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DESIGN OF OUTPUT FEEDBACK GUARANTEED COST CONTROLLERS WITH DISK CLOSED-LOOP POLE CONSTRAINTS FOR UNCERTAIN DISCRETE-TIME SYSTEMS

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Abstract：This paper is concerned with the design of robust output feedback controllers for a class of linear discrete-time systems with norm-bounded parameter uncertainty. The controller is designed to place the closed-loop poles of the uncertain system in a specified disk and guarantee an upper bound on a quadratic cost function. An existence condition for such controllers is derived. It is shown that the condition is equivalent to the feasibility of a linear matrix inequality (LMI) problem, and the procedure of constructing such a desired controller is presented in terms of the feasible solutions to this LMI. Furthermore, a convex optimization problem is introduced for the selection of a suitable output feedback controller minimizing the upper bound. *Copyright©2002 IFAC*

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1. INTRODUCTION

Among the numerous methods for solving the robust control design problem for uncertain systems, Lyapunov methods (quadratic stability in particular) have proven to be efficient (Barmish, 1985; Garcia, *et al*., 1994; Petersen, 1987). Although stability is a minimal requirement for control systems, it is not sufficient in most practical situations. A good controller should also deliver sufficiently fast and well-damped time responses. A customary way to guarantee satisfactory transients is to place the closed-loop poles in a suitable region of the complex plane (Chilali, *et al*., 1999; Gutman and Jury, 1981; Haddad and Bernstein, 1992), of particular interest is the circular region $D(\alpha, r)$ with centre at $(-\alpha, 0)$ and radius r. By placing the closed-loop poles within a suitable disk, not only can one guarantee an upper bound on the damping ratio, but also a bound on the natural frequency and damped natural frequency (Haddad and Bernstein, 1992) . In Garcia and Bernussou (1995), the problem of robust pole placement within a disk using state feedback has been solved using the concept of quadratic-d stabilizability, which is the extension of quadratic stabilizability.

From a practical viewpoint, enforcing the closed-loop poles in a suitable region is rarely sufficient because most design problems are essentially multi-objective (Chilali and Gahinet, 1996; Scherer, *et al*., 1997). Recently, the quadratic-d guaranteed cost control problem has been addressed for continuous time uncertain systems (Moheimani and Petersen, 1996) and discrete time uncertain ones (Garcia, 1997) by combining pole placement with a guaranteed cost control (Petersen and McFarlane, 1994; Petersen, *et al.,* 1998; Yu and Chu, 1999). This problem is to design a controller to place the closed-loop poles in a specified disk and guarantee an upper bound on a quadratic cost function. Riccati equation approaches are developed in (Garcia, 1997; Moheimani and Petersen, 1996) to design a state feedback quadratic-d guaranteed cost controllers. These results are based on the assumption that all states are available for

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feedback. In practice, this is rare. The problem of quadratic-d guaranteed cost control via output feedback is therefore interesting. To the best of our knowledge, this problem has not yet been investigated in the literature.

This paper is to investigate the design problem of output feedback quadratic-d guaranteed cost controllers for a class of discrete-time systems with norm-bounded parameter uncertainty. Using the LMI approach, necessary and sufficient conditions for the existence of quadratic-d guaranteed cost controllers are derived, and it is proved that this existence condition is equivalent to the feasibility of a certain matrix inequality which is jointly convex in all variables, and a set of desired output feedback controllers is characterized using the feasible solutions to this LMI. A convex optimization problem is introduced to select a suitable controller to minimize the upper bound of the closed-loop cost function.

It should be noted that the recent results on the multi-objective design (Chilali, *et al*., 1999; Scherer, *et al*., 1997) provide an alternative approach to the control problem considered in this paper. These results are based on the basic requirement that a single closed-loop Lyapunov function should account for all design specifications. The derived conditions, therefore, are sufficient and the results are conservative. In contrast, the results of this paper provide the necessary and sufficient conditions for the existence of quadratic-d guaranteed cost controllers and require only the solution of one LMI.

Notation: Throughout the paper the symbol I denotes the identity matrix of appropriate dimension. M' denotes the transpose of M. For symmetric matrices A and B, $A<(\le)B$ means that the matrix $A-B$ is negative definite (semidefinite). $D(\alpha, r)$ denotes the disk with the centre $-\alpha + j0$ and the radius r.

2. ROBUST PERFORMANCE ANALYSIS

Consider discrete-time systems described by

$$x(k+1) = (A + H\Delta E)x(k)\,, \qquad (1)$$

where $x \in R^n$ is the state, $\Delta \in R^{i\times j}$ is a matrix of uncertain parameters satisfying the bound $\Delta'\Delta \le I$, A, H and E are constant real matrices of appropriate dimensions, and the initial condition is specified as $x(0) = x_0$.

Associated with this system is the cost function

$$J = \sum_{k=0}^{\infty} x'(k)Qx(k)\,, \qquad (2)$$

where $Q = Q' > 0$.

For a given disk $D(\alpha, r)$, the notion of quadratic-d stability for the system (1) was introduced in Garcia and Bernussou (1995). If a system is quadratic-d stable, its poles belong to the disk $D(\alpha, r)$. This notion was extended for the system (1) with a quadratic performance index (2) in Garcia (1997) and Moheimani and Petersen (1996).

Definition 1: A symmetric matrix $P > 0$ is said to be a quadratic-d cost matrix for the system (1) and the cost function (2) if

$$\begin{bmatrix} -P^{-1} & (A + H\Delta E + \alpha I)/r \\ (A + H\Delta E + \alpha I)'/r & -P + Q \end{bmatrix} < 0 \qquad (3)$$

for all $\Delta : \Delta'\Delta \le I$.

Lemma 1: (Garcia, 1997)
Suppose $P > 0$ is a quadratic-d cost matrix for the uncertain system (1) and the cost function (2). Then the system is quadratically-d stable and the cost function satisfies the bound

$$J \le x_0' \frac{P}{r^2} x_0\,. \qquad (4)$$

Conversely, if the system (1) is quadratically-d stable there will exist a quadratic-d cost matrix for this system and cost function.

Note that the bound in (4) depends on the initial state x_0. To remove this dependence on the initial state, there are two approaches: one is the deterministic method (Petersen, *et al*., 1998) and the other is the stochastic approach (Petersen and McFarlane, 1994). In this paper, we adopt the deterministic approach. Suppose that the initial state of the system (1) is arbitrary but belongs to the set $S = \{x_0 \in R^n : x_0 = Uv, v'v \le 1\}$, where U is a given matrix. The cost bound (4) then leads to

$$J \le \frac{1}{r^2}\lambda_{\max}(U'PU)\,, \qquad (5)$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue.

The objective of this section is to derive verification existence conditions of quadratic-d cost matrices for the system (1) and cost function (2). To this end, we first introduce a useful lemma.

Lemma 2: (Xie, 1996)
Given matrices G, H and E of appropriate dimensions and with G symmetrical, then

$$G + H\Delta E + E'\Delta'H' < 0$$

holds for any admissible uncertain matrix Δ satisfying $\Delta'\Delta \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$G + \varepsilon HH' + \varepsilon^{-1}E'E < 0\,.$$

The following theorem is the main result of this section. It shows that the feasibility of an LMI problem is necessary and sufficient for the existence of a quadratic-d cost matrix.

Theorem 1: For a given disk $D(\alpha, r)$, there exists a quadratic-d cost matrix for the system (1) and cost function (2) if and only if there exist a scalar $\varepsilon > 0$ and a symmetric positive definite matrix V such that

$$\begin{bmatrix} -V & (A+\alpha I)V & 0 & H & 0 \\ V(A+\alpha I)' & -r^2V & VE' & 0 & rV(Q^{1/2})' \\ 0 & EV & -I & 0 & 0 \\ H' & 0 & 0 & -I & 0 \\ 0 & rQ^{1/2}V & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \cdot \quad (6)$$

Proof: It follows from Definition 1 that the system (1) admits a quadratic-d cost matrix P if and only if inequality (3) holds for all admissible parameter uncertainties. It is straightforward to verify that the inequality (3) is equivalent to

$$\begin{bmatrix} -P^{-1} & A+\alpha I+H\Delta E \\ (A+\alpha I+H\Delta E)' & -r^2P+r^2Q \end{bmatrix} < 0 .$$

This inequality can be rewritten as

$$\begin{bmatrix} -P^{-1} & A+\alpha I \\ (A+\alpha I)' & -r^2P+r^2Q \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} \Delta [0 \quad E] + [0 \quad E]' \Delta' \begin{bmatrix} H \\ 0 \end{bmatrix}' < 0 \quad . \quad (7)$$

By Lemma 2, the inequality (7) holds for any uncertain matrix Δ satisfying $\Delta'\Delta \le I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} -P^{-1} & A+\alpha I \\ (A+\alpha I)' & -r^2P+r^2Q \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} H \\ 0 \end{bmatrix} [H' \quad 0] + \varepsilon \begin{bmatrix} 0 \\ E' \end{bmatrix} [0 \quad E] < 0 ,$$

that is,

$$\begin{bmatrix} -P^{-1}+\varepsilon^{-1}HH' & A+\alpha I \\ (A+\alpha I)' & -r^2P+r^2Q+\varepsilon E'E \end{bmatrix} < 0 .$$

Pre- and post-multiplying this inequality by the matrix diag$\left\{\sqrt{\varepsilon}I, \sqrt{\varepsilon}P^{-1}\right\}$ yield

$$\begin{bmatrix} \varepsilon P^{-1}+HH' & \varepsilon(A+\alpha I)P^{-1} \\ \varepsilon P^{-1}(A+\alpha I)' & -r^2\varepsilon P^{-1}+r^2\varepsilon P^{-1}QP^{-1}+\varepsilon^2P^{-1}E'EP^{-1} \end{bmatrix} < 0$$

Now let $V = \varepsilon P^{-1}$. Then using the Schur complement, it is straightforward to verify that this inequality is equivalent to (6). This completes the proof of the theorem.

(6) is an LMI in the variables V and ε. Therefore, the existence problem of the quadratic cost matrix can be considered as an LMI feasibility problem, the latter is convex and can be efficiently solved by the existing LMI software (Gahinet, *et al.*, 1995).

Remark 1: It follows from the proof of Theorem 1 that if the condition (6) is satisfied with a scalar $\varepsilon > 0$ and a positive definite matrix V then the corresponding upper bound on the cost function (2) is given by

$$J \le \frac{\varepsilon}{r^2} \lambda_{\max}(U'V^{-1}U) .$$

3. DESIGN OF QUADRATIC-D GUARANTEED COST CONTROLLERS

Consider the following uncertain discrete-time systems

$$\begin{aligned} x(k+1) &= (A+H_1\Delta E_1)x(k)+(B+H_1\Delta E_2)u(k), \\ y(k) &= (C+H_2\Delta E_1)x(k)+(D+H_2\Delta E_2)u(k), \end{aligned} \quad (8)$$

where $x(k) \in R^n$, $u(k) \in R^m$ and $y(k) \in R^p$ are the state, control input and measurement output of the system, respectively, $A, B, C, D, E_1, E_2, H_1$ and H_2 are real constant matrices with appropriate dimensions, $\Delta \in R^{i \times j}$ is an unknown matrix satisfying $\Delta'\Delta \le I$, the initial state $x(0) = x_0$ is assumed to be arbitrary but belongs to the set S, in which S is described in Section 2.

Associated with this system is the cost function

$$J = \sum_{k=0}^{\infty} [x'(k)Qx(k)+u'(k)Ru(k)] , \quad (9)$$

where $Q > 0$ and $R > 0$ are given weighting matrices.

In this section, for a given disk $D(\alpha, r)$, we deal with the output feedback quadratic-d guaranteed cost control problem for the system (8) and cost function (9). To this end, let a full order output feedback controller be of the form

$$\begin{aligned} \hat{x}(k+1) &= A_c\hat{x}(k)+B_cy(k), \quad \hat{x}(0)=0, \\ u(k) &= C_c\hat{x}(k), \end{aligned} \quad (10)$$

where $\hat{x}(k) \in R^n$ is the state of the controller, and A_c, B_c and C_c are matrices with appropriate dimensions to be determined later. Applying this controller to the system (8) results in the closed-loop system

$$\overline{x}(k) = (\overline{A}+\overline{H}\Delta\overline{E})\overline{x}(k) , \quad (11)$$

where

$$\overline{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} A & BC_c \\ B_cC & A_c+B_cDC_c \end{bmatrix},$$
$$\overline{H} = \begin{bmatrix} H_1 \\ B_cH_2 \end{bmatrix}, \quad \overline{E} = [E_1 \quad E_2C_c]$$

The corresponding closed-loop cost function is

$$J = \sum_{k=0}^{\infty} \overline{x}'(k)\overline{Q}\overline{x}(k) , \quad (12)$$

where $\overline{Q} = \text{diag}\{Q, C_c' R C_c\}$.

Definition 2: Consider the uncertain system (8) with the cost function (9) and a given disk $\mathrm{D}(\alpha, r)$, a controller of the form (10) is said to be an output feedback quadratic-d guaranteed cost controller with cost matrix $\overline{P} > 0$ if the matrix $\overline{P} > 0$ is a quadratic-d cost matrix for the closed-loop system (11) and cost function (12).

Remark 2: Using the analysis results of the last section, it follows that if (10) is an output feedback quadratic-d guaranteed cost controller, then the resulting closed-loop system will be quadratically stable and the closed-loop poles are all in the disk $\mathrm{D}(\alpha, r)$, and furthermore, the closed-loop cost function is no more than a specified bound for all admissible uncertainties.

By applying Theorem 1 to the closed-loop system (11) and the corresponding cost function (12), an existence condition of the guaranteed cost controllers can be easily derived.

Theorem 2: For a given disk $D(\alpha, r)$, there exists an output feedback quadratic-d guaranteed cost controller of the form (10) for the uncertain system (8) and cost function (9) if and only if there exist a positive scalar ε and a positive definite matrix $\overline{V}$ such that

$$\begin{bmatrix} -\overline{V} & (\overline{A}+\alpha I)\overline{V} & \overline{H} & 0 & 0 \\ \overline{V}(\overline{A}+\alpha I) & -r^2\overline{V} & 0 & \overline{VE}' & r\overline{V}\hat{Q}' \\ \overline{H}' & 0 & -I & 0 & 0 \\ 0 & \overline{EV} & 0 & -I & 0 \\ 0 & r\hat{Q}\overline{V} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \,. \quad (13)$$

where $\overline{Q} = \hat{Q}'\hat{Q}, \quad \hat{Q} = \text{diag}\{Q^{1/2}, R^{1/2}C_c\}$.

In the matrix inequality (13), the matrix $\overline{V}$, the scalar ε, and the controller parameters A_c, B_c and C_c which enter the matrices $\overline{A}, \overline{H}, \overline{E}$ and $\hat{Q}$ are unknown and occur in nonlinear fashion. Therefore, (13) cannot be considered as an LMI problem. In the sequel, we will use a method of changing variables (Scher, *et al.*, 1997) such that (13) is reduced to an LMI in all variables.

First, partition the matrix $\overline{V}$ and its inverse as

$$\overline{V} = \begin{bmatrix} Y & N \\ N' & W \end{bmatrix}, \quad \overline{V}^{-1} = \begin{bmatrix} X & M \\ M' & Z \end{bmatrix},$$

where $X, Y \in R^{n \times n}$ are symmetric matrices. Note that the equality $\overline{V}^{-1}\overline{V} = I$ gives

$$MN' = I - XY \,. \quad (14)$$

Define

$$F_1 = \begin{bmatrix} X & I \\ M' & 0 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} I & Y \\ 0 & N' \end{bmatrix}.$$

Then it follows that

$$\overline{V}F_1 = F_2, \qquad F_1'\overline{V}F_1 = F_2'F_1 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}. \quad (15)$$

Now, define a new set of variables as follows

$$\begin{aligned} \hat{A} &= XAY + XB\hat{C} + \hat{B}CY + MA_cN' + \hat{B}D\hat{C}, \\ \hat{B} &= MB_c, \\ \hat{C} &= C_cN', \end{aligned} \quad (16)$$

Therefore, given positive definite matrices X, Y and invertible matrices M, N, the controller matrices A_c, B_c and C_c can be uniquely determined by $\hat{A}, \hat{B}$ and $\hat{C}$. By using matrix operation, it is straight-forward to verify that

$$F_1'\overline{A}\,\overline{V}F_1 = \begin{bmatrix} XA + \hat{B}C & \hat{A} \\ A & AY + B\hat{C} \end{bmatrix}, \quad F_1'\overline{VE}' = \begin{bmatrix} E_1' \\ YE_1' + \hat{C}'E_2' \end{bmatrix},$$
$$F_1'\overline{V}\hat{Q}' = \begin{bmatrix} Q^{1/2} & 0 \\ YQ^{1/2} & \hat{C}'R^{1/2} \end{bmatrix}, \quad F_1'\overline{H} = \begin{bmatrix} XH_1 + \hat{B}H_2 \\ H_1 \end{bmatrix}, \quad (17)$$

The following theorem gives the main result on output feedback quadratic-d guaranteed cost control based on the LMI approach.

Theorem 3: For a given disk $D(\alpha, r)$, there exists an output feedback quadratic-d guaranteed cost controller of the form (10) for the uncertain system (8) and cost function (9) if and only if there exist a scalar $\varepsilon > 0$, symmetric positive definite matrices X and Y, matrices $\hat{A}, \hat{B}$ and $\hat{C}$ such that

$$\begin{bmatrix} -X & -I & XA + \hat{B}C + \alpha X & \hat{A} + \alpha I & XH_1 + \hat{B}H_2 & 0 & 0 & 0 \\ * & -Y & A + \alpha I & AY + B\hat{C} + \alpha Y & H_1 & 0 & 0 & 0 \\ * & * & -r^2 X & -r^2 I & 0 & E' & rQ^{1/2} & 0 \\ * & * & * & -r^2 Y & 0 & YE_1' + \hat{C}'E' & rYQ^{1/2} & r\hat{C}'R^{1/2} \\ * & * & * & * & -I & G' & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (18)$$

where * replaces matrix blocks that are readily inferred by symmetry.

Proof: Post- and pre-multiplying the matrix inequality (13) by the matrix diag $\{F_1, F_1, I, I, I\}$ and its transpose, respectively, and using (15) and (17), the result of Theorem can be obtained.

Remark 3: Note that (18) is an LMI in all variables. Therefore, the existing LMI tool (Gahinet, *et al.*, 1995) can be used to find a feasible solution if exists.

Remark 4: Given any feasible solution to the LMI (18) in Theorem 3, an output feedback quadratic-d guaranteed cost controller can be constructed as follows:

1. Compute invertible matrices M and N by using the singular value decomposition of $I - XY$.
2. Obtain the controller matrices A_c, B_c and C_c by solving (16).

Remark 5: It follows from Theorem 3 that if the LMI (18) is feasible, then the system (8) has an output feedback quadratic-d guaranteed cost controller, and the corresponding closed-loop cost function is bounded by

$$J \le \frac{\varepsilon}{r^2}\lambda_{\max}(U'XU). \tag{19}$$

The upper bound (19) for the closed-loop cost function is apparently not a convex function in X and ε. Hence, finding the minimum of this upper bound cannot be considered as a convex optimization problem. Instead, the following optimization problem is introduced to find a suboptimal value for this bound.

$$\min z = \varepsilon + \lambda$$
$$\text{s.t.} \quad \begin{cases} \text{(i).} \quad (18) \\ \text{(ii).} \quad U'XU < \lambda \end{cases} \tag{20}$$

This is a convex optimization problem with LMI constraints, which can be effectively solved by existing LMI software.

4. ILLUSTRATIVE EXAMPLE

Consider an inverted pendulum system, the discretized model for this system with parameter uncertainty is given by Lewis (1992)

$$x(k+1) = \begin{bmatrix} 1.0544 & 0.1018 - 0.08\gamma & 0 & 0.0 - 0.08\gamma \\ 1.0975 & 1.0544 - 0.8\gamma & 0 & 0.0 - 0.8\gamma \\ -0.005 & -0.0002 + 0.08\gamma & 1 & 0.1 + 0.08\gamma \\ -0.0998 & -0.0049 + 0.8\gamma & 0 & 1.0 + 0.8\gamma \end{bmatrix} x(k) + \begin{bmatrix} -0.05 \\ -1.0 \\ 0.05 \\ 1.0 \end{bmatrix} u(k),$$
$$y(k) = \begin{bmatrix} 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k), \tag{21}$$

where $x \in R^4$ is the state, $u \in R$ is a force to be applied to the cart, $y \in R^2$ is the measurement output, $|\gamma| \le 0.5$ is an unknown parameter. The initial state x_0 may be unknown, but satisfies $x_0' x_0 \le 1$.

It is required to construct a suitable output feedback controller of the form (10) for this system such that for any admissible realization of the uncertain parameter γ, all the closed-loop poles locate in a disk of centre $0.5 + j0$ and radius 0.5, and a corresponding upper bound for the cost function

$$J = \sum_{k=0}^{\infty} [2x_1^2(k) + 2x_2^2(k) + 0.5x_3^2(k) + 0.5x_4^2(k) + 0.01u^2(k)] \tag{22}$$

is minimized. Thus, we will apply the approach proposed in this paper to find the optimal output feedback quadratic-d guaranteed cost controller. The system (21) is of the form given in (8) with

$$H_1 = [-0.08 \quad -0.8 \quad 0.08 \quad 0.8]',$$
$$E_1 = [0 \quad 0.5 \quad 0 \quad 0.5], \quad H_2 = E_2 = 0,$$
$$Q = \text{diag}\{2, 2, 0.5, 0.5\}, \quad R = 0.01.$$

It is found, using the software LMI Toolbox in Matlab, that the corresponding optimization problem (20) is feasible. Therefore, There exist the output feedback quadratic-d guaranteed cost controllers for the system (21) with the cost function (22), and using the optimal solution to the problem (20) and the constructive procedure for the controllers in Remark 4, a sub-optimal output feedback quadratic-d guaranteed cost controller is given by

$$\hat{x}(k+1) = \begin{bmatrix} -0.2129 & 0.0783 & 0.0830 & -0.7470 \\ -0.4942 & 0.6303 & -0.0292 & -0.0476 \\ 3.5815 & -1.1164 & 0.7969 & 0.0948 \\ 1.2121 & 0.1218 & 0.2057 & -1.3057 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 0.0257 & -2.9187 \\ 0.0559 & -4.9219 \\ -1.0069 & 0.1760 \\ -0.2434 & 4.5105 \end{bmatrix} y(k),$$
$$u(k) = [-1.0360 \quad -0.3522 \quad 0.2649 \quad -2.6485]\hat{x}(k).$$

The corresponding upper bound for the closed-loop cost function is $J \le 5.002 \times 10^8$. For constant values of the uncertain parameter γ, a plot of the closed-loop pole locations with this controller for the allowed range of γ is shown in Fig. 1.

5. CONCLUSIONS

In this paper the output feedback quadratic-d guaranteed cost control with disk pole constraint for uncertain discrete-time systems has been solved. The feasibility of a certain LMI has been proved to be necessary and sufficient for the existence of guaranteed cost controllers, and the feasible solutions to this LMI can be used to construct a controller with the desired properties. Furthermore, a convex optimization problem has been introduced to select a guaranteed cost controller which minimizes the upper bound on the closed-loop cost function. A numerical example showed the potential of the proposed approach.

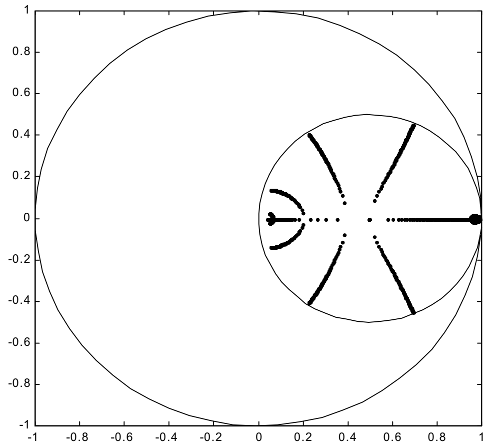

Fig. 1 Closed-loop poles

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