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AN OPTIMIZATION APPROACH TO FDF DESIGN FOR UNCERTAIN DISCRETE-TIME SYSTEMS

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Abstract: In this contribution, problems related to fault detection filter (FDF) design for uncertain discrete-time systems with both modelling errors and unknown input are studied. The basic idea of our study consists in the formulation of designing robust fault detection filters as a standard H_∞-model matching problem. To this end, the design scheme consists of two parts. We first propose an optimization approach to the design of fault detection filters for the discrete-time systems only with unknown inputs. The resulted fault detection filter acts then as a reference model. In the second part, the design problem is formulated as a standard H_∞-model matching problem which is solved using H_∞-optimization LMI techniques. A numerical example is finally presented to illustrate the developed approach.

Keywords: Fault detection filter, discrete-time system, modelling error, H_∞, model matching

1. INTRODUCTION AND PROBLEM FORMULATION

In this contribution, we study fault detection problems for discrete-time LTI systems described by

$$\begin{aligned} x(k+1) &= (A+\Delta A)x(k) + (B+\Delta B)u(k) \\ &\quad + B_{\mathsf{f}} f(k) + B_{\mathsf{d}} d(k) \qquad (1) \\ y(k) &= Cx(k) + Du(k) + D_{\mathsf{f}} f(k) \\ &\quad + D_{\mathsf{d}} d(k) \qquad (2) \end{aligned}$$

where $x(k) \in R^{\mathsf{n}}$ is the state vector, $u(k) \in R^{\mathsf{p}}$ the control input vector, $y(k) \in R^{\mathsf{q}}$ the measurement output vector. $d(k) \in R^{\mathsf{m}}$ the unknown input vector and $f(k) \in R^{\mathsf{l}}$ the fault vector to be detected and isolated. $A, B, C_1, C_2, D_1, B_{\mathsf{f}}, B_{\mathsf{d}}$, D_{f} and D_{d} are known matrices with appropriate dimensions. $\Delta A = E\Sigma(t)F_1$, $\Delta B = E\Sigma(t)F_2$ are modelling errors, and denote

$$\begin{aligned} \Omega_{\mathsf{A}} &:= \{\Delta A \mid \Delta A = E\Sigma(t)F_1,\ \Sigma^{\mathsf{T}}(t)\Sigma(t) \le I\} \\ \Omega_{\mathsf{B}} &:= \{\Delta B \mid \Delta B = E\Sigma(t)F_2,\ \Sigma^{\mathsf{T}}(t)\Sigma(t) \le I\} \end{aligned}$$

where E, F_1, F_2 are known matrices, $\Sigma(t)$ is unknown but $\Sigma^{\mathsf{T}}(t)\Sigma(t) \le I$, $A + \Delta A$ is assumed asymptotically stable for all $\Delta A \in \Omega_{\mathsf{A}}$. Without loss of generality, $d(k)$ is assumed to be l_2-norm bounded. For our purpose, the following assumptions are also made:

A1. (C, A) is detectable;

A2. $\begin{bmatrix} A - e^{\mathrm{j}\theta} I & B_{\mathsf{d}} \\ C & D_{\mathsf{d}} \end{bmatrix}$ has full row rank for all $\theta \in [0, 2\pi)$

The first step to a successful fault detection and isolation (FDI) is the so-called residual generation [(Chen J, 1999); (Frank P M, 1997);(Frank P M, 2000)]. In this contribution, our main attention is focused on the design of fault detection filters which are known as a valuable solution for the residual generation. For system (1)-(2), a fault detection filter is given by

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + H(y(k) - \hat{y}(k)) \quad (3)$$
$$\hat{y}(k) = C\hat{x}(k) + Du(k) \quad (4)$$
$$r(k) = V(y(k) - \hat{y}(k)) \quad (5)$$

where $\hat{x}, \hat{y}$ are estimation of state and output vectors respectively, r is the so-called residual signal. The observer gain matrix H and matrix V are the design parameters which will be determined for achieving required FDI performance.

Denote $e(k) = x(k) - \hat{x}(k)$, it follows from (1)-(5) that

$$\begin{aligned} e(k+1) = &(A - HC)e(k) + \Delta Ax(k) + \Delta Bu(k) \quad (6) \\ &+(B_{\mathsf{f}} - HD_{\mathsf{f}})\ f(k) + (B_{\mathsf{d}} - HD_{\mathsf{d}})d(k) \\ r(k) = &VCe(k) + VD_{\mathsf{f}}\ f(k) + VD_{\mathsf{d}}d(k) \quad (7) \end{aligned}$$

It thus becomes evident that the residual signal is not only influenced by f but also by d, $\Delta A, \Delta B$ and the control input u. For this reason, the robustness issue in FDI has received considerable attention in the past [(Chen J, 1999); (Frank P M, 1997);(Frank P M, 2000)]. The main objective of this contribution is to design fault detection filter (3)-(5) in such a way that the dynamics of the fault detection system governed by (6)-(7) is asymptotically stable, and r is as sensitive as possible to f, whilst is as robust as possible to $d, \Delta A, \Delta B$.

The paper is organized as follows. In Section 2, the basic idea of our study is presented. Section 3 contains the main results of this paper, the approach to the design of fault detection filters. In section 4, a numerical example is used to illustrate the approach proposed.

2. BASIC IDEAS OF OUR STUDY

There exist a number of schemes to approach the robustness issue in FDI. One of them is to introduce a reference model and further to formulate the design of residual generator as an $H_\infty-$filtering or $H_\infty-$model matching problem, see for instance [(Chen J, 2000); (Ding S X, 2001); (Frisk E, 1999);(Nobrega E G, 2000)], to mention only some of them. Another widely adopted way is to introduce a performance index which can be generally formulated as follows

$$\min \frac{\text{Influence of the unknown inputs}}{\text{Influence of the faults}}$$

[(Chen J, 1999); (Ding S X, 2000*a*); (Ding S X, 2000*b*); (Frank P M, 2000);(Patton R J, 1999)]. As demonstrated in [(Ding S X, 2000*a*); (Ding S X, 2000*b*)], the latter scheme can be interpreted as an optimal compromise between the maximal fault detection rate and minimal false alarm rate. On the other side, the most existing methods of this scheme only apply to the LTI with unknown inputs and an extension of them to deal with uncertain systems seems difficult. Different from this, the former scheme can, thanks to the standard $H_\infty-$model matching problem formulation, also be used to deal with uncertain systems using existing optimization tool like LMI-technique. The major difficulty with this scheme consists in the selection of a suitable reference model which is of significant physical meanings from the FDI viewpoint.

In this contribution, we are going to present a design scheme which is indeed a combination of the above-mentioned two schemes. It consists of two steps:

- design of an optimal fault detection filter for system (1)-(2) on assumption that $\Delta A = 0, \Delta B = 0$, aiming at an optimal compromise between the maximal fault detection rate and minimal false alarm rate;
- set the optimal fault detection filter achieved in Step 1 as the reference model and design the fault detection filter for system (1)-(2) (also $\Delta A \in \Omega_{\mathsf{A}}, \Delta B \in \Omega_{\mathsf{B}}$ are taken into account) by solving the standard H_∞-model matching problem using LMI-technique.

The basic idea behind this design scheme is that the reference model is an ideal solution which ensures an optimal compromise between the maximal fault detection rate and minimal false alarm rate in case that the system model is perfectly known (i.e. $\Delta A = 0, \Delta B = 0$). Thus, the design objective is to find a fault detection filter which results in a minimal different between the reference model (ideal solution) and the fault detection filter (real solution) to be designed. Mathematically, we are able to formulate the design scheme as follows.

Suppose that the optimal fault detection filter achieved after the first design step is given by

$$x_o(k+1) = A_o x_o(k) + B_{of} f(k) + B_{od} d(k) \quad (8)$$
$$r_o(k) = C_o x_o(k) + D_{of} f(k) + D_{od} d(k) \quad (9)$$

In the second step, we consider system (1)-(2) and try to design fault detection filter (3)-(5) by selecting H and V in such a way that the difference between the real residual generator and the reference model (8)-(9) is minimized in the sense of H_∞-norm. Denote

$$r_e(k) = r(k) - r_o(k)$$

The overall system is described by (1),(6), (8) and

$$r_e(k) = -C_o x_o(k) + VCe(k) + (VD_f - D_{of})f(k) + (VD_d - D_{od})d(k) \quad (10)$$

The RFDF design for uncertain LTI system (1)-(2) can be furthermore defined as: to find the observer gain matrix H and matrix V such that system $A - HC$ is asymptotically stable and for all $\Delta A \in \Omega_A$, $\Delta B \in \Omega_B$, the following H_∞-norm constraint is satisfied:

$$\| [\, G_{r_e f} \;\; G_{r_e d} \;\; G_{r_e u} \,] \|_\infty < \gamma \quad (11)$$
$$\text{or} \;\; \| [\, G_{r_e f} \;\; G_{r_e d} \;\; G_{r_e u} \,] \|_\infty \longrightarrow \min \quad (12)$$

where$G_{r_e f}, G_{r_e d}$ and $G_{r_e u}$ denote the transfer function matrix from f, d and u to r_e respectively.

For our purpose, following two problems should evidently be solved

- Design of reference model
- Solving optimization problem (11) or (12).

These are also the main tasks of this contribution.

3. MAIN RESULTS

In this section, solutions for the above-formulated two problems will be derived.

3.1 Design of reference model

Suppose $\Delta A = 0, \Delta B = 0$. Then the dynamics of fault detection filter (6)-(7) is given by

$$r(z) = G_{rf}(z) f(z) + G_{rd}(z) d(z) \quad (13)$$

where

$$G_{rf}(z) = V(C(zI - A + HC)^{-1}(B_f - HD_f) + D_f)$$
$$G_{rf}(z) = V(C(zI - A + HC)^{-1}(B_d - HD_d) + D_d)$$

For the design of the reference model, the following performance index is introduced

$$J = \frac{\|G_{rd}(z)\|_\infty}{\sigma_i(G_{rf}(e^{j\theta}))}, \;\; \theta \in [0, 2\pi), i = 1, ..., l \quad (14)$$

where $\sigma_i(\cdot)$ denotes a nonzero singular values of a transfer function matrix (analogous to [(Ding S X, 2000b)]). The formulation of performance index J also means that all nonzero singular values of transfer function matrix $G_{rf}(z)$ are taken into account over the whole frequency domain.

Now, the reference model design problem can be formulated as follows: to find observer gain matrix H, matrix V such that $A - HC$ is asymptotically stable and performance index J is minimized. As demonstrated in [(Ding S X, 2000a); (Ding S X, 2000b)], minimization of J can be interpreted as an optimal compromise between the maximal fault detection rate and minimal false alarm rate.

The solution of the above-defined optimization problem is similar to the one given in [(Ding S X, 2000b)], where continuous-time systems are considered. We begin with the concept of co-inner matrix. Transfer function matrix $G(z)$ is called co-inner matrix when

$$G(e^{j\theta})G^T(e^{-j\theta}) = I, \quad \theta \in [0, 2\pi)$$

The following lemma presents a state space characterization of co-inner transfer functions.

Lemma 1. Suppose $G(z) = C(zI - A)^{-1}B + D$ is stable and minimal, and $X = X^T \geq 0$ satisfies

$$AXA^T + BB^T - X = 0 \quad (15)$$

Then $G(z)$ is a co-inner if and only if

(a) $AXC^T + BD^T = 0$

(b) $DD^T + CXC^T = I$

Proof: refer to the case of continuous-time system given in [(Zhou K, 1998)], here omitted.

With the aid of lemma 1, the conditions under which $G_{rd}(z)$ is co-inner can be derived, which is given in the following theorem.

Theorem 1: Given system (1)-(2) with the assumptions $A1$ and $A2$, then transfer function matrix $G_{rd}(z)$ satisfying

$$Q = CXC^T + D_d D_d^T, V = Q^{-1/2} \quad (16)$$
$$H = (AXC^T + B_d D_d^T)Q^{-1} \quad (17)$$

where $X \geq 0$ solves the Discrete-time Algebraic Ricaati Equation (DARE)

$$AXA^T - (AXC^T + B_d D_d^T)(CXC^T + D_d D_d^T)^{-1}(CXA^T + D_d B_d^T) + B_d B_d^T - X = 0 \quad (18)$$

is co-inner.

Proof. by using lemma 1 it is easy to finish the proof, since the limitation of space, omitted.

Based on the above results, the solution of the reference model design problem is given in the following theorem

Theorem2: Given process (1)-(2) with $\Delta A = 0, \Delta B = 0$ and suppose assumptions $A1-A2$ hold, then

$$Q = CXC^{\mathsf{T}} + D_{\mathsf{d}}D_{\mathsf{d}}^{\mathsf{T}}, V^* = Q^{-1/2}$$
$$H^* = (AXC^{\mathsf{T}} + B_{\mathsf{d}}D_{\mathsf{d}}^{\mathsf{T}})Q^{-1}$$

solve the optimization problem

$$\min_{\mathsf{H},\mathsf{V}} J = \frac{\|G_{\mathsf{rd}}(z)\|_\infty}{\sigma_{\mathsf{i}}(G_{\mathsf{rf}}(e^{\mathsf{j}\theta}))}, \theta \in [0, 2\pi), i = 1, ..., l$$

where $X \geq 0$ is a solution of DARE (18).The optimization value of J is given by

$$J^* = \frac{1}{\sigma_{\mathsf{i}}(V^*(D_{\mathsf{f}} + C(e^{\mathsf{j}\theta}I - A + H^*C)^{-1}(B_{\mathsf{f}} - H^*D_{\mathsf{f}})))}$$

with $G_{\mathsf{rd}}(z)$ being co-inner, i.e.

$$\|G_{\mathsf{rd}}(z)\|_\infty = 1$$

A similar proof is given in [(Ding S X, 2000b)] for the continuous-time systems. The proof here is thus omitted.

Recall that the basic idea of our study is to define the optimal fault detection filter for system (1)-(2) with $\Delta A = 0, \Delta B = 0$ as reference model, hence (8)-(9) should be

$$\begin{aligned} x_{\mathsf{o}}(k+1) &= (A - H^*C)\ x_{\mathsf{o}}(k) \\ &\quad + (B_{\mathsf{f}} - H^*D_{\mathsf{f}})\ f(k) \\ &\quad + (B_{\mathsf{d}} - H^*D_{\mathsf{d}})\ d(k) \\ r_{\mathsf{o}}(k) &= V^*(Cx_{\mathsf{o}}(k) + D_{\mathsf{f}}\ f(k) \\ &\quad + D_{\mathsf{d}}\ d(k)) \end{aligned}$$

3.2 The design of RFDF

After having the reference model, the next step is to formulate the RFDF design problem as an H_∞-optimization problem and then solve it. For this purpose, the following lemma is needed.

Lemma 2. Consider uncertain LTI discrete-time system

$$\begin{aligned} x(k+1) &= (A + \Delta A)x(k) + (B + \Delta B)w(k) \\ y(k) &= Cx(k) + Dw(k) \\ &\Delta A \in \Omega_{\mathsf{A}}, \Delta B \in \Omega_{\mathsf{B}} \end{aligned}$$

Given $\gamma > 0$, if there exist positive numbers $\varepsilon_1 > 0, \varepsilon_2 > 0$ and positive-definite matrix P such that the following LMI

$$\begin{bmatrix} -P & PA & PB & PE & PE & 0 \\ A^{\mathsf{T}}P & -P + \varepsilon_1 F_1^{\mathsf{T}}F_1 & 0 & 0 & 0 & C^{\mathsf{T}} \\ B^{\mathsf{T}}P & 0 & -\gamma^2 I + \varepsilon_2 F_2^{\mathsf{T}}F_2 & 0 & 0 & D^{\mathsf{T}} \\ E^{\mathsf{T}}P & 0 & 0 & -\varepsilon_1 I & 0 & 0 \\ E^{\mathsf{T}}P & 0 & 0 & 0 & -\varepsilon_2 I & 0 \\ 0 & C & D & 0 & 0 & -I \end{bmatrix} < 0$$

holds, then the uncertain LTI system is robust stable and satisfies $\|G_{\mathsf{zw}}\|_\infty < \gamma$.

Based on the discrete-time bounded real lemma [(Geronel J C, 1994)], it is easy to finish the proof, omitted.

We now re-formulate and solve the RFDF design problem. Re-write (1),(6), (8) and (10) into

$$\begin{aligned} \tilde{e}(k+1) &= (\tilde{A} + \tilde{E}\Sigma\tilde{F}_1)\tilde{e}(k) \\ &\quad + (\tilde{B}_{\mathsf{w}} + \tilde{E}\Sigma\tilde{F}_{\mathsf{w}})w(k) \end{aligned} \tag{19}$$

$$r_{\mathsf{e}}(k) = \tilde{C}\tilde{e}(k) + \tilde{D}_{\mathsf{w}}w(k) \tag{20}$$

where

$$\begin{aligned} \tilde{e}(k) &= \begin{bmatrix} x^{\mathsf{T}}(k) & e^{\mathsf{T}}(k) & x_{\mathsf{o}}^{\mathsf{T}}(k) \end{bmatrix}^{\mathsf{T}} \\ w(k) &= \begin{bmatrix} u^{\mathsf{T}}(k) & f^{\mathsf{T}}(k) & d^{\mathsf{T}}(k) \end{bmatrix}^{\mathsf{T}} \\ \tilde{A} &= \begin{bmatrix} A & 0 & 0 \\ 0 & A - HC & 0 \\ 0 & 0 & A_{\mathsf{o}} \end{bmatrix} \\ \tilde{B}_{\mathsf{w}} &= \begin{bmatrix} B & B_{\mathsf{f}} & B_{\mathsf{d}} \\ 0 & B_{\mathsf{f}} - HD_{\mathsf{f}} & B_{\mathsf{d}} - HD_{\mathsf{d}} \\ 0 & B_{\mathsf{of}} & B_{\mathsf{od}} \end{bmatrix} \\ \tilde{E} &= \begin{bmatrix} E \\ E \\ 0 \end{bmatrix}, \tilde{F}_1 = \begin{bmatrix} F_1 & 0 & 0 \end{bmatrix} \\ \tilde{F}_{\mathsf{w}} &= \begin{bmatrix} F_2 & 0 & 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0 & VC & C_{\mathsf{o}} \end{bmatrix} \\ \tilde{D}_{\mathsf{w}} &= \begin{bmatrix} 0 & VD_{\mathsf{f}} - D_{\mathsf{of}} & VD_{\mathsf{d}} - D_{\mathsf{od}} \end{bmatrix} \end{aligned}$$

Then, the RFDF problem is reduced to determine H and V such that system (19)-(20) is stable and the H_∞-norm of transfer function matrix $G_{\mathsf{r_e w}}$ from w to r_{e} is minimized or smaller than a given $\gamma > 0$. The following theorem provide us with a sufficient condition for designing RFDF and the solution of observer gain matrix H and matrix V.

Theorem 3. Considering uncertain discrete-time LTI system (1)-(2) and for given $\gamma > 0$, if there exist positive numbers $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, positive-definite matrices P_1, P_2, P_3 and matrix Y_2 such that LMI

$$[N_{\mathsf{i,j}}]_{12\times 12} < 0 \tag{21}$$

is satisfied, then the above-defined RFDF design problem is solvable and the corresponding observer gain matrix is given by

$$H = P_2^{-1} Y_2 \tag{22}$$

where

$$
\begin{aligned}
&N_{1,1} = -P_1, N_{1,4} = P_1 A, N_{1,7} = P_1 B\\
&N_{1,8} = P_1 B_{\mathsf{f}}, N_{1,9} = P_1 B_{\mathsf{d}}, N_{1,10} = P_1 E\\
&N_{1,11} = P_1 E, N_{2,2} = -P_2, N_{2,5} = P_2 A - Y_2\\
&N_{2,8} = P_2 B_{\mathsf{f}} - Y_2 D_{\mathsf{f}}, N_{2,9} = P_2 B_{\mathsf{d}} - Y_2 D_{\mathsf{d}}\\
&N_{2,10} = P_2 E,\ N_{3,3} = -P_3,\ N_{3,6} = P_3 A_{\mathsf{o}}\\
&N_{3,8} = P_3 B_{\mathsf{of}}, N_{3,9} = P_3 B_{\mathsf{od}}\\
&N_{4,4} = -P_1 + \varepsilon_1 F_1^{\mathsf{T}} F_1, N_{5,5} = -P_2\\
&N_{5,12} = C^{\mathsf{T}} V^{\mathsf{T}}, N_{6,6} = -P_3, N_{6,12} = -C_{\mathsf{o}}^{\mathsf{T}}\\
&N_{7,7} = -\gamma^2 I + \varepsilon_2 F_2^{\mathsf{T}} F_2, N_{8,12} = D_{\mathsf{f}}^{\mathsf{T}} V^{\mathsf{T}} - D_{\mathsf{of}}^{\mathsf{T}}\\
&N_{9,12} = D_{\mathsf{d}}^{\mathsf{T}} V^{\mathsf{T}} - D_{\mathsf{od}}^{\mathsf{T}}, N_{8,8} = N_{9,9} = -\gamma^2 I\\
&N_{10,10} = -\varepsilon_1 I,\ N_{11,11} = -\varepsilon_2 I, N_{12,12} = -I
\end{aligned}
$$

others, $N_{\mathsf{i,j}} = 0$

Proof. Using lemma 2 and through some variable changes, it is easy to finish the proof. omitted here.

Remark 1. For a given constant $\gamma > 0$, theorem 3 provide us with a sufficient conditions for the solution of RFDF design as well as an approach to finding observer gain matrix H and matrix V.

In summary, RFDF design can be carried out following the algorithm given below:

- Step 1: Design of reference model with the aid of Theorem 2;
  Step 2: Set initial value $\gamma > 0$ and iterative step length $\Delta\gamma > 0$;
  Step 3: Solve LMI (21);
  Step 4: if LMI (21) is solvable, let $\gamma = \gamma - \Delta\gamma$, go to step 3, till the minimum of γ; else, let $\gamma = \gamma + \Delta\gamma$, go to step 3, till LMI (21) is solvable;
  Step 5: matrix V can be obtained directly by solving LMI (21), while the observer gain matrix H can be calculated by equation (22).

4. A NUMERICAL EXAMPLE

Consider discrete-time uncertain LTI system (1)-(2) with parameter matrices

$$A = \begin{bmatrix} 0.5 & 0 & 0\\ 0 & 0.6 & 0\\ 0 & 0 & 0.8 \end{bmatrix}, B = \begin{bmatrix} 2 & 0\\ 3 & -1\\ -1 & 2 \end{bmatrix}$$

$$B_{\mathsf{f}} = \begin{bmatrix} 2 & 0\\ 3 & -1\\ -1 & 2 \end{bmatrix}, B_{\mathsf{d}} = \begin{bmatrix} 1 & 2 & 1\\ 2 & 0 & 3\\ 1 & 3 & 0 \end{bmatrix}$$

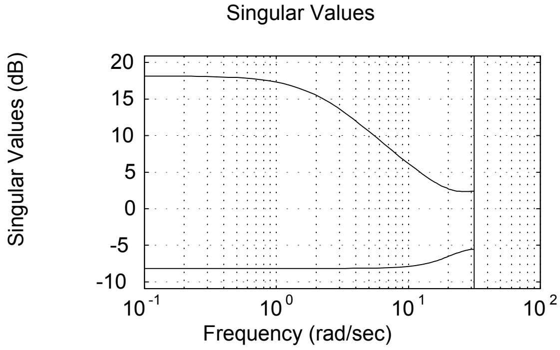

Fig. 1. the singular values plot of $W_{\mathsf{f}}(e^{\mathsf{j}\theta\mathsf{T}})$

$$C = D_{\mathsf{d}} = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 & 0\\ 0.1 & 0.1\\ 0.1 & 0.1 \end{bmatrix}$$

$$D_{\mathsf{f}} = 0, E = diag[\,0.01,\ 0.01,\ 0.01\,]$$
$$F_1 = diag\,[\,0.1,\ 0.1,\ 0.1\,]$$

Using the above proposed RFDF design algorithm gives:

- the reference model

$$
\begin{aligned}
&x_{\mathsf{o}}(k+1)\\
&= \begin{bmatrix} 0.4553 & -0.3047 & -0.3897\\ -0.3047 & 0.0902 & -0.1038\\ -0.3897 & -0.1038 & 0.2553 \end{bmatrix} x_{\mathsf{o}}(k)\\
&+ \begin{bmatrix} 2 & 0\\ 3 & -1\\ -1 & 2 \end{bmatrix} f(k)\\
&+ \begin{bmatrix} 0.9553 & 1.6953 & 0.6103\\ 1.6953 & -0.5098 & 2.8962\\ 0.6103 & 2.8962 & -0.5447 \end{bmatrix} d(k)
\end{aligned}
$$

$$
\begin{aligned}
r_{\mathsf{o}}(k) &= \begin{bmatrix} 0.7479 & -0.1585 & -0.3100\\ -0.1585 & 0.3172 & 0.1057\\ -0.3100 & 0.1057 & 0.4640 \end{bmatrix} x_{\mathsf{o}}(k)\\
&+ \begin{bmatrix} 0.7479 & -0.1585 & -0.3100\\ -0.1585 & 0.3172 & 0.1057\\ -0.3100 & 0.1057 & 0.4640 \end{bmatrix} d(k)
\end{aligned}
$$

- the solution of the H_∞−model matching problem: $\gamma_{\min} \simeq 8.2$, and the observer gain matrix H , matrix V

$$H = \begin{bmatrix} 0.2386 & 0.2814 & 0.2440\\ 0.1652 & 0.5990 & 0.1387\\ -0.2451 & 0.1871 & 0.9050 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.1140 & -0.0927 & -0.0570\\ -0.0281 & 0.0222 & 0.0130\\ -0.0731 & 0.0591 & 0.0360 \end{bmatrix}$$

Figure 1 is the singular values plot of $W_{\mathsf{f}}(e^{\mathsf{j}\theta\mathsf{T}})$ for $T = 0.1\,\mathrm{sec}$. $W_{\mathsf{d}}(z)$ is co-inner. For $\Sigma(t) = I$, $T = 0.1\,\mathrm{sec}$ the singular values plot of $G_{\mathsf{rf}}(e^{\mathsf{j}\theta\mathsf{T}}), G_{\mathsf{rd}}(e^{\mathsf{j}\theta\mathsf{T}})$ and $G_{\mathsf{ru}}(e^{\mathsf{j}\theta\mathsf{T}})$ are given in Figure 2, Figure 3 and Figure 4 respectively.

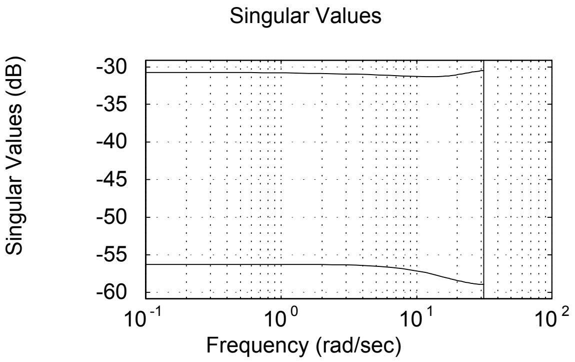

Fig. 2. the singular values plot of $G_{\mathsf{rf}}(e^{\mathsf{j}\theta\mathsf{T}})$;

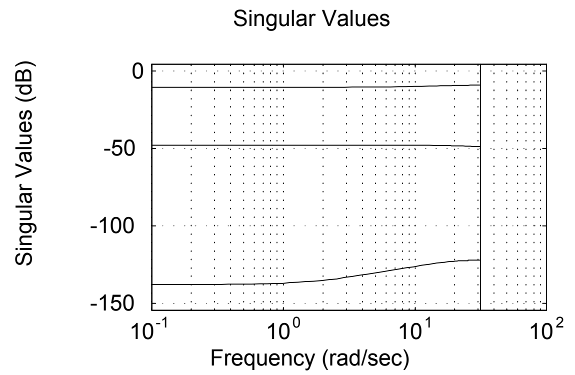

Fig. 3. the singular values plot of $G_{\mathsf{rd}}(e^{\mathsf{j}\theta\mathsf{T}})$;

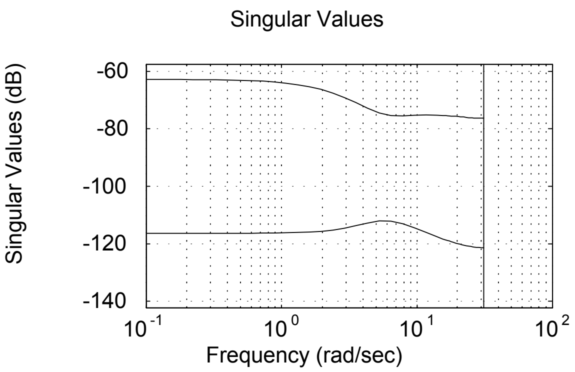

Fig. 4. the singular values plot of $G_{\mathsf{ru}}(e^{\mathsf{j}\theta\mathsf{T}})$;

5. CONCLUSION

This contribution dealt with the design of robust fault detection filters for discrete-time LTI systems with both unknown input and modelling errors. A design scheme has been proposed, which consists of two major steps: design of a reference model and formulation of the design problem as an $H_\infty-$model matching problem. For the solution, a method of designing the reference model has been developed; also, an LMI approach to solve the formulated $H_\infty-$ model matching problem has been demonstrated. An illustrated example has finally been used to illustrate the proposed scheme.

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