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SCHEDULING AND DESIGN FOR COST MINIMIZATION OF MULTIPURPOSE BATCH PLANT

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Abstracts: In this paper, multipurpose scheduling and design problems are treated simultaneously in order to minimize investment cost of plant with taking into account completion times, cycle times and equipment sizes. Usually, equipment sizes are considered as continuous variable. And processing time of task is usually assumed to be independent of batch size. However, most of manufactured equipments have their standard volume. And processing times are dependent on batch sizes. In this research, the volumes of equipments are considered as discrete variables and processing time is a function of batch size in order to apply these optimization problems to real industries. *Copyright © 2002 IFAC*

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1. INTRODUCTION

In recent years, customer demands are getting diverse and life cycles of products becoming shorter. So multiproduct and multipurpose batch plants are interested in the industries, which are suitable for flexibility of plants.

There are three categories of approaches for the batch plant optimization. Short term scheduling determines completion times of tasks in equipment units which minimize makespan. Operations management of production within time horizon is determined by long term planning. And design problem determines plant configuration in order to produce required products with minimum investment cost. It selects equipment types, their sizes, pipe network and so on. Many papers that treat scheduling, planning or design were published. However, there are only a few papers that treat design and scheduling or design and planning simultaneously. Plant configuration must be previously determined for the purpose of process scheduling and the scheduling is to be determined for the plant configuration. Suhami and Mah (1982) developed the first formulation for design problem with MINLP(Mixed Integer Nonlinear Programming). Papageorgaki and Reklaitis (1990) decomposed the MINLP problem. MILP(Mixed Integer Linear Programming) is a master problem, which determines the values of the binary assignment variables for fixed campaign lengths, and an NLP subproblem, which determines equipment sizes and the values of the campaign lengths. Voudouris and Grossmann (1992) modified MINLP to MILP introducing discrete equipment sizes. Fuchino, *et al*. (1994) decomposed the multipurpose problem into plant configuration and cyclic scheduling. They suggested *Evolutionary Design Method*, which finds an initial plant configuration and increases the number of equipments. Heo, *et al*. (2001) suggested three MILP models for scheduling and equipment sizing simultaneously with separable programming under assumption of continuous variable of volume.
However, manufacturing companies of equipments would manufacture them according to standardized sizes. So this research considers equipment units as discrete volumes. And processing times of tasks are assumed to be dependent on batch sizes. This paper intends that proposed mathematical approach could be applied to real industry.

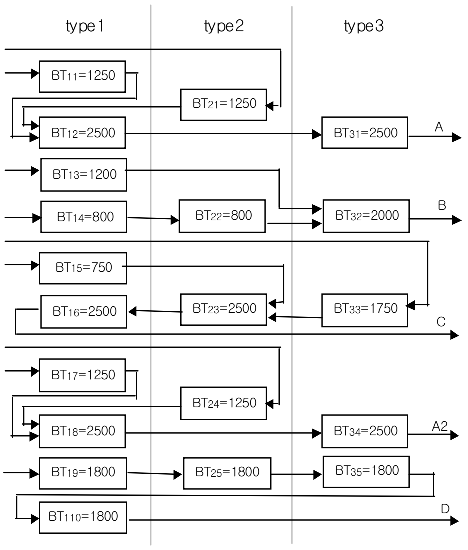

Figure 1. Recipe data on example 1
(BT_i: total batch size of the task i)

2. PROBLEM DESCRIPTION

Given data is (1) a set of N products and the requirement for each product within the available time horizon; (2) a set of available equipment units classified by equipment types according to their common usage of tasks; (3) recipe information for each product, a set of mean processing times and a corresponding set of size factors; (4) transfer times are fixed and a set of sequence dependent cleanup times; and (5) cost data.

Determinant variables are (a) a feasible equipment configuration and the number of units required for each equipment type; (b) sizes of processing units and amount of task operated in each equipment; and (c) starting and completion time of each task, so as to optimize the investment cost for the plant configuration besides satisfying the constraints on the production requirements and other constraints.

Intermediate storage policy is NIS(No Intermediate Storage) as like (Fuchino, *et al.*, 1994) solved. The common usage of equipment of same type is possible. Product requirements must be satisfied during time horizon. Total batch size of each task can be produced dividedly. However, relative volume of tasks among different types must be satisfied. Other assumptions are same as described by (Fuchino, *et al.*, 1994). Each task can be operated in different equipment unit of same type as in-phase operation. Each product is produced once in a cycle.

3. MILP MODEL FOR DETERMINING EQUIPMENT SIZES

The objective function of this problem is the total equipment cost which is a function of volumes. It must be determined which of equipment among several given volumes are introduced. Binary variable y_j means that equipment j is used or not. V_j is given parameter of discrete equipment volume.

$$\text{Minimize} \quad \sum_j y_j \alpha_j V_j^{\beta_j} \tag{1}$$

α_j and β_j is cost data according to equipment types. Usually, the larger is the batch size the longer become its processing time. Ierapetritou and Floudas (1998) represent task processing time as a linear function of the amount of batch.

$$PCT_i = a_i + b_i B_i\,, \quad \text{for } \forall i \tag{2}$$

$$a_i = \tfrac{2}{3} PT_i^{mean} \quad , \quad \text{for } \forall i \tag{3}$$

$$b_i = \frac{PT_i^{\max} - PT_i^{\min}}{B_i^{\max} - B_i^{\min}} \quad , \quad \text{for } \forall i \tag{4}$$

$$PT_i^{\max} = \tfrac{4}{3} PT_i^{mean}, \quad \text{for } \forall i \tag{5}$$

$$PT_i^{\min} = \tfrac{2}{3} PT_i^{mean}, \quad \text{for } \forall i \tag{6}$$

There are many cycles during time horizon, so total batch sizes can be divided by the number of cycles.

$$BT_i \frac{CT}{H} = B_i \quad , \quad \text{for } \forall i \tag{7}$$

Another binary variable W_{ij} is introduced, which is one if task i is processed in unit j. Binary variable $Z_{ipi'p'}$ is also used. When task i of product p is followed by task i' of product p' the value is one and zero when task i follows task i'. The number of these binary variables can be reduced by restraining the order of p and p', for example, if p is product B, p' could be only product C, then A is invalid. When Z is 1, product B is followed by product C. If zero, product C is followed by product B. Then, while the number of binary variables are reduced, similar constraints, i.e. (8') and (10'), are necessary.

Constraints for cyclic scheduling follow.

$$C_i + tr_i + CLT_{pp'} \le C_{i'} + tr_i - ht_{i'} - PCT_{i'} + M_2\left(2 - W_{ij} - W_{i'j}\right) + M_2\left(1 - Z_{pip'i'}\right)$$

, for $(i,j) \in IJ$, $(p,i) \in PI$ (8)

$$C_{i'} + tr_{i'} + CLT_{p'p} \le C_i + tr_i - ht_i - PCT_i + M_2\left(2 - W_{ij} - W_{i'j}\right) + M_2 Z_{pip'i'}$$

, for $(i,j) \in IJ$, $(p,i) \in PI$ (8')

$$C_i + tr_i = C_{i'} - ht_{i'} - PT_{i'}$$

, for $(i,i') \in II$ (9)

In these equations C_i implies completion time, tr_i transfer time, PCT_i processing time and ht_i means holding time of task i. If storage policy is ZW(Zero Wait), then holding time is fixed as zero. M_2 is a sufficiently large positive number. Equations (8) and

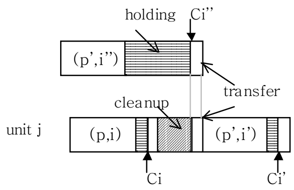

Figure 2. Schematic Gantt Chart for example

(8') mean that when different tasks are processed in the same unit *j* cleanup time must be taken into account. *II* is a set of two tasks that process directly next. *IJ* is a set of task and unit where the task is processed. *PI* is a set of product and its task.

$$C_{i''} \geq C_i + tr_i + CLT_{pp'} - M_2\left(2 - W_{ij} - W_{i'j}\right) - M_2\left(1 - Z_{pip'i'}\right) ,$$

for(i'',i')∈ II,(i,j),(i',j)∈ *IJ*,(p,i),(p',i'),(p',i'')∈ *PI* (10)

$$C_{i''} \geq C_{i'} + tr_{i'} + CLT_{p'p} - M_2\left(2 - W_{ij} - W_{i'j}\right) - M_2 Z_{pip'i'} ,$$

for(i'',i)∈ II,(i,j),(i',j)∈ *IJ*,(p,i),(p,i'') ,(p',i')∈ *PI* (10')

$$SLF_j \leq C_i - ht_i - PCT_i - tr_i + M_2\left(1 - W_{ij}\right) \quad (11)$$

$$SLL_j \leq MS - tr_i - C_i + M_2\left(1 - W_{ij}\right) ,$$

for (i,j)∈ *IJ* (12)

$$SLD \leq SLF_j + SLL_j \text{, for } \forall j \quad (13)$$

$$CT \geq MS - SLD + CLT_{pp'} \quad (14)$$

Equations (10) and (10') are required in NIS policy in that task *i'* which is completed in the direct former unit cannot transferred to unit *j* until previous task *i* is over. SLF_j is head and SLL_j is tail. The usage of these two variables is convenient in that makespan minus sum of the two terms represents cycle time.

$$MTD \cdot V_j \leq B_{ij} + M_1 \cdot \left(1 - W_{ij}\right) \text{, for } (i,j) \in IJ \quad (15)$$

$$B_{ij} \leq M_1 W_{ij} \text{, for } (i,j) \in IJ \quad (16)$$

$$B_{ij} \leq V_j + M_1\left(1 - W_{ij}\right) \text{, for } (i,j) \in IJ \quad (17)$$

$$y_j \geq W_{ij} \text{, for } (i,j) \in IJ \quad (18)$$

In the equation (15) *MTD* means minimum operating capacity. When a task is processed in unit its batch size must be over minimum operating capacity. B_{ij} batch size and M_1 is a sufficiently large positive number.

This MILP model consists of objective function, equation (1) and constraints, equation (2)-(18). This model is coded with AMPL(Robert, *et al.*, 1993) and solved by CPLEX 7.0.0, hardware is Pentium III 1000MHz PC.

4. EXAMPLES AND RESULTS

4.1 EXAMPLE 1

In the example 1, sizes of equipment of type 1 are 3, 5, 10, 15, 20, 30, type 2 and 3 are 5, 10, 15, 20, 30. And minimum operating ratio is 50%. Other data is given in Figure 1, Table 1 and 2. Transfer time is 0.05. Cost data of α is 250, 350 and 300 with respect to their types and β is 0.6 for all types. Available time horizon is 300.
Solutions are obtained as the minimum cost of 10,612 at cycle time of 2.4 although the minimum cycle time

Table 1. Mean processing time of example 1

task *i*	PT_i	task *i*	PT_i
i11	0.2	i21	0.25
i12	0.15	i22	0.25
i13	0.2	i23	0.25
i14	0.15	i24	0.25
i15	0.15	i25	0.25
i16	0.15	i31	0.2
i17	0.2	i32	0.15
i18	0.15	i33	0.15
i19	0.15	i34	0.2
i110	0.15	i35	0.2

Table 2. Sequence dependent cleanup time of example 1

From/To	A	A2	B	C	D
A	-	-	0.05	0.1	0.1
A2	-	-	0.05	0.1	0.1
B	0.1	0.1	-	0.15	0.1
C	0.05	0.05	0.15	-	0.1
D	0.15	0.15	0.1	0.05	-

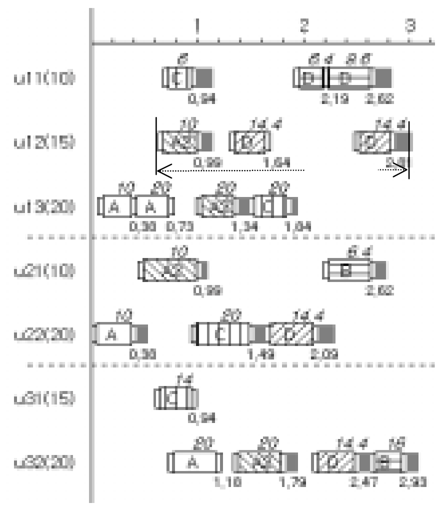

Figure 3. Result of example 1 (CT=2.4, dot line: type, number in bracket: volume of equipment, italic number: batch size, small number: completion time)

is 1.5. Figure 3 shows this result. Gantt chart is illustrated in Figure 3. There are 125 cycles (300/2.4) within the time horizon.

4.2 EXAMPLE 2

In the example 2, sizes of type1 are 10, 20, 30, 45 and 50, 10, 20, 35, 40 and 50 for type 2, 10, 20, 30, 40 and 50, 10, 20, 30, 40 and 50 for type 3 and 5 and 10, 15, 30, 40 and 50 for equipment type 4. Other data is given in Figure 4, Table 3 and 4. Transfer time is 0.5. Cost data of α is 250, 350, 300, 400 and 350. Available time horizon is 800.

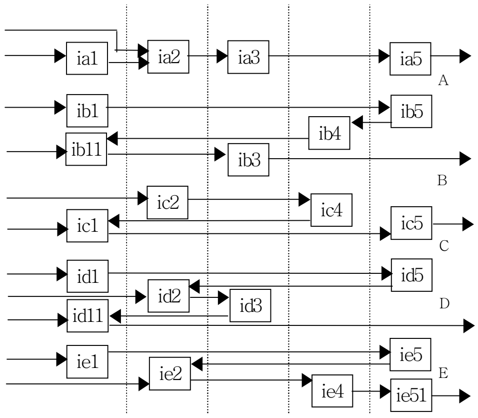

Figure 4. Recipe data on example 2

Table 3. Mean processing time and total batch size of example 2

task *i*	PT_i	BT_i	task *i*	PT_i	BT_i
ia1	3	450	ic5	5	1000
ia2	5	900	id1	3	500
ia3	2	900	id11	7	1200
ia5	6	900	id2	6	1000
ib1	5	400	id3	4	1000
ib11	4	600	id5	4	500
ib3	2	600	ie1	7	200
ib4	3	400	ie2	6	800
ib5	3	400	ie4	8	800
ic1	4	1000	ie5	4	200
ic2	2	750	ie51	5	800
ic4	2	750			

Table 4. Sequence dependent cleanup time of example 2

From/To	A	B	C	D	E
A	-	1.5	1	0.5	1
B	1	-	1.5	1	1
C	0.5	1	-	1.5	0.5
D	1	1	0.5	-	1
E	2	1.5	0.5	1	-

The minimum cost is 31,329.627 at cycle time 26.67 although minimum cycle time is 23. Details are illustrated in Figure 5. There are 30 cycles (800/26.67) within the time horizon.

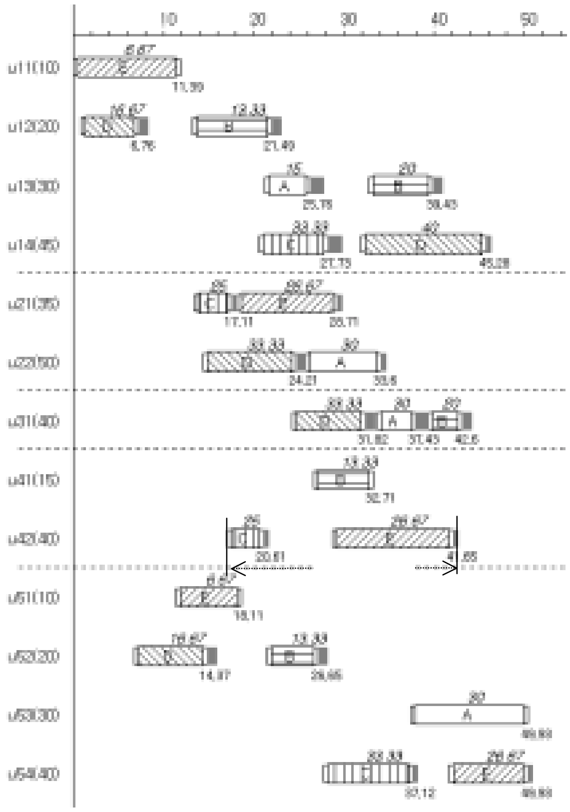

Figure 5. Result of example 2 (CT=26.67)

5. CONCLUSIONS

Design parameters or variables are very interactive with each other, so the problem sizes are usually large. Parameters of scheduling and plant configuration are usually so interactive that the schedule must be previously determined to set the plant configuration. And the plant configuration has to be known for the sake of scheduling. Therefore, it is known that simultaneous treatment of both categories is so complicated.

In this study, scheduling and equipment sizing are considered simultaneously. Sizes of chemical process equipments like reactor, mixer, column or condenser and so on are assumed discrete variables. The proposed model gives the solution of the plant configuration at once, while the former researches, i.e. (Fuchino, *et al.*, 1994) and (Heo, *et al.*, 2000), proposed two or three models in sequence. And MILP model is suggested by expression of processing time as a function of batch size. Usually, there are many assumptions in order to express real process as mathematical model. On the contrary, it may obtain unreasonable solutions. In that point, this approach is expected to contribute the real industries.

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