15th Triennial World Congress, Barcelona, Spain

LYAPUNOV-BASED FORCE CONTROL OF A FLEXIBLE ARM CONSIDERING BENDING AND TORSIONAL DEFORMATION

Yoshifumi Morita * **Fumitoshi Matsuno** **
Yukihiro Kobayashi * **Motohisa Ikeda** * **Hiroyuki Ukai** *
Hisashi Kando *

* *Nagoya Institute of Technology, Gokiso, Showa, Nagoya, 466-8555, JAPAN*
** *Tokyo Institute of Technology, Nagatsuta, Midori, Yokohama, 226-8502, JAPAN*

Abstract: This paper describes the Lyapunov-based force control suppressing the coupled bending and torsional vibrations of a one-link flexible arm with a rigid tip body. On the basis of the distributed parameter model, the output feedback control law is constructed using Lyapunov method, and the asymptotic stability of the closed-loop system is proved. The proposed controller consists of the PD feedback of the motor angle and a feedback of the bending strain at the root of the flexible arm. Some simulations are performed to show the effectiveness of the proposed controller.

Keywords: Flexible arms, Force control, Lyapunov methods, Output feedback, Distributed parameter systems, Vibration dampers, Asymptotic stability

1. INTRODUCTION

Modeling and vibration control of flexible structures have received a great deal of attention in recent years. Vibration control of a flexible arm especially with a concentrated mass has been studied(Sakawa, Matsuno and Fukushima, 1985; Luo, 1993). In the case of the flexible arm with a rigid tip body, for example when the end-effecter is attached at the end tip of the arm and it grasps an object, however, coupled bending and torsional vibrations occur. For this reason, the robust controller design of a flexible arm with a rigid tip body have been discussed (Sakawa and Luo, 1989; Matsuno, Murachi and Sakawa, 1994; Morita and Matsuno et al., 2001). However, the order of the resulting controller becomes large owing to the elimination of the spillover effect. So the controller cannot help being complicated.

On the other hands, the Lyapunov-based control has been proposed for some simple distributed parameter systems(Matsuno and Ohno, 1997; Kasai and Matsuno, 2000). The resulting control law is a kind of direct sensor feedback control, and is simple and robust against parameter uncertainty. However, the applicability of the Lyapunov-based control to the coupled bending and torsional vibrations system has never been discussed.

In this paper, robust force control based on Lyapunov method is discussed considering the absorption of the coupled bending and torsional vibrations of a one-link flexible arm with a rigid tip body. In Section 2, an ordinary differential equation of a motor angle and distributed parameter systems of coupled bending and torsional vibrations are derived by using Hamilton's principle. In Section 3, an output feedback control law is constructed by applying Lyapunov method to the distributed parameter model. In Section 4, the asymptotic stability of the closed-loop system is proved on the basis of the distributed parameter model. The simulation results are given in Section 5.

2. DISTRIBUTED PARAMETER MODEL

The constrained one-link flexible arm with a rigid tip body considered in this paper is shown in Figure 1. The flexible arm of length L, having uniform mass density ρ per unit length, mass polar moment of inertia $\rho\kappa^2$ per unit length, uniform bending rigidity EI, and uniform torsional rigidity GJ, is clamped on the vertical shaft of the rotor of the motor at one end, and has the rigid body at the other end. Let O_2 denote the connected point of the arm and the rigid tip body, and let Q and R denote the center of gravity and the end point of the rigid tip body, respectively. To simplify the discussion, we assume that O_2, Q and R are positioned on the same straight line.

Let X, Y, Z denote an inertial Cartesian coordinate frame. Let $X_1, Y_1, Z_1(= Z)$ denote a coordinate frame rotating with the motor. Let X_2, Y_2, Z_2 denote a coordinate frame attached to the point O_2, where the X_2 axis is the beam's tip tangent, the Z_2 axis is on the straight line passing through O_2 and Q when there is no torsional deformation of the arm, and the Y_2 axis is chosen according to the right-handed coordinate system. By the torsional deformation, the coordinate frame X_2, Y_2, Z_2 is rotated around the X_2 axis, and let $X_3(= X_2), Y_3, Z_3$ be the rotated coordinate frame. Let X_4, Y_4, Z_4 be a coordinate frame attached to the point Q, where the X_4, Y_4, Z_4 axes are parallel to the X_3, Y_3, Z_3 axes, respectively. The rigid tip body has the mass m and the moment of inertia J_x around the X_4 axis.

Let $\theta(t)$, $\tau(t)$ and J_m be the angle of rotation of the motor, the torque developed by the motor, and the moment of inertia of the rotor of the motor. Let $w(t, r)$ and $\varphi(t, r)$ denote the bending and torsional displacements at time t and at a spatial point r $(0 \le r \le L)$. Because the point R is constrained, a reaction force $f(t)$ is generated along the normal direction of the constraint surface. Let g be the acceleration of gravity. $(\dot{\ })$ and $(\ ')$ denote the time derivative and the derivative with respect to the spatial variable r, respectively.

As force control is considered as small motion around an equilibrium state, it is assumed that $\theta(t) \cong 0$, $w(t, r) \cong 0$, $w'(t, r) \cong 0$, $\varphi(t, r) \cong 0$, and their product terms are neglected. Let $\boldsymbol{r}$ and $\boldsymbol{P}$ be a position vector of an arbitrary point on the arm and the point Q, which can be expressed as

$$\boldsymbol{r} = [\, r \quad r\theta(t) - w(t, r) \quad 0\,]^T, \tag{1}$$

$$\boldsymbol{P} = [\, L \quad L\theta(t) - w_E(t) - \varphi_E(t)e_z \quad e_z\,]^T, \tag{2}$$

where $w_E(t) = w(t, L)$, $\varphi_E(t) = \varphi(t, L)$ and $e_z = |\overline{O_3Q}|$. In Figure 1, the constraint surface is assumed to be described as

$$\Phi(X,\ Y,\ Z) = Y = 0. \tag{3}$$

The surface equation (3) can be rewritten as the constraint condition:

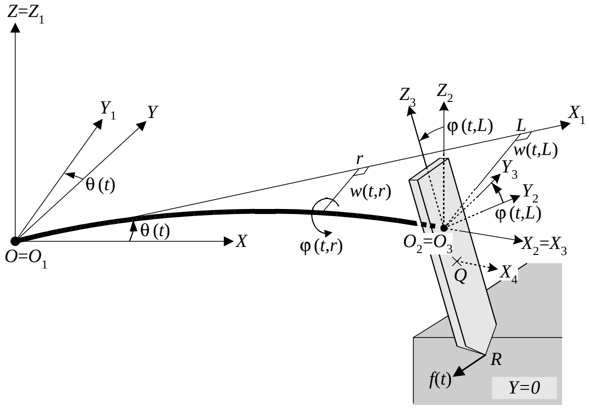


Fig. 1. Constrained flexible arm with a rigid tip body

$$\phi(\theta(t), w_E(t), \varphi_E(t)) = L\theta(t) - (w_E(t) + l_z\varphi_E(t)) = 0, \tag{4}$$

where $l_z = |\overline{O_3R}|$.

The total kinetic energy $\mathcal{T}$ and the potential energy $\mathcal{V}$ are given by

$$\mathcal{T} = \mathcal{T}_{motor} + \mathcal{T}_{arm} + \mathcal{T}_{tip}, \tag{5}$$

$$\mathcal{T}_{motor} = \frac{1}{2}J_m\dot{\theta}^2(t),$$

$$\mathcal{T}_{arm} = \frac{1}{2}\int_0^L \rho\dot{\boldsymbol{r}}^T\dot{\boldsymbol{r}}dr + \frac{1}{2}\int_0^L \rho\kappa^2\dot{\varphi}^2(t, r)dr,$$

$$\mathcal{T}_{tip} = \frac{1}{2}m\dot{\boldsymbol{P}}^T\dot{\boldsymbol{P}} + \frac{1}{2}J_x\dot{\varphi}_E^2(t),$$

$$\mathcal{V} = \frac{1}{2}\int_0^L [EIw''^2(t, r) + GJ\varphi'^2(t, r)]dr + mge_z \tag{6}$$

The virtual work $\delta\mathcal{W}$ of the motor is given by

$$\delta\mathcal{W} = \tau\delta\theta. \tag{7}$$

Let $\lambda(t)$ be the Lagrange multiplier associated with the constraint condition (4). The constraint force $f(t)$ at the end point of the arm, i.e., the contact force between the end-effector and the constraint surface, can be expressed in terms of the Lagrange multiplier as $f(t) = -\lambda(t)$. By using Hamilton's principle and Lagrange multiplier method, the equations of motion are derived as follows:
the equation of the motor angle:

$$J_m\ddot{\theta}(t) + EIw''(t, 0) = \tau(t) + \lambda(t)\left(\frac{\partial\phi}{\partial\theta} + \frac{\partial\phi}{\partial w_E}L\right), \tag{8}$$

the equation of the bending vibration of the arm:

$$\ddot{w}(t, r) + \frac{EI}{\rho}w''''(t, r) = r\ddot{\theta}(t), \tag{9}$$

$$EI\left[\frac{m}{\rho}w_E''''(t) + w_E'''(t)\right] - \frac{mGJe_z}{\rho\kappa^2}\varphi_E''(t)$$

$$= -\lambda(t)\frac{\partial \phi}{\partial w_E}, \quad (10)$$

$$EIw''_E(t) = 0, \quad w(t,0) = 0, \quad w'(t,0) = 0, \quad (11)$$

the equation of the torsional vibration of the arm:

$$\ddot{\varphi}(t,r) - \frac{GJ}{\rho\kappa^2}\varphi''(t,r) = 0, \quad (12)$$

$$-\frac{mEIe_z}{\rho}w''''_E(t) + (me_z^2 + J_x)\frac{GJ}{\rho\kappa^2}\varphi''_E(t)$$
$$+GJ\varphi'_E(t) = \lambda(t)\frac{\partial \phi}{\partial \varphi_E}, \quad (13)$$
$$\varphi(t,0) = 0. \quad (14)$$

The boundary conditions (10) and (13) are coupled each other. It is found that the bending and torsional vibrations are coupled.

3. LYAPUNOV-BASED FORCE CONTROL

3.1 *The relation between a desired force and the state variables*

Let f_d be the desired constraint force, λ_d the related Lagrange multiplier, θ_d the related static motor angle, $w_d(r)$ the related static bending displacement and $\varphi_d(r)$ the related static torsional displacement at the desired state. In the situation where the system is controlled to accomplish the desired contact force f_d, the relation between f_d and the desired Lagrange multiplier λ_d is given by

$$f_d = -\lambda_d. \quad (15)$$

By substituting the condition for the desired states:

$$\begin{cases} \theta(t) = \theta_d, \quad \dot{\theta}(t) = \ddot{\theta}(t) = 0, \\ w(t,r) = w_d(r), \quad \dot{w}(t,r) = \ddot{w}(t,r) = 0, \\ \varphi(t,r) = \varphi_d(r), \quad \dot{\varphi}(t,r) = \ddot{\varphi}(t,r) = 0, \\ f(t) = f_d, \quad \lambda(t) = \lambda_d, \end{cases}$$

into the equations of vibration (9)–(14) and the constraint condition (4), and solving them for the static bending displacement $w_d(r)$ and the static torsional displacement $\varphi_d(r)$, the relations of $w_d(r)$, $\varphi_d(r)$ and f_d are given by

$$w_d(r) = \frac{1}{6EI}\left(3Lr^2 - r^3\right)f_d, \quad (16)$$

$$\varphi_d(r) = \frac{l_z}{GJ}rf_d, \quad (17)$$

and the related desired motor angle θ_d to the desired contact force f_d is obtained as

$$\theta_d = \frac{h_0}{3EIGJ}f_d, \quad (18)$$

where $h_0 = GJL^2 + 3EIl_z^2 (> 0)$ is a constant.

3.2 *Controller Design*

The following Lyapunov function candidate:

$$V(\boldsymbol{x}) = k_1\mathcal{T}_{motor} + k_2(\mathcal{T}_{arm} + \mathcal{T}_{tip} + \overline{\mathcal{V}})$$
$$+\frac{1}{2}k_3(\theta(t) - \theta_d)^2, \quad (19)$$

is investigated, where k_1, k_2 and k_3 are positive parameters, and

$$\boldsymbol{x} = [\ \theta(t), \dot{\theta}(t), w(t,r), \dot{w}(t,r), \varphi(t,r), \dot{\varphi}(t,r),$$
$$w_E(t), \dot{w}_E(t), \varphi_E(t), \dot{\varphi}_E(t),$$
$$w'_E(t), \dot{w}'_E(t), w''(t,r), \varphi'(t,r)\ ]^T,$$

$$\overline{\mathcal{V}} = \frac{1}{2}\int_0^L EI[w''(t,r) - w''_d(r)]^2 dr$$
$$+\frac{1}{2}\int_0^L GJ[\varphi'(t,r) - \varphi'_d(r)]^2 dr.$$

The last term in (19) has been added as a pseudo-energy to insure that the desired final states:

$$\boldsymbol{x}_d = [\ \theta_d, 0, w_d(r), 0, \varphi_d(r), 0, w_d(L), 0, \varphi_d(L), 0,$$
$$w'_d(L), 0, w''_d(r), \varphi'_d(r)\ ]^T, \quad (20)$$

is a unique minimum of V. It is straightforward to check that the positiveness of the parameters k_1, k_2 and k_3 in (19) guarantees $V(\boldsymbol{x}) \geq 0$ and that indeed the global minimum of $V(\boldsymbol{x}) = 0$ is attained only at the desired states $\boldsymbol{x}_d$.

Differentiating (19) with respect to t, employing the equations of motion (8)-(14), and using the constraint condition (4) yield

$$\frac{dV(\boldsymbol{x})}{dt} = \dot{\theta}(t)\big\{k_1\tau(t) + (k_2 - k_1)EIw''(t,0)$$
$$+k_3(\theta(t) - \theta_d) - k_2EIw''_d(0)\big\}. \quad (21)$$

If the following control law:

$$\tau(t) = -\{\frac{k_3}{k_1}(\theta(t) - \theta_d) + \frac{k_4}{k_1}\dot{\theta}(t)$$
$$+\frac{k_2 - k_1}{k_1}EI(w''(t,0) - w''_d(0))$$
$$-EIw''_d(0)\}, \quad (22)$$

is used, where $k_4 > 0$, then substituting (22) into (21) yields

$$\frac{dV(\boldsymbol{x})}{dt} = -k_4\dot{\theta}^2(t) \leq 0, \quad (23)$$

which implies the boundedness $V(\boldsymbol{x}(T)) \leq V(\boldsymbol{x}(0)) < \infty$ of the Lyapunov function candidate (19) for all $T \geq 0$ and the stability of the solutions of the closed-loop system (8)-(14) and (22).

4. ASYMPTOTIC STABILITY

The invariance principle (Henry, 1981) is applicable to this system. By virtue of this principle, the trajectories of the closed-loop system (8)-(14) and (22) tend to the maximal invariant subset of a set of solutions of (8)-(14) and (22) for $\dot{V}(\boldsymbol{x}) = 0$. The latter equality implies that

$$\dot{\theta}(t) = 0, \quad (\theta(t) = \theta_s = const.), \quad \ddot{\theta}(t) = 0. \quad (24)$$

Under the condition (24), the constraint condition (4) is represented by

$$w_E(t) + l_z \varphi_E(t) = L\theta_s = const., \quad (25)$$

which is regarded as a boundary condition along the manifold where $\dot{V}(\boldsymbol{x}) = 0$. Substituting (24) into the closed-loop system of (8) and (22) yields

$$w''(t,0) = -\frac{k_3}{k_2 EI}(\theta_s - \theta_d) + w_d''(0). \quad (26)$$

Moreover, by using the conditions (24) and (25) and introducing new variables:

$$v_1(t,r) = w(t,r) + \frac{GJ}{2h_0}\theta_s(r^3 - 3Lr^2), \quad (27)$$

$$v_2(t,r) = \varphi(t,r) - \frac{3EIl_z}{h_0}\theta_s r, \quad (28)$$

the following equations of vibration of the arm along the manifold where $\dot{V}(\boldsymbol{x}) = 0$ are obtained, the equation of bending vibration:

$$\ddot{v}_1(t,r) + \frac{EI}{\rho}v_1''''(t,r) = 0, \quad (29)$$

$$EIv_{1E}''(t) = 0, \quad (30)$$

$$v_{1E}(t) + l_z v_{2E}(t) = 0, \quad (31)$$

$$v_1(t,0) = 0, \quad v_1'(t,0) = 0, \quad (32)$$

the equation of torsional vibration:

$$\ddot{v}_2(t,r) - \frac{GJ}{\rho\kappa^2}v_2''(t,r) = 0, \quad (33)$$

$$EI[\frac{m}{\rho}(l_z - e_z)v_{1E}''''(t) + l_z v_{1E}'''(t)] + GJv_{2E}'(t) + \{me_z(e_z - l_z) + J_x\}\frac{GJ}{\rho\kappa^2}v_{2E}''(t) = 0, \quad (34)$$

$$v_2(t,0) = 0, \quad (35)$$

where $v_{1E}(t) = v_1(t,L)$, $v_{2E}(t) = v_2(t,L)$.

By using the eigenvalues γ_n and the corresponding eigenfunctions $\boldsymbol{\psi}_n(r) = [\,\psi_{1n}(r) \quad \psi_{2n}(r)\,]^T$ described in Appendix A, the solution of (29)–(35) is known to be expanded into the series:

$$\begin{bmatrix} v_1(t,r) \\ v_2(t,r) \end{bmatrix} = \sum_{n=1}^{\infty} \frac{1}{\gamma_n} \begin{bmatrix} \psi_{1n}(r) \\ \psi_{2n}(r) \end{bmatrix} \nu_n(t), \quad (36)$$

where the variables $\nu_n(t)$ satisfy

$$\ddot{\nu}_n(t) = -\gamma_n \nu_n(t), \quad n = 1, 2, \cdots. \quad (37)$$

Since the solution of (37) is given by

$$\nu_n(t) = c_n \sin(\sqrt{\gamma_n}t + \phi_n), \quad n = 1, 2, \cdots, \quad (38)$$

where c_n and ϕ_n are constants, the expansion (36) results in

$$\begin{bmatrix} v_1(t,r) \\ v_2(t,r) \end{bmatrix} = \sum_{n=1}^{\infty} \frac{1}{\gamma_n} \begin{bmatrix} \psi_{1n}(r) \\ \psi_{2n}(r) \end{bmatrix} c_n \sin(\sqrt{\gamma_n}t + \phi_n). \quad (39)$$

Differentiating $v_1(t,r)$ twice with respect to the spatial variable r and setting $r = 0$ yield

$$v_1''(t,0) = \sum_{n=1}^{\infty} \frac{1}{\gamma_n}\psi_{1n}''(0) c_n \sin(\sqrt{\gamma_n}t + \phi_n). \quad (40)$$

On the other hand, (27) ensures that

$$v_1''(t,0) = w''(t,0) - \frac{3GJL}{h_0}\theta_s. \quad (41)$$

Cancelling $w''(t,0)$ from (26) and (41) yields

$$v_1''(t,0) = -\frac{k_3}{k_2EI}(\theta_s - \theta_d) + w_d''(0) - \frac{3GJL}{h_0}\theta_s = -c_0 = const. \quad (42)$$

Now, combining (40) and (42) gives

$$c_0 + \sum_{n=1}^{\infty} c_n \frac{1}{\gamma_n}\psi_{1n}''(0)\sin(\sqrt{\gamma_n}t + \phi_n) = 0, \quad (43)$$

while from (A.5) we can find that

$$\psi_{1n}''(0) = -\frac{2}{\widehat{C}_n}\mu_1(\beta_n)\left(\frac{\beta_n}{L}\right)^2 \neq 0, \quad n = 1, 2, \cdots. \quad (44)$$

Due to the property of the eigenvalues ($0 < \beta_1 < \beta_2 < \cdots$) and by virtue of the linear independence of the functions $\{1, \sin\sqrt{\gamma_1}t, \sin\sqrt{\gamma_2}t, \cdots\}$, it is concluded from (43) that $c_0 = c_1 = c_2 = \cdots = 0$ and the ordinary differential equation (38) has the trivial solution:

$$\nu_n(t) = 0, \quad n = 1, 2, \cdots \quad (45)$$

only. From these results, Eqs.(36) and (42) are expressed as

$$v_1(t,r) = 0, \quad v_2(t,r) = 0, \quad (46)$$

$$-\frac{k_3}{k_2EI}(\theta_s - \theta_d) + w_d''(0) - \frac{3GJL}{h_0}\theta_s = 0. \quad (47)$$

Thus, the relation (47) ensures from (16), (18) and (25) that

$$\theta_s = \theta_d. \tag{48}$$

By using (25), (46) and (48), Eqs.(27) and (28) are expressed as

$$w(t,r) = -\frac{GJ}{2h_0}\theta_d(r^3 - 3Lr^2), \tag{49}$$

$$\varphi(t,r) = \frac{3EIl_z}{h_0}\theta_d r. \tag{50}$$

In turn, the relations (49) and (50) ensure from (16), (17) and (18) that $w(t,r) = w_d(r)$, $\varphi(t,r) = \varphi_d(r)$. Thus, the equality $\dot{V}(\boldsymbol{x}) = 0$ holds just at the equilibrium point, and in accordance with the invariance principle the proposed control law (22) stabilizes the distributed parameter system asymptotically.

To this end, the control law (22) can be represented in the form:

$$\tau(t) = -K_p(\theta(t) - \theta_d) - K_d\dot{\theta}(t) \\ -K_s EI(w''(t,0) - w_d''(0)) + EIw_d''(0) \tag{51}$$

where $K_p = k_3/k_1$, $K_d = k_4/k_1$ and $K_s = (k_2 - k_1)/k_1$. This control law consists of the PD feedback of the motor angle and a bending strain(S) feedback of the flexible arm, which is called PDS control. The last term in the right side of (51) is the feedforward of the desired bending moment torque to achieve the desired force control. It is clear that the controller (51) admits simple implementation since no state estimation is required. When applying this controller to the system, the spillover does not occur. The joint angle and the angular velocity as well as the root bending strain can be measured by using conventional sensors. Letting $k_1 = k_2$ in (51), the control law becomes the local PD feedback. It implies that the PD control law also ensures the asymptotic stability of the closed-loop system. Moreover, it is noted that the PDS control law ensures the asymptotic stability of the closed-loop system of the coupled bending and torsional vibration system, although no information on the torsional vibration is used in the control law. The PDS control law is effective even if the arm separates from the constraint surface, because the arm approaches the constraint surface by the first and the last terms in the right side of (51).

5. SIMULATION

Some simulations are performed to confirm the effectiveness of the PDS controller.

As the simulation model, the finite dimensional modal model derived by using the eigenfunction expansion is used, and the first four modes are considered. The physical parameters are taken as follows: $L = 0.85$ [m], $EI = 5.43$ [Nm^2], $GJ = 7.61$ [Nm^2], $\rho = 0.307$ [kg/m], $\rho\kappa^2 = 4.11 \times 10^{-5}$ [kgm], $J_m = 0.137$ [kgm^2], $J_x = 4.50 \times 10^{-3}$ [kgm^2], $m = 1.34$[kg], $l_z = 0.13$ [m], $e_z = -0.05$ [m]. The desired contact force is given by $f_d = 2.0$[N]. The corresponding desired values are $\theta_d = 0.0693$[rad], $EIw_d''(0) = 1.76$[Nm], and $GJ\varphi_d'(0) = -0.260$[Nm].

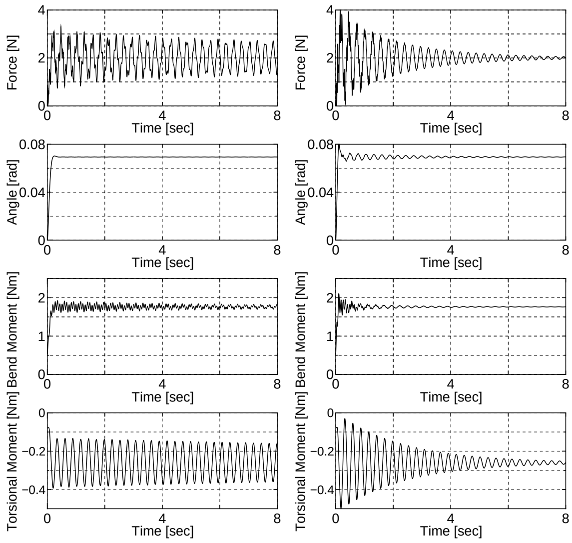
Figures 2 (a) and (b) show the step responses for the PD controller and for the PDS controller, respectively. Figures 3 (a) and (b) show the input disturbance responses for the PD controller and for the PDS controller, respectively. The input disturbance 2.0[Nm] is applied for 0.05[s] from $t = 1$[s] to $t = 1.05$[s]. In the simulations, the controller gains are used as follows: PD control: $K_p = 20$, $K_d = 10$, $K_s = 0$ and PDS control: $K_p = 20$, $K_d = 10$, $K_s = 5$.

From the figures, it is found that the bending strain feedback is effective for not only bending vibration absorption but also torsional vibration absorption. From the theoretical point of view, both the PD and PDS feedback control law ensure the asymptotic stability of the closed-loop system. From simulation results, the advantage of the bending strain feedback is found.

6. CONCLUSION

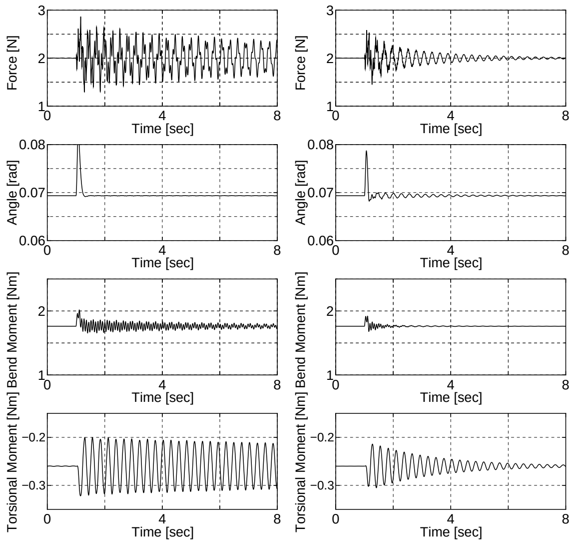
In this paper the force control of a one-link flexible arm with a rigid tip body has been discussed using Lyapunov method . On the basis of the distributed parameter model, a simple and robust controller has been constructed. By using Lyapunov method and the invariance principle, the controller was proved to ensure the asymptotic stability. The simulation results demonstrated the effectiveness of the proposed PDS feedback controller.

In order to accomplish the force control, in general, the force is directly fed back. However, in the case of a flexible arm, as the force sensor and the actuator are located at the tip and the root of the arm, respectively, the system is non-collocated, hence the closed-loop system becomes a nonminimum phase one. So, the direct force feedback cannot necessarily ensure the closed-loop stability. The result has been proved in the case that the system is represented as the finite-dimensional model(Morita and Matsuno et al., 2001). Even in the case of the distributed parameter system treated in this paper, one can guess the similar result. Now the proposed force control based on the position and root strain feedback is collocated control. So the closed-loop stability can be ensured in this paper. On the other hand, since the proposed force control is based on the position and strain feedback, parameter uncertainty possibly leads to the steady state error in force time response. However, although the force error occurs, the closed-loop stability is ensured. The discussion



(a) PD control (b) PDS control

Fig. 2. Step Responses



(a) PD control (b) PDS control

Fig. 3. Responses for input disturbance

about compensation of the parameter uncertainty for achieving precise force control is the next work.

The future work is to demonstrate the validity of the proposed controller by experiments and to extend the results to cooperative control of flexible arms for grasping an object.

7. REFERENCES

Henry, D. (1981). Geometric theory of semilinear parabolic equation. *Lecture Notes in Mathematics.*

Kasai, S. and F. Matsuno (2000). Force control of one-link flexible arms mounted on a linear motor with PD and shear force (PDSF) feedback. *Proc. 2000 IEEE Int. Conf. on IECON*, pp. 195–200.

Luo, Z. H (1993). Direct strain feedback control of flexible robot arms : New theoretical and experimental results. *IEEE Trans. Automatic Control.* **AC-38**(11), 1610–1622.

Matsuno, F. and T. Ohno (1997). Distributed parameter control of large space structures with lumped and distributed flexibility. *Proc. the 36th IEEE CDC*, pp. 269–274.

Matsuno, F., T. Murachi and Y. Sakawa (1994). Feedback control of decoupled bending and torsional vibrations of flexible beams. *J. Robotic Syst.* **11**(5), 341–353.

Morita, Y., F. Mastuno, Y. Kobayashi, T. Kanzawa, H. Ukai and H. Kando (2001). Robust force control of a flexible arm with a nonsymmetric rigid tip body. *J. Robotic Syst.* **18**(5), 221–235.

Sakawa, Y. and Z. H. Luo (1989). Modeling and control of coupled bending and torsional vibrations of flexible beams. *IEEE Trans. Automatic Control* **AC-34**(9), 970–977.

Sakawa, Y., F. Matsuno and S. Fukushima (1985). Modeling and feedback control of a flexible arm. *J. Robotic Syst.* **2**(4), 453–472.

Appendix A. EIGENVALUE PROBLEM

An eigenvalue problem related to the distributed parameter system (29)-(35) is as follows:

$$\begin{bmatrix} \dfrac{EI}{\rho}\psi_1''''(r) \\ -\dfrac{GJ}{\rho\kappa^2}\psi_2''(r) \end{bmatrix} = \gamma \begin{bmatrix} \psi_1(r) \\ \psi_2(r) \end{bmatrix}, \qquad \text{(A.1)}$$

$$\psi_1(0) = 0,\ \psi_1'(0) = 0,\ \psi_2(0) = 0,$$

$$EI\psi_1''(L) = 0,\ \psi_1(L) + l_z\psi_2(L) = 0,$$

$$m\gamma(l_z - e_z)\psi_1(L) - \gamma\{me_z(e_z - l_z) + J_x\}\psi_2(L) + EIl_z\psi_1'''(L) + GJ\psi_2'(L) = 0.$$

A parameter β is introduced such that $\gamma = (EI/\rho)(\beta/L)^4$. Let β_n be the solution of eigenequation such that $0 < \beta_1 < \beta_2 < \cdots < \beta_n < \cdots$. Then the eigenvalues γ_n are given by

$$\gamma_n = \frac{EI}{\rho}\Big(\frac{\beta_n}{L}\Big)^4, \quad n = 1, 2, \cdots, \qquad \text{(A.2)}$$

and the eigenfunctions are given by

$$\psi_{1n}(r) = \frac{1}{\widehat{C}_n}\{\sin\frac{\beta_n}{L}r - \sinh\frac{\beta_n}{L}r + \mu_1(\beta_n)(\cos\frac{\beta_n}{L}r - \cosh\frac{\beta_n}{L}r)\}, \qquad \text{(A.3)}$$

$$\psi_{2n}(r) = \frac{1}{\widehat{C}_n}\mu_2(\beta_n)\sin\frac{\alpha\beta_n^2}{L^2}r, \qquad \text{(A.4)}$$

where $\widehat{C}_n \neq 0$ are bounded arbitrary constants, and

$$\mu_1(\beta_n) = -\frac{\sinh\beta_n + \sin\beta_n}{\cosh\beta_n + \cos\beta_n}, \qquad \text{(A.5)}$$

$$\mu_2(\beta_n) = \frac{2(\sinh\beta_n\cos\beta_n - \cosh\beta_n\sin\beta_n)}{l_z(\cosh\beta_n + \cos\beta_n)\sin(\alpha\beta_n^2/L)}.$$