can easily be derived by using the algorithm provided in Theorem 1. The H_2 optimal control design of a FIR controller Φ with the N_p-previewed signal $h(k)$ corresponds to solve a finite horizon linear quadratic problem with impulse disturbance of the type stated in Problem 3. In fact, the H_2 norm of the transfer function of the overall system from $h_p(k)$ to $y(k)$, see fig. 1, can be written as

$$
\begin{aligned}
\|W\|_2 &= \left(\frac{1}{2\pi} \operatorname{tr} \left[\int_{-\pi}^{\pi} W(e^{j\omega}) W^*(e^{j\omega}) d\omega \right] \right)^{\frac{1}{2}} \\
&= \left(\operatorname{tr} \left[\sum_{k=0}^{\infty} w(k) w'(k) \right] \right)^{\frac{1}{2}} \\
&= \left(\operatorname{tr} \left[\sum_{k=0}^{\infty} w'(k) w(k) \right] \right)^{\frac{1}{2}} ,
\end{aligned}
\tag{16}
$$

where $w(k)$ denotes the impulse response matrix corresponding to $W(e^{j\omega})$. By using (16) Problem 1 is easily re-stated in terms of Problem 3, and solved with Theorem 1.

Theorem 2. A solution of Problem 1 is provided by

$$
\begin{aligned}
&\Phi(j) = \left[\phi(0) \ \phi(1) \ \cdots \ \phi(N-1) \right] W_N , \\
&\text{with } \begin{cases} \phi(i) = 0_s & \text{for } i \neq j \\ \phi(i) = I_s & \text{for } i = j \end{cases} ,
\end{aligned}
\tag{17}
$$

where W_N is defined as in Theorem 1, eq. (15). Matrices B_N and H_N in eq. (11) are computed with $Z = \sqrt{S_\infty}$, where S_∞ denotes the solution of the Liapunov equation

$$
S - A'SA = C'C . \tag{18}
$$

Proof: Owing to (16), it is immediate to verify that $\|W\|_2^2$ is equal to the cost index (8) in the statement of Problem 3 under the assumptions

$$
x(0) = 0_{n,s} , \ \ \Gamma = 0_n , \ \ y_f = 0_{n,s} ,
$$

$$
Z = \sqrt{S_\infty} , \ \ \bar{h} = I_s .
$$

Note that $x(N) S_\infty x(N)$ accounts for the cost from $k = N$ to $k = \infty$ and is evaluated through the Liapunov equation (18), since the control input becomes zero from $k = N$ on. ■

5. EXTENSION OF THE CONTROL INTERVAL

Referring to Fig. 3, assume that the overall control interval N_t is divided in a finite number of subintervals whose length N satisfies the computational constraint and is greater than the the controllability index of (A, B). Three subarcs can easily be distinguished: it will be shown that the costs on the intervals 1 and 3

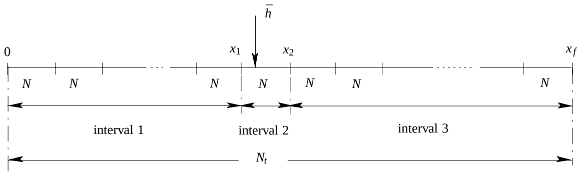

Fig. 3. Subarcs within the H_2-optimal decoupling problem.

are expressed by quadratic forms of x_1 and x_2, respectively. Once the corresponding cost matrices S_1 and S_2 have been determined as described in Section 5.2 below, it is possible to derive x_1 and x_2 as follows.

5.1 *Solution for interval 2*

The cost for the whole interval $[0, N_t]$ can be expressed as

$$
c = x_1' S_1 x_1 + x_2' S_2 x_2 + \begin{bmatrix} x_1 \\ x_2 \\ \bar{h} \end{bmatrix}' \begin{bmatrix} C_N' C_N & C_N' D_N & C_N' E_N \\ D_N' C_N & D_N' D_N & D_N' E_N \\ E_N' C_N & E_N' D_N & E_N' E_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \bar{h} \end{bmatrix} \tag{19}
$$

or, by setting $\xi := [x_1' \ x_2']'$ and consequently defining new matrices,

$$
c = \xi' M \xi + \begin{bmatrix} \xi \\ \bar{h} \end{bmatrix}' \begin{bmatrix} R_1 & R_2 \\ R_2' & R_3 \end{bmatrix} \begin{bmatrix} \xi \\ \bar{h} \end{bmatrix} , \tag{20}
$$

The optimal value of ξ is derived as

$$
\xi^o = -(M + R_1)'^{\#} R_2 \bar{h} . \tag{21}
$$

Being x_1 and x_2 known, the control sequences of intervals 1 and 3 can be derived as $(V_N)_1 x_1$ and $(T_N)_3 x_2$, respectively, where $(V_N)_1$ and $(T_N)_3$ denote the global matrices obtained with the iterative procedure described in the subsequent Section 5.2. The control sequence of interval 2 is computed as $T_N x_1 + V_N x_2 + W_N \bar{h}$, according to (10).

5.2 *Welding subarcs in intervals 1 and 3*

Let us consider Problem 3 with $\bar{h} = 0$. Recall that, if both the initial state x_0 and the final state x_f are given, Theorem 1, which refers to interval $[0, N]$, defines matrices T_N, V_N, C_N and D_N that provide an optimal control sequence expressed as

$$
u_N^o = T_N x_0 + V_N x_f , \tag{22}
$$

and the corresponding optimal cost as

$$
J^o = \begin{bmatrix} x_0 \\ x_f \end{bmatrix}' \begin{bmatrix} M_1 & M_2 \\ M_2' & M_3 \end{bmatrix} \begin{bmatrix} x_0 \\ x_f \end{bmatrix} , \tag{23}
$$

with $M_1 = C_N' C_N$, $M_2 = C_N' D_N$, $M_3 = D_N' D_N$. We shall show that the control interval can arbitrarily be enlarged by welding optimal subarcs. Let us suppose that the generic time interval $[0, \bar{N}]$ has been divided into two subsequent subintervals, $[0, N_1]$ and $[N_1, N_1 + N_2]$ and that the corresponding control input and cost matrices $(T_{N_1}, V_{N_1}, M_{1,1}, M_{2,1}, M_{3,1})$ and $(T_{N_2}, V_{N_2}, M_{1,2}, M_{2,2}, M_{3,2})$ have already been computed. Assume $x(0)=x_0$, $x(N_1)=x_1$, $x(\bar{N})=x(N_1+N_2)=x_f$. The overall cost is

$$\begin{aligned} c = {} & x_0' M_{1,1} x_0 + 2 x_0' M_{2,1} x_1 + x_1' M_{3,1} x_1 \\ & x_1' M_{1,2} x_1 + 2 x_1' M_{2,2} x_f + x_f' M_{3,2} x_f \,. \end{aligned} \tag{24}$$

The value of x_1 minimizing c is derived by nulling ∇c_{x_1} as

$$x_1 = Q_1 x_0 + Q_2 x_f \,, \tag{25}$$

where

$$\begin{aligned} Q_1 &:= -(M_{3,1}+M_{1,2})'^{\#} M_{2,1} \,, \\ Q_2 &:= -(M_{3,1}+M_{1,2})'^{\#} M_{2,2} \,. \end{aligned} \tag{26}$$

By substitution in (24) we obtain the cost matrices referring to the overall interval $[0, \bar{N}]$ as

$$\begin{aligned} \bar{M}_1 &:= M_{1,1} + 2M_{2,1}Q_1 + Q_1'(M_{3,1}+M_{1,2})Q_1 \,, \\ \bar{M}_2 &:= M_{2,1}Q_2 + Q_1'(M_{3,1}+M_{1,2})Q_2 + Q_1' M_{2,2} \,, \\ \bar{M}_3 &:= Q_2'(M_{3,1}+M_{1,2})Q_2 + 2Q_2' M_{2,2} + M_{3,2} \,. \end{aligned} \tag{27}$$

and the corresponding control input matrices as

$$T_{\bar{N}} = \begin{bmatrix} T_{N_1} + V_{N_1} Q_1 \\ T_{N_2} Q_1 \end{bmatrix}, \quad V_{\bar{N}} = \begin{bmatrix} V_{N_1} Q_2 \\ T_{N_2} Q_2 + V_{N_2} \end{bmatrix}. \tag{28}$$

The above described procedure can be iterated to achieve the solution of the problem in an arbitrarily large control interval, starting from two intervals for which direct computation as provided in Theorem 1 is feasible. At the last iteration, if the final state is not sharply assigned but just weighted (like in the case of interval 3), matrices V_{N_2}, $M_{2,2}$ and $M_{3,2}$ should be omitted in eqs. (26), (27) and (28) since they are not defined.

6. AN EXAMPLE

Let

$$A = \begin{bmatrix} 0.5 & 1 & -0.4 & 0 \\ 0.1 & 0.7 & 0 & -0.5 \\ 0 & 0 & 0.4 & 0 \\ 0.2 & 0 & 0 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 4 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The system (A,B,C,D) is both left and right invertible. The plant is nonminimum phase (with invariant zeros 0.8 and 1.1). Condition (6) is satisfied because of right invertibility, so that perfect decoupling could be achieved at the limit as both N_p and $N-N_p$ approach infinity. Suppose that, due to restricted preview time of the signal to be decoupled, a preaction time $N_p=20$ is only possible and assume $N=40$ for the FIR compensator window. Fig. 4 shows the FIR gains that optimally decouple a previewed unit impulse $h(k)=\delta(k-N_p)$ occurring at $k=N_p$ and the corresponding optimal responses. The square of the H_2 optimal norm computed in this case is $\|W\|_2^2=0.246$.

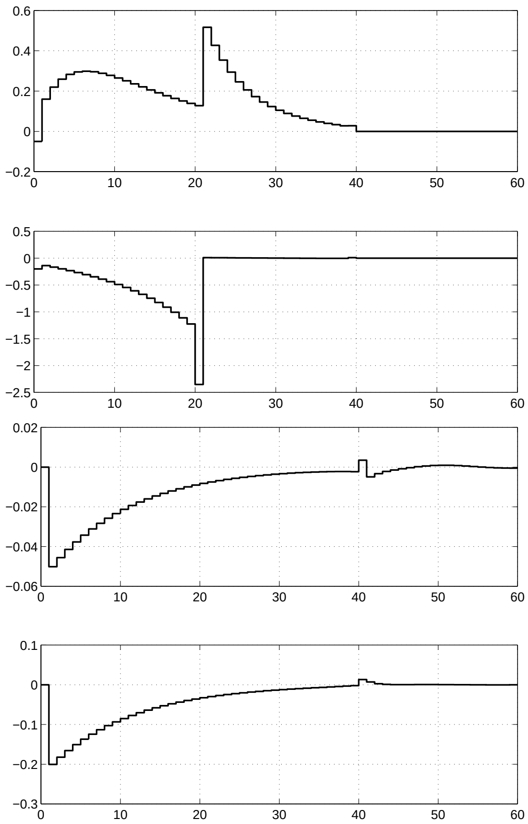

Fig. 4. FIR gains (left) and optimally decoupled outputs (right) for $N=40$, $N_p=20$.

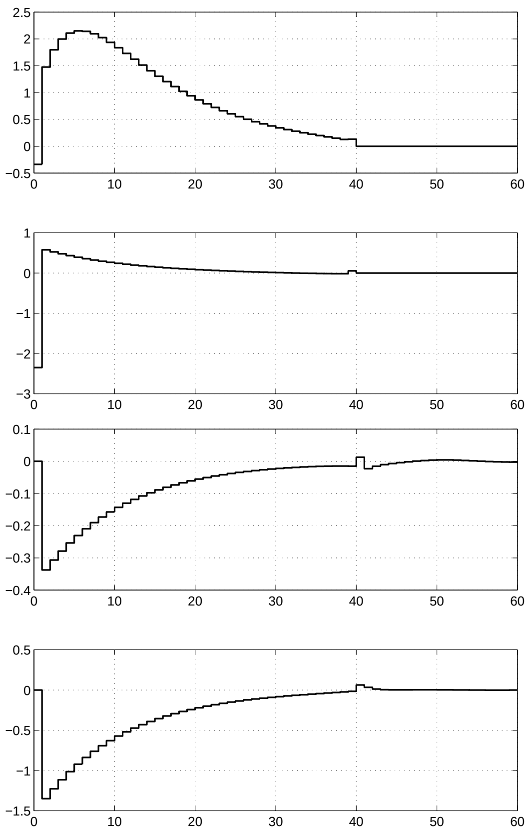

Fig. 5. FIR gains (left) and optimally decoupled outputs (right) for $N=40$, $N_p=0$.

Suppose now that no preview of signal $h(k)$ is available. A FIR compensator with $N_p = 0$ and $N = 40$ is derived. The FIR gains and corresponding optimal responses referring to this case are shown in Fig. 5. The square of the H_2 optimal norm is $\|W\|_2^2 = 11.333$ in this case, hence significantly greater than the preview case.

7. CONCLUDING REMARKS

A design procedure for a FIR system with given window N, providing H_2-optimal decoupling of a N_p-step previewed signal has been described. The use of a FIR compensator instead of a dynamic unit, although not extensively treated in the literature, is advisable since preaction reduces the H_2 norm of the transfer function matrix from the input to be decoupled to the controlled output also in the minimum phase case if the geometric conditions (5) and (6) recalled in Section 2.1 are not satisfied. Since feedthrough matrices D and G are present in the system equations, the disturbance decoupling problem also includes as a particular case H_2 optimal right inversion (or tracking). The results obtained are directly applicable to the dual problem, H_2 optimal unknown-input observation of a linear function of the state with N_p-step postknowledge and, as a particular case, left inversion (estimation of an unknown input).

8. ACKNOWLEDGEMENTS

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