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ESTIMATION AND REJECTION OF DISTURBANCES IN SERVO SYSTEMS

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Abstract: This paper proposes a new approach to disturbance estimation based on a curvature model to improve the disturbance rejection performance of a servo system. The main feature is that the stability of the control system is guaranteed when the disturbance estimation is incorporated directly into the designed control law. Simulation results show that disturbances are rejected efficiently when this approach is used.

Keywords: servo system, circle of curvature, disturbance estimation, disturbance rejection.

1. INTRODUCTION

The perfect rejection of a disturbance in a servo system can be achieved by inserting an internal model of the disturbance generator into the servo controller. However, if the disturbance is unknown, it is difficult to provide the desired rejection performance. While several methods of rejecting disturbances have been proposed for the purpose of improving the performance, they require some *a priori* information about the disturbances; otherewise the design of the controller becomes complicated.

In this paper, a new approach to disturbance estimation based on a curvature model is proposed to improve the performance of disturbance rejection in a servo system. The characteristics of this method are that disturbances are reproduced satisfactorily even though the estimation model is very simple; the stability of the system is guaranteed when disturbance estimation is incorporated directly into the designed servo control law; and no *a priori* information about disturbances, such as the peak value, is needed.

Throughout this paper, $||A||$ is the Euclidean norm of matrix or vector A; and $\mathcal{O}_a^b(\tau^k)$ is an infinitesimal with the same order as τ^k. For a vector-valued sequence $x(k), k = 0, 1, \cdots$, $||x||_\infty = \sup_k ||x(k)||$; and for a system G, $||G||_1 = \sup_{||w||_\infty=1} ||Gw||_\infty$.

2. DISTURBANCE ESTIMATION

The configuration of a conventional servo system is shown in Fig. 1. An exogenous disturbance, $d(k)$, is assumed to be added to the input channel. The plant and the servo controller are respectively given by

$$\begin{cases} x_P(k+1) = A_P x_P(k) + B_P[u(k) + d(k)], \\ y(k) = C_P x_P(k), \end{cases} \quad (1)$$

and

$$\begin{cases} x_K(k+1) = A_K x_K(k) + A_{KP} x_P(k) + B_K e(k), \\ u(k) = C_K x_K(k) + C_{KP} x_P(k) + D_K e(k), \end{cases} \quad (2)$$

where $x_P(k) \in \mathbf{R}^{n_P}$, $x_K(k) \in \mathbf{R}^{n_K}$, $y(k) \in \mathbf{R}$, $u(k) \in \mathbf{R}$, $d(k) \in \mathbf{R}$ and $e(k) \in \mathbf{R}$ are the states of the plant, servo controller, output,

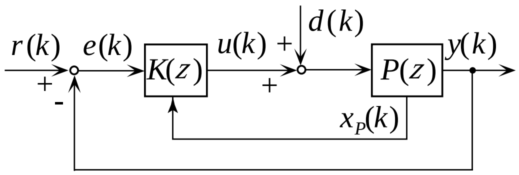

Fig. 1. Conventional servo system.

control input, disturbances and tracking error, respectively. We assume that the servo controller has been designed so that the internal stability of the servo system is guaranteed, and also make the following assumptions.

ASSUMPTION 1. (A_P, B_P) is controllable.

ASSUMPTION 2. The disturbance $d(k)$ is bounded and smooth enough.

Perfect disturbance rejection is obtained for signals for which an internal model is contained in the controller, $K(z)$. However, for other disturbances, good rejection performance cannot be expected. Generally speaking, the peak value of the tracking error is proportional to the peak value of the disturbance. If some *a priori* information about disturbances, e.g. the peak value, is known, a nonlinear control law can be designed to reject the disturbances (Young *et al.*, 1999). In this paper, we do not use such *a priori* information. The only assumption about the disturbances is that the sampling frequency is high enough that the disturbances are smooth enough.

Ohnishi *et al.* (1994) have proposed a method called disturbance observer to estimate a disturbance, and the method has been applied to several electro-mechanical systems (Komada and Ohnishi, 1990; White *et al.*, 2000). In their method, the disturbance is first described by

$$d(k) = \frac{1}{P(z)} y(k) - u(k). \tag{3}$$

Since $\frac{1}{P(z)}$ is not proper, the disturbance cannot obtained directly from Eq. (3). A low-pass filter $F(z)$ is used to make $\frac{F(z)}{P(z)}$ proper, and the disturbance is estimated by

$$\hat{d}(k) = \frac{F(z)}{P(z)} y(k) - u(k). \tag{4}$$

Note that Eq. (4) cannot be used for a continuous plant with unstable zeros/poles because unstable pole-zero cancellations would occur. Even if a continuous plant has no unstable zeros/poles, Eq. (4) still cannot be used when the relative degree of the plant is higher than two because unstable limiting zeros occur in the pulse-transfer function of the plant. So, special techniques are required to use a discrete-time disturbance observer

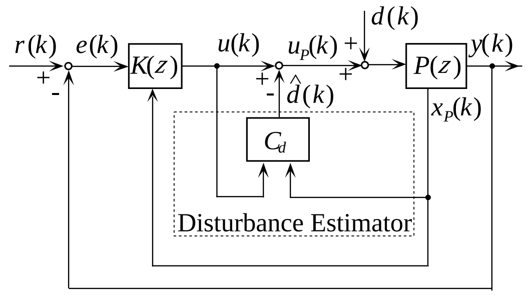

Fig. 2. Configuration of proposed servo system.

to estimate disturbances. Furthermore, since the stability of the system is not guaranteed when the estimated disturbance is incorporated directly into the designed control law, the issue of the stability of the whole system must be taken into account in the design of the low-pass filter $F(z)$. So, the construction of $F(z)$ may be complicated. In contrast, one feature of the method proposed in this paper is that the stability of the whole system is guaranteed when the estimation is incorporated directly into the designed control law.

In this paper, a low-order nonlinear disturbance-estimation model called a curvature model is proposed for the estimation of disturbances, and is used to reduce the tracking error. The configuration of the proposed servo system is shown in Fig. 2. It results from plugging a nonlinear disturbance estimator, C_d, into a conventional servo system, and has a structure similar to that of a two-degree-of-freedom servo system (Hara, 1987). So roughly speaking, the rejection of disturbances is mainly handled by the controller C_d and the reference tracking is primarily handled by the controller $K(z)$.

A circle of curvature approximation approximates the curve around the point $(k-1)\tau$ using an arc of the circle of curvature at $(k-1)\tau$. Here, this method is employed to estimate the disturbance. If the circle of curvature at $(k-1)\tau$ is known, then the value on this circle at $k\tau$ can be considered to be an estimate of the disturbance at $k\tau$ (see Fig. 3). This estimate has the following characteristics:

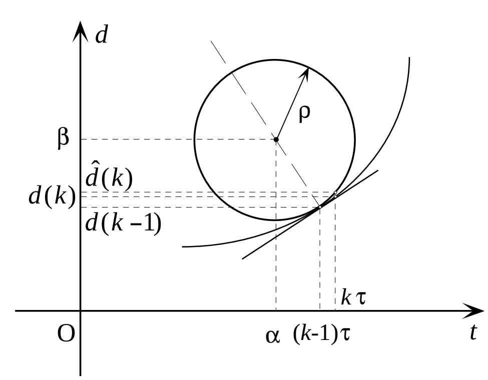

Fig. 3. Curvature circle model for disturbance estimation.

1) The circle of curvature shares the same tangent line with the disturbance at $(k-1)\tau$.
2) The circle of curvature has the same concavity or convexity as the disturbance at $(k-1)\tau$.
3) The curvature of the circle of curvature equals that of the disturbance at $(k-1)\tau$.

So, the characteristics of the disturbance are reflected in the estimate; and by making use of them, the disturbance can effectively be suppressed. The details are given below.

According to Assumption 1, there exists a nonsingular matrix $T \in \mathbf{R}^{n_P \times n_P}$ that converts the plant (1) into the following controllability canonical form:

$$\begin{cases} \bar{x}_P(k+1) = \bar{A}_P \bar{x}_P(k) + \bar{B}_P[u(k) + d(k)], \\ y(k) = \bar{C}_P \bar{x}_P(k), \end{cases} \tag{5}$$

where

$$\begin{cases} \bar{A}_P = T^{-1} A_P T \\ \quad = \left[\begin{array}{c|ccc} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ \hline -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n_P} \end{array}\right], \\ \bar{B}_P = T^{-1} B_P = \left[\begin{array}{c|c} 0 \cdots 0 & 1 \end{array}\right]^T, \\ \bar{C}_P = C_P T = \left[\begin{array}{ccc} c_1 \ \cdots \ c_{n_P-1} \ c_{n_P} \end{array}\right]. \end{cases}$$

Multiplying both sides of (5) by $\bar{B}_P^T$ gives

$$\begin{cases} \bar{B}_P^T \bar{x}_P(k+1) = D\bar{x}_P(k) + u(k) + d(k), \\ D := \left[-\alpha_1 \ -\alpha_2 \ \cdots \ -\alpha_{n_P} \right]. \end{cases} \tag{6}$$

So, the disturbance $d(k)$ can be expressed as

$$d(k) = \bar{B}_P^T \bar{x}_P(k+1) - D\bar{x}_P(k) - u(k), \tag{7}$$

and the following equations hold:

$$\begin{cases} d(k-1) = \bar{B}_P^T \bar{x}_P(k) - D\bar{x}_P(k-1), \\ \qquad\qquad -u(k-1) \\ d(k-2) = \bar{B}_P^T \bar{x}_P(k-1) - D\bar{x}_P(k-2), \\ \qquad\qquad -u(k-2) \\ d(k-3) = \bar{B}_P^T \bar{x}_P(k-2) - D\bar{x}_P(k-3). \\ \qquad\qquad -u(k-3) \end{cases} \tag{8}$$

For a sampling period, τ, if the first and second derivatives of the disturbance $d(k)$ at $(k-1)\tau$ are approximated by

$$\begin{cases} \hat{d}'(k-1) \approx \dfrac{d(k-1) - d(k-2)}{\tau}, \\ \hat{d}''(k-1) \approx \dfrac{d(k-1) - 2d(k-2) + d(k-3)}{\tau^2}, \end{cases} \tag{9}$$

then the radius of the circle of curvature, ρ, is

$$\rho^2 = \frac{\left[1 + \hat{d}'(k-1)^2\right]^3}{\hat{d}''(k-1)^2}, \tag{10}$$

and the coordinates of the center are

$$\begin{cases} \alpha = (k-1)\tau - \dfrac{\hat{d}'(k-1)\left[1 + \hat{d}'(k-1)^2\right]}{\hat{d}''(k-1)}, \\ \beta = d(k-1) + \dfrac{1 + \hat{d}'(k-1)^2}{\hat{d}''(k-1)}. \end{cases} \tag{11}$$

Thus, the disturbance estimate $\hat{d}(k)$ is obtained from the following lemma.

Lemma 1. The disturbance estimate $\hat{d}(k)$ is given by

$$\hat{d}(k) = \begin{cases} \beta - \sqrt{\rho^2 - (k\tau - \alpha)^2}, \ \hat{d}''(k-1) > 0, \\ d(k-1) + \tau\hat{d}'(k-1), \ \hat{d}''(k-1) = 0, \\ \beta + \sqrt{\rho^2 - (k\tau - \alpha)^2}, \ \hat{d}''(k-1) < 0, \end{cases} \tag{12}$$

where ρ, α and β are given by (10) and (11).

3. DISTURBANCE REJECTION

Combining the designed servo control law with the disturbance estimate yields the control law

$$u_P(k) = u(k) - \hat{d}(k). \tag{13}$$

The following theorem holds for this law.

Theorem 2. The control law (13) guarantees the stability of the control system and suppresses disturbances when the sampling period, τ, is small enough.

Proof. Assume that the internal model contained in $K(z)$ is $\frac{1}{\phi(z^{-1})}$, where $\phi(z^{-1})$ is a polynomial in z^{-1}. According to Assumption 2, there exists a positive number, d_{eM}, such that

$$||\phi(z^{-1})d(k)||_\infty = d_{eM} < \infty. \tag{14}$$

Since the designed servo system without disturbance estimation is stable, there exists a positive number $\mathcal{K} < \infty$ such that

$$\left\| \begin{array}{c} \phi(z^{-1})x_K(k) \\ \phi(z^{-1})x_P(k) \end{array} \right\|_\infty = \mathcal{K}||\phi(z^{-1})d(k)||_\infty = \mathcal{K}d_{eM}. \tag{15}$$

On the other hand, the Taylor expansion of $d(k-2)$ at $(k-1)\tau$ is

$$d(k-2) = d(k-1) - d'(k-1)\tau + \mathcal{O}_1^1(\tau^2),$$

or equivalently

$$d'(k-1) = \hat{d}'(k-1) + \mathcal{O}_1^1(\tau). \tag{16}$$

In the same manner,

$$d'(k-2) = \hat{d}'(k-2) + \mathcal{O}_1^2(\tau). \tag{17}$$

And the Taylor expansion of $d'(k-2)$ at $(k-1)\tau$,

$$d^{'}(k-2) = d^{'}(k-1) - d^{''}(k-1)\tau + \mathcal{O}_2(\tau^2),$$

gives

$$d^{''}(k-1) = \tilde{d}^{''}(k-1) + \mathcal{O}_2(\tau). \qquad (18)$$

When $\tilde{d}^{''}(k-1) > 0$, the disturbance estimate is

$$\begin{aligned}
&\hat{d}(k) = \beta - \sqrt{\rho^2 - (k\tau - \alpha)^2} \\
&= d(k-1) + \frac{1+\hat{d}^{'}(k-1)^2}{\hat{d}''(k-1)} \\
&-\sqrt{\frac{\left[1+\hat{d}'(k-1)^2\right]^3}{\hat{d}''(k-1)^2} - \left\{\tau + \frac{\hat{d}'(k-1)\left[1+\hat{d}'(k-1)^2\right]}{\hat{d}''(k-1)}\right\}^2} \\
&= d(k-1) + \frac{1+\hat{d}^{'}(k-1)^2}{\hat{d}''(k-1)} \times \\
&\left(1 - \sqrt{1 - \left\{\frac{2\hat{d}'(k-1)\hat{d}''(k-1)}{1+\hat{d}'(k-1)^2} + \frac{\hat{d}''(k-1)^2}{[1+\hat{d}'(k-1)^2]^2}\tau\right\}\tau}\right) \\
&= d(k-1) + \hat{d}^{'}(k-1)\tau + \frac{1}{2}\hat{d}^{''}(k-1)\tau^2 + \mathcal{O}_{est}(\tau^3),
\end{aligned}$$

where the following relationship is used in the derivation:

$$\sqrt{1-\chi} = 1 - \frac{1}{2}\chi - \frac{1}{2\cdot 4}\chi^2 - \frac{1\cdot 3}{2\cdot 4\cdot 6}\chi^3 - \cdots, \quad |\chi| \le 1.$$

The condition $|\chi| \le 1$ is guaranteed for a small τ. Since the Taylor expansion of $d(k)$ at $(k-1)\tau$ is

$$d(k) = d(k-1) + d^{'}(k-1)\tau + \frac{1}{2}d^{''}(k-1)\tau^2 + \mathcal{O}_0(\tau^3),$$

then

$$\begin{aligned}
\Delta d(k) := d(k) - \hat{d}(k) = &\left\{d^{'}(k-1) - \hat{d}^{'}(k-1)\right\}\tau \\
&+ \frac{1}{2}\left\{d^{''}(k-1) - \hat{d}^{''}(k-1)\right\}\tau^2 + \mathcal{O}_{err}(\tau^3).
\end{aligned}$$

From (16) and (18) we obtain

$$||\Delta d(k)||_\infty = \mathcal{O}(\tau^2). \qquad (19)$$

The above equation also holds for $\hat{d}^{''}(k-1) < 0$ and $\hat{d}^{''}(k-1) = 0$. So, if a small enough τ is chosen, then $\Delta d(k)$ will be bounded. In general, if the effects of the disturbances cannot be ignored, then $||d(k)||_\infty >> \mathcal{O}(\tau^2)$. Therefore,

$$||\Delta d(k)||_\infty << ||d(k)||_\infty \qquad (20)$$

is satisfied, and

$$\begin{aligned}
\left|\left|\phi(z^{-1})\Delta d(k)\right|\right|_\infty &= \left|\left|\phi(z^{-1})\right|\right|_1 ||\Delta d(k)||_\infty \\
&<< \left|\left|\phi(z^{-1})\right|\right|_1 ||d(k)||_\infty \\
&= \left|\left|\phi(z^{-1})d(k)\right|\right|_\infty
\end{aligned}$$

holds. The above yields

$$\left|\left|\phi(z^{-1})\Delta d(k)\right|\right|_\infty << d_{eM}. \qquad (21)$$

So, in the improved servo system in Fig. 2, the equivalent disturbance added to the plant is $\Delta d(k)$, which is much smaller than the actual disturbance $d(k)$. If we incorporate the estimated disturbance into the servo control law, the following holds:

$$\begin{aligned}
\left\|\begin{matrix}\phi(z^{-1})x_K(k) \\ \phi(z^{-1})x_P(k)\end{matrix}\right\|_\infty &= \mathcal{K}\left|\left|\phi(z^{-1})\Delta d(k)\right|\right|_\infty \qquad (22)\\
&<< \mathcal{K}d_{eM}.
\end{aligned}$$

The above equation means that the control system is stable and the effects of disturbances are suppressed when the estimated disturbance is combined with the designed servo control law. □

4. NUMERICAL EXAMPLE

Consider the following second-order plant:

$$\begin{aligned}
&P(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}, \qquad (23)\\
&\omega = 1 \text{ rad/s}; \quad \zeta = 0.5.
\end{aligned}$$

The sampling period

$$\tau = 0.1 \text{ s} \qquad (24)$$

is used to discretize the continuous plant. The reference input

$$r(k) = \sin\frac{2\pi}{21}k + \sin\frac{4\pi}{21}k \qquad (25)$$

is added. The periodicity of the reference input makes a repetitive control scheme suitable (Hara *et al.*, 1988; Tomizuka *et al.*, 1989) and the internal model is given by

$$\phi(z^{-1}) = 1 - z^{-L}. \qquad (26)$$

The number of steps of the repetitive controller is

$$L = 21. \qquad (27)$$

First, choosing

$$Q = \begin{bmatrix} 100 \times I_{21} & 0 \\ 0 & I_2 \end{bmatrix}, \qquad (28)$$

and optimizing the following performance index

$$\begin{aligned}
&J := \sum_{k=0}^{\infty}\left[x_e^T(k)Qx_e(k) + u_e(k)^2\right], \\
&x_e(k) := \left[e(k) \ \cdots \ e(k-L+1) \ \ (1-z^{-L})x_P^T(k)\right]^T, \\
&u_e(k) := (1-z^{-L})u(k)
\end{aligned}$$

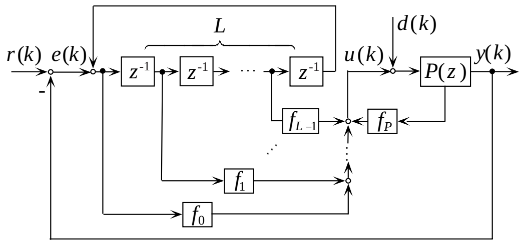

Fig. 4. Optimal tracking control system.

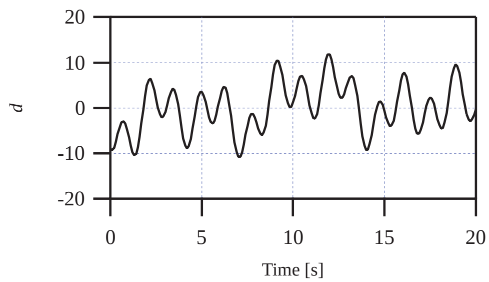

Fig. 5. Disturbances.

gives the optimal repetitive control law

$$u_e(k) = F_G x_e(k) = [\, f_0 \quad \cdots \quad f_{L-1} \quad f_P \,]\, x_e(k). \quad (29)$$

The optimal repetitive control system is shown in Fig. 4 (Tsuchiya and Egami, 1992).

The disturbance

$$d(k) = -5\cos\frac{3\pi}{21}k - 4\sin\frac{3\pi}{50}k - 3\cos\frac{3\pi}{110}k, \\ -2\sin\frac{3\pi}{230}k - \cos\frac{3\pi}{410}k \\ 0 \le k \le 200 \quad (30)$$

which is non-periodic up to 27 sec (270 steps), was input (Fig. 5). The simulation results for the optimal system are shown in Fig. 6. Since an

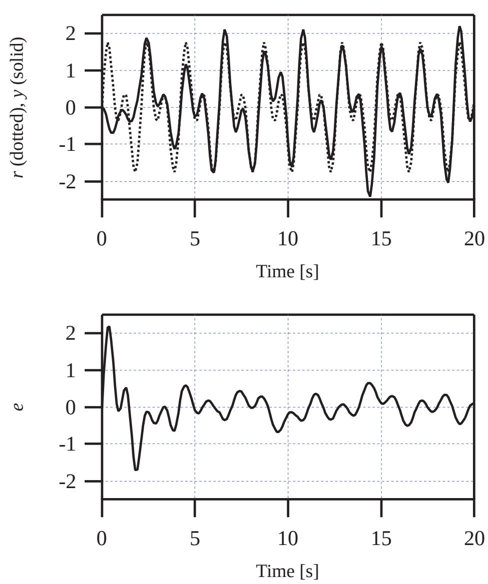

Fig. 6. Response of optimal tracking control system.

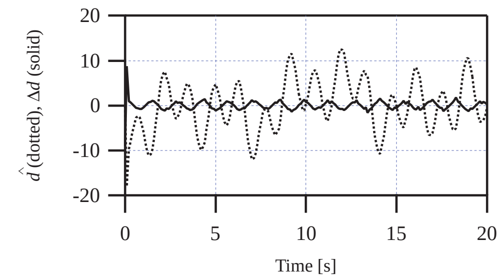

Fig. 7. Disturbance estimates and estimation error.

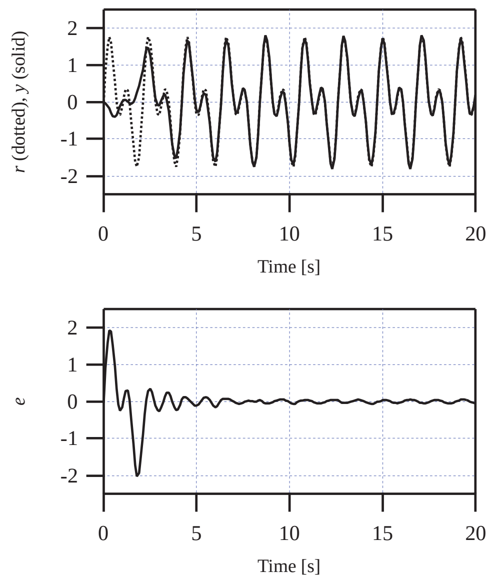

Fig. 8. Response of optimal repetitive control system with disturbance estimation.

internal model of the disturbance is not contained in the repetitive controller, the disturbance cannot be rejected completely. In the steady state, the peak-to-peak value of the tracking error is about 1. Next, the disturbance was estimated using the method proposed in this paper. The disturbances and the corresponding estimates are shown in Fig. 7. It is clear from the figure that the estimates reproduce the disturbance satisfactorily. The simulation results for a control law that makes use of the estimates are shown in Fig. 8. A comparison of Figs. 6 and 8 reveals that making use of the estimated disturbance significantly reduces the tracking error.

5. CONCLUSIONS

To improve the disturbance rejection performance of a servo system, this paper proposes a curvature model for disturbance estimation, and an improved servo control law that makes use of the estimate. Unlike other approaches, we do not assume that any information about the disturbances, such as the peak value, is known. The main features of this method are:

1) disturbances are reproduced satisfactorily even though the estimation model is very simple; and
2) the stability of the servo system is guaranteed when disturbance estimation is incorporated directly into the designed servo control law.

The validity of the proposed method has been demonstrated through simulations.

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