

## A GEOMETRIC APPROACH TO DIAGNOSIS APPLIED TO A SHIP PROPULSION PROBLEM<sup>1</sup>

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**Abstract:** A geometric approach to FDI diagnosis for input-affine nonlinear systems is briefly described and applied to a ship propulsion benchmark. The analysis method is used to examine the possibility of detecting and isolating predefined faults in the system. The considered faults cover sensor, actuator, and parameter faults.

**Keywords:** Geometric approach, input-affine nonlinear systems, fault detection, fault isolation, ship propulsion benchmark.

### 1. INTRODUCTION

Fault detection and isolation (FDI) for nonlinear systems has generated a significant amount of awareness in recent years. The design of FDI is motivated by the need for knowledge about occurring faults in fault-tolerant control systems (FTCS); (Patton, 1997). A major part of designing and developing FTCS systems consists of developing/applying appropriate methods to detect, isolate, and accommodate occurring faults in such a way that the systems can still perform in a required manner. One prefers reduced performance after occurrence of a fault to the shut down of sub-systems; (Blanke, 1999). Hence, the idea of fault-tolerance can be applied to ordinary industrial processes that are not categorized as high risk applications, but where high availability is desirable; e.g. propulsion systems in commercial marine vessels. In the past mainly linear FDI methods have been developed, but as most plants show nonlinear behavior, nonlinear methods are preferred (Frank *et al.*, 1999).

Among different approaches for FDI the geometric methods are of high interest. Geometric theory offers

various advantages as it gives a general formulation of the FDI problem. It is more compact and more transparent for more general systems (like the nonlinear systems) than the algebraic approach. In recent years the existing geometric theory for the residual generation in linear systems based on the original work by Massoumnia (Massoumnia *et al.*, 1989; Massoumnia, 1986) has been extended. Formulations for different classes of nonlinear systems were derived in order to handle state-affine nonlinear systems (Hammouri *et al.*, 1998) and lately also the class of input-affine systems (DePersis and Isidori, 2000a; Hammouri *et al.*, 1999). A detailed geometric description of how to tackle the residual generation problem for nonlinear systems is given by DePersis and Isidori; (DePersis and Isidori, 2000a).

Only few results from applying the geometric approach have been obtained so far. This paper contributes with application results where the geometric approach for input-affine nonlinear systems is applied to a ship propulsion benchmark. The used geometric approach is briefly outlined in Section 2 followed by a description of the ship propulsion system in Section 3. For each subsystem different fault scenarios are considered. Using the geometric approach in Section 4, the possibility of detecting and isolating different

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faults is investigated considering the different fault scenarios. Finally, the obtained results are presented and discussed.

## 2. GEOMETRIC APPROACH TO NONLINEAR FDI

This section reviews briefly the geometric approach to nonlinear FDI by (DePersis, 1999; DePersis and Isidori, 2000a) for systems of the form:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^m g_i(x)u_i + \sum_{i=1}^s p_i(x)w_i + l(x)v \\ y_j &= h_j(x), \quad j \in \mathbf{l} \end{aligned} \quad (1)$$

where the states  $x$  are defined on a neighborhood  $\mathcal{N}$  of the origin in  $\mathbb{R}^n$ .  $u_i$ ,  $i \in \mathbf{m} = \{1, \dots, m\}$ , denotes the  $m$  inputs and  $y_j$ ,  $j \in \mathbf{l}$ , the  $l$  outputs of system (1).  $v \in \mathbb{R}$  describes a scalar fault signal with the nonlinear fault signature  $l(x)$ .  $f(x)$ ,  $g_i(x)$ ,  $i \in \mathbf{m}$ ,  $l(x)$ , and  $p_i(x)$ ,  $i \in \mathbf{s}$  are smooth vector fields and  $h_j(x)$ ,  $j \in \mathbf{l}$  is a smooth function. Furthermore, let  $f(0) = 0$  and  $h(0) = 0$ . The vector  $w = [w_1 \ w_2 \ \dots \ w_s]^T \in \mathbb{R}^s$  denotes the disturbances and fault signals from which the fault  $v$  has to be isolated.

In order to detect the fault  $v$  and isolate it from the disturbances and other faults ( $w_i$ ) in system (1) the following problem definition is formulated as described in (DePersis, 1999):

*Definition 1. Considering a system of the form (1) the local nonlinear fundamental problem of residual generation (l-NLFPRG) is to find, if possible, a filter:*

$$\begin{aligned} \dot{z} &= \tilde{f}(y, z) + \sum_{i=1}^m \tilde{g}_i(y, z)u_i \\ r &= \tilde{h}(y, z) \end{aligned} \quad (2)$$

where  $z \in \mathbb{R}^q$ ,  $r \in \mathbb{R}^p$ ,  $1 \leq p \leq l$ .  $\tilde{f}(y, z)$ ,  $\tilde{g}_i(y, z)$ ,  $i \in \mathbf{m}$ , and  $\tilde{h}(y, z)$  are smooth vector fields, with  $\tilde{f}(0, 0) = 0$  and  $\tilde{h}(0, 0) = 0$ , such that on a neighborhood  $\mathcal{N}^e$  of  $x^e = (x, z) = (0, 0)$ , where  $x_e$  denotes the state vector of the cascaded system formed by (1) and (2), the following properties hold:

- (i) if  $v = 0$ , then  $r$  is unaffected by  $u_i$ ,  $w_j$ ,  $\forall i, j$ ;
- (ii)  $r$  is affected by  $v$ ;
- (iii)  $\lim_{t \rightarrow \infty} \|r(t; x^0, z^0; u, v=0, w)\| = 0$  for any initial condition  $x^0, z^0$  in a suitable set containing the origin  $(x, z) = (0, 0)$ .

For linear systems Definition 1 reduces exactly to the linear Fundamental problem of residual generation (FPRG) defined in (Massoumnia *et al.*, 1989). Both describe the problem of detecting a fault and isolating it from disturbances and other faults. Condition (i) in Definition 1 assures that the control signals  $u$  and the disturbances (and other faults)  $w$  do not affect (i.e. do not become visible in) the residual  $r$  in the fault-free case ( $v = 0$ ). If fault  $v$  occurs Condition (ii)

assures that the fault affects the residual. Condition (iii) considers the stability of the filter (2). Note that the convergence to zero of the residual is only required in absence of the fault ( $v = 0$ ).

Using the method presented in (DePersis and Isidori, 2000b) a solution for the l-NLFPRG can be obtained. It is based on the calculation of the largest observability codistribution (o.c.a.  $((\Sigma_*^P)^\perp)$ ) contained in  $P^\perp$  the annihilator of  $P$  (i.e.  $P^\perp = \{x' ; x'x = 0, \forall x \in P\}$ ), where  $P$  is the distribution spanned by the disturbance vectors  $p_i$ ,  $i \in \mathbf{s}$ :  $P = \text{span}\{p_1, \dots, p_s\}$ . For System (1) one can calculate o.c.a.  $((\Sigma_*^P)^\perp)$  by the following two algorithms (details are given in (DePersis and Isidori, 2000b)):

*Computing  $\Sigma_*^P$ :*

$$\begin{aligned} S_0 &= \bar{P} \\ S_{k+1} &= \bar{S}_k + \sum_{i=0}^m [g_i, \bar{S}_k \cap \text{Ker}\{dh\}] \end{aligned} \quad (3)$$

where  $\bar{\Delta}$  denotes the involutive closure of a distribution  $\Delta$ . For every constant distribution  $\Delta$  it holds that  $\bar{\Delta} = \Delta$ . The notation  $[\xi, \zeta]$  with  $\xi, \zeta \in \mathbb{R}^n$  denotes the Lie bracket.  $g_0 \dots g_m$  stand for the column vectors of  $g(x)$  and for  $f(x)$ , which is written as  $f(x) = g_0(x)$  to ease the notation.  $\text{Ker}\{dh\}$  denotes the distribution annihilating the differentials of the rows of the mapping  $h(x)$ . If there exists a  $k^*$  such that:

$$S_{k^*+1} = \bar{S}_{k^*}$$

then one sets  $\Sigma_*^P = \bar{S}_{k^*}$  and continues with the following algorithm.

*Computing o.c.a.  $((\Sigma_*^P)^\perp)$ :*

$$\begin{aligned} Q_0 &= (\Sigma_*^P)^\perp \cap \text{span}\{dh\} \\ Q_{k+1} &= (\Sigma_*^P)^\perp \cap \left( \sum_{i=0}^m L_{g_i} Q_k + \text{span}\{dh\} \right) \end{aligned} \quad (4)$$

where  $\text{span}\{dh\}$  is the codistribution spanned by the differentials of the rows of the mapping  $h(x)$  and  $L$  denotes the Lie derivative. Suppose that all codistributions  $Q_k$  of this sequence are nonsingular, so that there is an integer  $k^* \leq n - 1$  such that  $Q_k = Q_{k^*}$  for all  $k > k^*$ , then

$$\text{o.c.a.}((\Sigma_*^P)^\perp) = Q_{k^*}.$$

When the distribution  $\Sigma_*^P$  is well-defined and nonsingular, and  $\Sigma_*^P \cap \text{Ker}\{dh\}$  is a smooth distribution, then o.c.a.  $((\Sigma_*^P)^\perp)$  is the maximal (in the sense of codistribution inclusion) observability codistribution, which is locally spanned by exact differentials and contained in  $P^\perp$ . The corresponding unobservability distribution  $Q$  can be obtained by:

$$Q = (\text{o.c.a.}((\Sigma_*^P)^\perp))^\perp$$

For more details about the o.c.a. algorithm and the calculation of  $Q$  the reader is referred to (DePersis and Isidori, 2000a).

As a result of the algorithms  $Q$  is the smallest involutive unobservability distribution that contains  $P$  (i.e.

the disturbance and unwanted fault effects) due to the maximality of o.c.a.  $((\Sigma_*^P)^\perp)$ .

In (DePersis and Isidori, 2000b) it is shown that if

$$\text{span}\{l(x)\} \not\subseteq (\text{o.c.a.}((\Sigma_*^P)^\perp))^\perp = Q$$

then it is possible, under certain conditions, to find a change of state coordinates  $\tilde{x} = \Phi(x)$  and a change of output coordinates  $\tilde{y} = \Psi(y)$ , defined locally around  $x = 0$  and, respectively,  $y = 0$ , such that, in the new coordinates, the system (1) admits the following normal form (Proposition 3 in (DePersis and Isidori, 2000b)):

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{f}_1(\tilde{x}_1, \tilde{x}_2) + \tilde{g}_1(\tilde{x}_1, \tilde{x}_2)u + \tilde{l}_1(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)v \\ \dot{\tilde{x}}_2 &= \tilde{f}_2(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) + \tilde{g}_2(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)u \\ &\quad + \tilde{p}_2(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)w + \tilde{l}_2(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)v \\ \dot{\tilde{x}}_3 &= \tilde{f}_3(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) + \tilde{g}_3(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)u \\ &\quad + \tilde{p}_3(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)w + \tilde{l}_3(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)v \\ \tilde{y}_1 &= h_1(\tilde{x}_1) \\ \tilde{y}_2 &= \tilde{x}_2\end{aligned}$$

where the states  $\tilde{x}_2$  are all measured. Output  $\tilde{y}_1$  is a function of the states  $\tilde{x}_1$ , but not of the other states  $\tilde{x}_2$  and  $\tilde{x}_3$ . Hence, the following subsystem can be subtracted:

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{f}_1(\tilde{x}_1, \tilde{y}_2) + \tilde{g}_1(\tilde{x}_1, \tilde{y}_2)u + \tilde{l}_1(\tilde{x}_1, \tilde{y}_2, \tilde{x}_3)v \\ \tilde{y}_1 &= h_1(\tilde{x}_1)\end{aligned}\quad (5)$$

which, obviously, when it admits an observer can be used to solve the corresponding *l-NLFRG*. As mentioned in chapter 10 of (Åström *et al.*, 2000) the observability can be guaranteed depending on how the conditioned invariant distribution  $Q$  is generated. The resulting estimation error  $e = \tilde{y}_1 - \hat{y}_1$  is only affected by the unknown fault signal  $v$  but not by the  $w$ . Hence, it can by construction be used as residual  $r$  that fulfills all conditions given in Definition 1 as long as  $\tilde{l}_1(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \neq 0$ .

### 3. SHIP PROPULSION BENCHMARK

A complete mathematical model for the benchmark is described in (Izadi-Zamanabadi and Blanke, 1998). In the following the dynamics and the considered fault scenarios are briefly described. The subsystems of interest in this paper include dynamics for ship speed ( $U$ ), shaft speed ( $n$ ), propeller pitch ( $\theta$ ), and the prime mover ( $Q_{eng}$ ). The essence is a model where developed thrust and torque are functions of diesel throttle position  $Y$ , propeller pitch  $\theta$ , shaft speed  $n$  and ship speed  $U$ . The corresponding measured variables are  $Y_m, \theta_m, n_m$  and  $U_m$ .

Diesel engine and shaft dynamic equations are:

$$Q_{eng} = k_y Y \quad (6)$$

$$I_m \dot{n} = Q_{eng} - Q_{prop}, \quad (7)$$

where  $Q_{eng}$  is the engine's generated torque,  $k_y$  the constant engine gain,  $I_m$  the shaft inertia, and  $Q_{prop}$  denotes the propellers developed torque.

The dynamics of the propeller pitch  $\theta$  can be described as follows:

$$\dot{\theta} = k_t (\theta_{ref} - \theta_m) \quad (8)$$

where  $k_t$  is a constant,  $\theta_{ref}$  denotes the pitch reference, and  $\theta_m$  stands for the pitch measurement.

Developed propeller thrust  $T_{prop}$  and torque  $Q_{prop}$  are given by the following (approximate) quadratic relations (for forward movement)

$$T_{prop} = T_{|n|n} \theta n^2 + T_{|n|U} n U \quad (9)$$

$$Q_{prop} = Q_{|n|n} \theta n^2 + Q_{|n|U} \theta n U \quad (10)$$

The coefficients  $T_{|n|n}, T_{|n|U}, Q_{|n|n}$  and  $Q_{|n|U}$  are in fact complex functions of propeller pitch  $\theta$  (see (Izadi-Zamanabadi and Blanke, 1998) for details). They are calculated from tables of data, which are obtained from sea trial.

Ship speed dynamics with corresponding hull resistance is described by the first order equation

$$m \dot{U} = R(U) + (1 - t_T) T_{prop} \quad (11)$$

The Ship's resistance to motion through the water, denoted  $R(U)$ , can be described by a resistance curve, which is a third to fifth order polynomial in  $U$ .  $m$  is the the ship weight and  $t_T$  is the thrust deduction number (and is a known value).

For the ship propulsion benchmark different faults have been defined including sensor, actuator, and parameter faults. A complete list is given in Table 1. For more details see (Izadi-Zamanabadi and Blanke, 1998).

Table 1. Implemented faults.

Fault	Type
shaft speed sensor faults $\Delta n_{sensor}$ - pos./neg.	additive - abrupt
pitch sensor faults $\Delta \theta_{sensor}$ - pos./neg.	additive - abrupt
hydraulic leak $\Delta \theta_{inc}$ - neg.	additive - incipient
diesel fault $\Delta k_y$ - neg.	multiplicative - abrupt

### 4. DIAGNOSIS USING THE GEOMETRIC APPROACH

In order to apply the geometric approach, described in Section 2, to the ship benchmark, it is necessary to define a set of *l-NLFRGs* and rewrite the considered system dynamics. The dynamics should have a form that corresponds to the required form given by Equation (1).

Investigating the system dynamics shows that there exist three different (sub-)systems: (a) the complete

Table 2. 1-NLFPRGs for the complete system.

1-NLFPRG	Fault ( $v$ )	Disturbances ( $w$ )
1-NLFPRG 1	$\Delta k_y Y$	$\Delta n_{sensor}, \Delta \theta_{sensor}, \Delta \theta_{inc}$
1-NLFPRG 2	$\Delta n_{sensor}$	$\Delta k_y Y, \Delta \theta_{sensor}, \Delta \theta_{inc}$
1-NLFPRG 3	$\Delta \theta_{sensor}$	$\Delta k_y Y, \Delta n_{sensor}, \Delta \theta_{inc}$
1-NLFPRG 4	$\Delta \theta_{inc}$	$\Delta k_y Y, \Delta n_{sensor}, \Delta \theta_{sensor}$

Table 3. 1-NLFPRGs for the pitch loop.

1-NLFPRG	Fault ( $v$ )	Disturbance ( $w$ )
1-NLFPRG 5	$\Delta \theta_{sensor}$	$\Delta \theta_{inc}$
1-NLFPRG 6	$\Delta \theta_{inc}$	$\Delta \theta_{sensor}$

Table 4. 1-NLFPRGs for the shaft speed loop.

1-NLFPRG	Fault ( $v$ )	Disturbances ( $w$ )
1-NLFPRG 7	$\Delta k_y Y$	$\Delta n_{sensor}$
1-NLFPRG 8	$\Delta n_{sensor}$	$\Delta k_y Y$

system, (b) the pitch loop, and (c) the shaft speed loop. Hence, the following *1-NLFPRGs* can be formulated:

In (Lootsma, 2001) detailed calculations for the different 1-NLFPRGs are given. It was shown that only 1-NLFPRG 7 and 1-NLFPRG 8 can be solved when designing an additional fault detector for a possible pitch fault. Note that in that case it is only required to detect pitch faults ( $\Delta \theta_{sensor}$  or  $\Delta \theta_{inc}$ ) not to isolate them from each other, which can be achieved by a simple observer design as shown in (Lootsma, 2001). This result is in compliance with earlier results for the ship benchmark, obtained by applying other FDI analysis methods (Åström *et al.*, 2000)[chapter 13].

Detailed calculations for 1-NLFPRG 7 are provided in the following.

First the dynamics of the shaft speed loop need to be formulated corresponding to Equation (1):

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + p(x)w + l(x)v \\ y_1 &= n_m = n + x_{\Delta n} \\ y_2 &= U_m = U \end{aligned} \quad (12)$$

where

$$\begin{aligned} x &= (n \ U \ x_{\Delta n})^T, \quad u_1 = Y, \quad u_2 = \theta_m \\ v &= \Delta k_y Y, \quad w = \Delta n_{sensor} \end{aligned}$$

and

$$f(x) = \begin{pmatrix} 0 \\ \frac{1}{m}R(U) + \frac{1-t_T}{m}T_{|n|U}nU \\ 0 \end{pmatrix},$$

$$\begin{aligned} g_1(x) &= \begin{pmatrix} \frac{1}{I_m}k_y \\ 0 \\ 0 \end{pmatrix} \\ g_2(x) &= \begin{pmatrix} -\frac{1}{I_m} \left[ Q_{|n|n}n^2 + Q_{|n|U}nU \right] \\ \frac{1-t_T}{m}T_{|n|n}n^2 \\ 0 \end{pmatrix} \\ p(x) &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \text{ and } l(x) = \begin{pmatrix} \frac{1}{I_m} \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

In (12) the shaft speed sensor fault is implemented as a pseudo-actuator fault  $\Delta n_{sensor}$  in order to obtain the form (1). This is done following a procedure described in (Hashtrudi-Zad and Massoumnia, 1999) by adding the following additional linear dynamics to the original system:

$$\begin{aligned} \dot{x}_{\Delta n} &= A_{\Delta n}x_{\Delta n} + L_{\Delta n}v_{\Delta n} \\ y_{\Delta n} &= C_{\Delta n}x_{\Delta n} = \Delta n_{sensor} \end{aligned}$$

where  $v_{\Delta n} = \Delta n_{sensor} = w$ ,  $A_{\Delta n} = 0$ , and  $L_{\Delta n} = C_{\Delta n} = 1$ .

The diesel engine gain fault is in its nature a multiplicative fault affecting the system parameter  $k_y$ . In (12) it is modeled as an additive fault by considering  $v = \Delta k_y Y$ . Its magnitude depends on the diesel throttle position  $Y$ . This is natural as the gain fault's impact becomes bigger as the higher the diesel intake to the engine becomes. If  $Y = 0$  the diesel engine is not running, hence, the fault would not affect the system's operation anyway.

The goal is now to solve the 1-NLFPRG for system (12), i.e. to detect the diesel engine gain fault  $v = \Delta k_y Y$  and isolate it from the shaft speed measurement fault  $\Delta n_{sensor}$ . The algorithms (3) and (4) are initiated with:

$$P = \text{span}\{p_1\} = \text{span}\{(0 \ 0 \ 1)^T\}$$

and lead to the following result (detailed calculations can be found in (Lootsma, 2001)):

$$Q = (\text{o.c.a.}((\Sigma_*^P)^\perp))^\perp = P = \text{span}\{(0 \ 0 \ 1)^T\}$$

hence

$$\text{span}\{l(x)\} \not\subseteq Q \quad \text{and} \quad \text{span}\{p(x)\} \subseteq Q$$

Following Proposition 3 in (DePersis and Isidori, 2000b) one can then obtain the following subsystem, which corresponds to (5) and obviously is not affected by the shaft speed sensor fault:

$$\begin{aligned} \dot{n} &= \frac{1}{I_m}(k_y Y + v) - \frac{1}{I_m}Q_{|n|U}nU\theta_m \\ &\quad - \frac{1}{I_m}Q_{|n|n}n^2\theta_m \end{aligned} \quad (13)$$

$$\dot{U} = \frac{1}{m}R(U) + \frac{1-t_T}{m} \left[ T_{|n|U}nU + T_{|n|n}n^2\theta_m \right] \quad (14)$$

$$y = U$$

The obtained subsystem (13) and (14) is a good starting point to obtain successful FDI that enables detection of the diesel engine gain fault and isolation from the shaft speed sensor fault. In (Lootsma, 2001) and (Lootsma *et al.*, 2001) a dedicated nonlinear observer has been designed corresponding to the filter stated by Equation 2. The proposed observer has the following form:

$$\dot{\hat{n}} = \frac{1}{I_m} k_y Y_m - \frac{1}{I_m} Q_{|n|U} \hat{n} \hat{U} \theta_m - \frac{1}{I_m} Q_{|n|n} \hat{n}^2 \theta_m + K_{\Delta k_y}^{\hat{n}} (U_m - \hat{U}) \quad (15)$$

$$\dot{\hat{U}} = \frac{1}{m} R(\hat{U}) + \frac{1-t_T}{m} \left[ T_{|n|U} \hat{n} \hat{U} + T_{|n|n} \hat{n}^2 \theta_m \right] + K_{\Delta k_y}^{\hat{U}} (U_m - \hat{U}) \quad (16)$$

$$\hat{y} = \hat{U}$$

with the diesel throttle position  $Y_m$  and the pitch measurement  $\theta_m$  as external inputs. For the stability proof the interested reader is referred to Section 5.2 in (Lootsma, 2001). It is based on the fact that the observer structure corresponds to a form like:

$$\begin{aligned} \dot{\hat{x}} &= \hat{f}(\hat{x}) + \hat{g}(\hat{x})u + K(y - \hat{y}) \\ \hat{y} &= h(\hat{x}) \end{aligned}$$

The function  $\hat{f}(x) + \hat{g}(x)u$  is globally Lipschitz for the complete operating range:  $\Omega_x = \{x | 0 < n < n_{max}; 0 < U < U_{max}\}$  and  $\Omega_u = \{u | -1 < \theta < 1; 0 < Y < 1\}$ ; i.e.  $\|\hat{f}(x) + \hat{g}(x)u - \hat{f}(\hat{x}) - \hat{g}(\hat{x})u\| \leq \Lambda \|x - \hat{x}\|$ , with Lipschitz constant  $\Lambda \in \mathbb{R}$  and  $x, \hat{x} \in \Omega_x$ . This is due to the physical limitations and the upper-level control of the propulsion system. It is designed to keep the signals  $(n, \theta)$  inside certain boundaries (corresponding to  $\Omega_x$ ) to achieve desired operation and to avoid overload situations for the shaft and the pitch. Furthermore, there are the following physical limitations: The pitch signal is physically limited by construction  $-1 < \theta < 1$  like the fuel index  $0 < Y < 1$ . The ship speed  $U$  is limited by the top speed of the ship. The shaft speed  $n$  is limited by an emergency shut-down.

Subsystem (13) and (14) is observable over the complete operating range  $\Omega_x$ . This can be seen when looking at the system and its corresponding observability codistribution (Nijmeijer and van der Schaft, 1990, Theorem 3.32). The observability codistribution can be obtained as follows (Nijmeijer and van der Schaft, 1990): For the considered subsystem it can be seen that

$$\begin{aligned} dh(x) &= (0 \quad 1) \\ dL_f h(x) &= \left( \frac{1-t_T}{m} T_{|n|U} U \right. \\ &\quad \left. \frac{1}{m} \frac{\partial R(U)}{\partial U} + \frac{1-t_T}{m} T_{|n|U} n \right) \end{aligned}$$

$$\Rightarrow \dim d\mathcal{O}(x) = 2 = \dim \Omega_x \quad \text{for } u \in \Omega_u$$

Hence, the subsystem is observable over the complete operating range  $\Omega_x$ .

Using the facts that  $\hat{f}(x) + \hat{g}(x)u$  is globally Lipschitz, the subsystem (13) and (14) is observable over the complete operating range  $\Omega_x$ , and that the inputs are bounded ( $u \in \Omega_u$ ) the stability of the proposed observer can be proven by using the result of (Gauthier *et al.*, 1992). In (Gauthier *et al.*, 1992) it is also shown how the observer gain  $K$  has to be chosen.

The residual obtained by using observer (15) and (16) is shown in figure 1. A sample sequence of 600 sec. from the total sequence of 3500 sec. is sufficient to illustrate the applicability of the observer. Measurement noise is not simulated to enhance visibility. The residual is generated as:

$$r = U_m - \hat{U}$$

where  $\hat{U}$  is the observer output. The gains and initial conditions are chosen as:

$$K_{\Delta k_y}^{\hat{n}} = 0.001, K_{\Delta k_y}^{\hat{U}} = 0.01, \hat{n}(t=0) = 9 \text{ rad/s}, \text{ and } \hat{U}(t=0) = 0.1 \text{ m/s.}$$

The dynamic transient effect in Fig. 1 arises as a result of mismatch between the real values of their involved parameters in Eqs. 9 and 10 and the computed values due to the shaft speed measurement fault as well as the fast change in set-points. This transient response is shown to have minimum impact on the residual and can be handled by choosing an appropriate threshold. The gain fault can be detected within the required *time-to-detect* proposed in (Izadi-Zamanabadi and Blanke, 1998). Measurement noise can be dealt with by using statistical methods such as CUSUM.

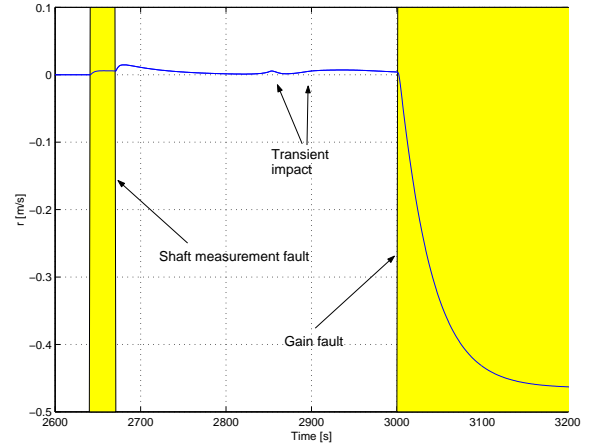


Fig. 1. Residual,  $r = U_m - \hat{U}$ . Both the shaft speed measurement and gain fault are present.

## 5. CONCLUSIONS

A brief review of a geometric approach to nonlinear fault detection and isolation was presented. The applicability of the method was illustrated on a ship propulsion system.

The results illustrate the strong ability of the geometric approach to analyze a system in a systematic way. The

prerequisite is that a complete nonlinear analytical model of the system is available. The outputs of the diagnosis are dedicated subsystems that can be used as starting point for designing observer-based FDI.

## 6. REFERENCES

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