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ROBUST ESTIMATION OF MECHANICAL VARIABLES IN PERMANENT MAGNET SYNCHRONOUS MOTORS

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Abstract: This paper deals with rotor position and speed estimation of permanent magnet synchronous motors. A reduced order observer based on back electromotive force estimation is proposed for estimating mechanical variables. Robustness against model uncertainty is analyzed. Under exact model assumption the estimation error converges to zero in exponential way, while a residual estimation error appears in presence of uncertainties. For this reason, estimation error bounds are calculated when uncertainty in both the mechanical and the electrical submodels are taken into account. The observer performance is tested through simulations.

Keywords: electrical machines, robust estimation, observers

1. INTRODUCTION

Recently, a great deal of attention has been drawn for improving the performance of electrical drives that use asynchronous and synchronous motors. Advancements in the technology of magnetic materials, electronic devices and VLSI circuits provide the tools for designing high performance adjustable speed drives (Novotny and Lipo, 1996). These drives need a precise knowledge of rotor position to synchronize the stator currents with the rotor. For different reasons, in some applications it is desirable to avoid mechanical sensors. In such drives rotor position and speed must be estimated and the estimated values used to compute the control law. Rotor position and speed of permanent magnet synchronous motors (PMSMs) may be estimated with observers based upon measurements of the electrical variables of the motor. Different approaches for estimating PMSMs state variables can be found in the literature (Johnson *et al.*, 1999). Among other techniques, nonlinear full order observers were employed for speed estimation and then rotor position was obtained integrating the speed estimate in open-loop (Jones and Lang, 1989; Low *et al.*, 1993). An algorithm for estimating flux and current by the integration of differential equations was proposed by Ertugrul and Acarnley (Ertugrul and Acarnley, 1994). An open loop model of the motor back electromotive force (EMF) under electrical steady-state operation is considered by Kim and Sul (Kim and Sul, 1997) for rotor position and speed estimation. In other works (Bolognani *et al.*, 1999; Dhaouadi *et al.*, 1991), the Extended Kalman Filter (EKF) was used for obtaining speed and rotor position

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estimates. Nevertheless, it must be noted that reduced order observers are a good alternative for decreasing the computational burden. Several researchers have developed algorithms based on reduced order observers. Among others, Shouse and Taylor proposed a design using singular perturbation theory (Shouse and Taylor, 1998); whereas PLL theory was used by Harnefors and Nee (Harnefors and Nee, 2000). In other approaches (Orlowska-Kowalska, 1998; Tomita *et al.*, 1998; Solsona *et al.*, 1996), linear and nonlinear reduced order observers were used for estimating the EMF and rotor position and speed estimates were obtained from the relationship between EMF and rotor variables. In these cases, the EMF must be estimated with low error, since the EMF estimation error is propagated to rotor variables. Usually, EMF estimation errors appear when the motor model is not exactly known. In such a case, bounds for estimation errors can be calculated assuming that uncertainties are bounded. These bounds guarantee that the rotor position and speed estimation error is lesser than a value depending on the uncertainty bound.

In this paper, rotor position and speed estimation of PMSMs taking into account parameters uncertainties is considered. The reduced order observer based on the nominal model proposed by Solsona *et al.*, 1996 is analyzed in presence of uncertainty. In addition, a bound for estimation error is calculated and it is illustrated how the introduced formulation can be used for analyzing several observers which were presented in other works. The paper is organized as follows. In section 2, the motor model and the observer are reviewed. In section 3, an estimation error bound is calculated. In section 4, some implementation aspects are considered. In section 5, the observer performance in presence of uncertainty is illustrated. Finally, in section 6 conclusions are drawn.

2. POSITION AND SPEED ESTIMATION

2.1 *Motor model*

The motor is described in a stationary two-axes reference frame, and the mechanical variables are converted to electrical angles. The machine model can be written as follows:

$$\dot{\theta}_{re} = \omega_{re} \tag{1}$$

$$J\dot{\omega}_{re} = K_T\left[-i_\alpha \sin\theta_{re} + i_\beta \cos\theta_{re}\right] - B\omega_{re} \tag{2}$$

$$L\dot{i}_\alpha = -R\, i_\alpha + K_E\, \omega_{re} \sin\theta_{re} + v_\alpha \tag{3}$$

$$L\dot{i}_\beta = -R\, i_\beta - K_E\, \omega_{re} \cos\theta_{re} + v_\beta \tag{4}$$

where i_α, i_β and v_α, v_β are currents and voltages in the stationary two-axes reference frame. The electrical parameters R, L, and K_E are resistance, inductance and EMF constant, respectively. The mechanical variables and parameters, θ_{re}, ω_{re}, B, J, and K_T, are rotor position and rotor speed in electrical radians, viscosity, inertia and torque constant, respectively.

2.2 *The proposed observer*

Taking into account the motor electrical submodel ((3) and (4)), it is clear that the rotor position and speed information is contained in the back electromotive force terms (f_α and f_β), given by:

$$f_\alpha = -K_E\, \omega_{re} \sin\theta_{re} \tag{5}$$

$$f_\beta = K_E\, \omega_{re} \cos\theta_{re} \tag{6}$$

Therefore, the EMF time derivatives become:

$$\dot{f}_\alpha = -K_E[\dot{\omega}_{re} \sin\theta_{re} + \omega_{re}^2 \cos\theta_{re}] \tag{7}$$

$$\dot{f}_\beta = K_E[\dot{\omega}_{re} \cos\theta_{re} - \omega_{re}^2 \sin\theta_{re}] \tag{8}$$

By using (2), (5) and (6), EMF time derivatives ((7) and (8)) can be written as follows:

$$J\dot{f}_\alpha = \frac{K_T K_E f_\alpha}{f_\alpha^2 + f_\beta^2}\left[i_\alpha f_\alpha + i_\beta f_\beta\right] - \frac{J}{K_E} f_\beta \sqrt{f_\alpha^2 + f_\beta^2} - B f_\alpha \tag{9}$$

$$J\dot{f}_\beta = \frac{K_T K_E f_\beta}{f_\alpha^2 + f_\beta^2}\left[i_\alpha f_\alpha + i_\beta f_\beta\right] + \frac{J}{K_E} f_\alpha \sqrt{f_\alpha^2 + f_\beta^2} - B f_\beta \tag{10}$$

A nonlinear observer can be designed for estimating EMF terms (f_α and f_β). The observer obtains the dynamic equations for the EMF from (9) and (10) and adds correction terms using stator current measurements. Therefore, observer equations are given by:

$$\dot{\hat{f}}_\alpha = \frac{K_{T_0} K_{E_0} \hat{f}_\alpha}{J_0(\hat{f}_\alpha^2 + \hat{f}_\beta^2)}\left[i_\alpha \hat{f}_\alpha + i_\beta \hat{f}_\beta\right] - \frac{1}{K_{E_0}} \hat{f}_\beta \sqrt{\hat{f}_\alpha^2 + \hat{f}_\beta^2} - \frac{B_0}{J_0}\hat{f}_\alpha + L_0\, g\,(\dot{\hat{i}}_\alpha - \dot{i}_\alpha) \tag{11}$$

$$\dot{\hat{f}}_\beta = \frac{K_{T_0} K_{E_0} \hat{f}_\beta}{J_0(\hat{f}_\alpha^2 + \hat{f}_\beta^2)}\left[i_\alpha \hat{f}_\alpha + i_\beta \hat{f}_\beta\right] + \frac{1}{K_{E_0}} \hat{f}_\alpha \sqrt{\hat{f}_\alpha^2 + \hat{f}_\beta^2} - \frac{B_0}{J_0}\hat{f}_\beta + L_0\, g\,(\dot{\hat{i}}_\beta - \dot{i}_\beta) \tag{12}$$

with

$$L_0 \dot{\hat{i}}_\alpha = -R_0\, i_\alpha - \hat{f}_\alpha + v_\alpha \tag{13}$$

$$L_0 \dot{\hat{i}}_\beta = -R_0\, i_\beta - \hat{f}_\beta + v_\beta \tag{14}$$

Mismatches between machine model and observer parameters are used for representing uncertainties. Thus, subscript $_0$ stands for observer parameters. The constant value g can be chosen to guarantee that the estimation error converges semiglobally to zero in exponential way when the model is perfectly known (Solsona *et al.*, 1996).

The rotor position and speed can be reconstructed from EMF estimates as follows:

$$\hat{\theta}_{re} = tg^{-1}\left(\frac{-\hat{f}_\alpha}{\hat{f}_\beta}\right) \tag{15}$$

$$\hat{\omega}_{re} = \frac{1}{K_{E_0}}\sqrt{\hat{f}_\alpha^2 + \hat{f}_\beta^2} \tag{16}$$

It must be noted that the points $(\theta_{re}, \omega_{re})$ and $(\theta_{re} + (2k+1)\pi, -\omega_{re})$ are mapped to the same point (f_α, f_β). Due to this fact, these pairs cannot be distinguished. In particular, when ω_{re} is equal to zero, some method based on irregularities can be used (see Harnefors and Nee, 2000 and references therein) for estimating the rotor position and speed.

The following vectors are defined in order to calculate the estimation errors and their bounds in a compact form. Let μ, $\hat{\mu}_0$ and r be

$$\mu = [m_\alpha/J \qquad m_\beta/J]^T \tag{17}$$

$$\hat{\mu}_0 = [\hat{m}_{\alpha_0}/J_0 \qquad \hat{m}_{\beta_0}/J_0]^T \tag{18}$$

$$r = [r_\alpha \qquad r_\beta]^T \tag{19}$$

$$r_\alpha = i_\alpha(R - R_0) + (L - L_0)\dot{i}_\alpha \tag{20}$$

$$r_\beta = i_\beta(R - R_0) + (L - L_0)\dot{i}_\beta \tag{21}$$

$$m_\alpha = \frac{K_T K_E f_\alpha}{f_\alpha^2 + f_\beta^2}\left[i_\alpha f_\alpha + i_\beta f_\beta\right] - \\ - \frac{J}{K_E} f_\beta \sqrt{f_\alpha^2 + f_\beta^2} - B f_\alpha \tag{22}$$

$$m_\beta = \frac{K_T K_E f_\beta}{f_\alpha^2 + f_\beta^2}\left[i_\alpha f_\alpha + i_\beta f_\beta\right] + \\ + \frac{J}{K_E} f_\alpha \sqrt{f_\alpha^2 + f_\beta^2} - B f_\beta \tag{23}$$

$$\hat{m}_{\alpha_0} = \frac{K_{T_0} K_{E_0} \hat{f}_\alpha}{\hat{f}_\alpha^2 + \hat{f}_\beta^2}\left[i_\alpha \hat{f}_\alpha + i_\beta \hat{f}_\beta\right] - \\ - \frac{J_0}{K_{E_0}} \hat{f}_\beta \sqrt{\hat{f}_\alpha^2 + \hat{f}_\beta^2} - B_0 \hat{f}_\alpha \tag{24}$$

$$\hat{m}_{\beta_0} = \frac{K_{T_0} K_{E_0} \hat{f}_\beta}{\hat{f}_\alpha^2 + \hat{f}_\beta^2}\left[i_\alpha \hat{f}_\alpha + i_\beta \hat{f}_\beta\right] + \\ + \frac{J_0}{K_{E_0}} \hat{f}_\alpha \sqrt{\hat{f}_\alpha^2 + \hat{f}_\beta^2} - B_0 \hat{f}_\beta \tag{25}$$

It must be remarked that linear observers proposed in other works (Orlowska-Kowalska, 1998; Tomita *et al.*, 1998) coincide with the proposed observer when $\hat{\mu}_0 = 0$ (*i.e.* prediction equals to zero). Note that in (Harnefors and Nee, 2000) acceleration prediction is equal to zero as well; whereas an observer based on singular perturbation theory (Shouse and Taylor, 1998) can be obtained by setting $L_0 = 0$.

3. A BOUND FOR THE ESTIMATION ERROR

Let e be the EMF estimation error. Therefore, $e = [e_\alpha \quad e_\beta]^T = [\hat{f}_\alpha - f_\alpha \quad \hat{f}_\beta - f_\beta]^T$. Consider the following Lyapunov candidate function $V = e^T e$. Therefore, the time derivative of Lyapunov candidate function is given by:

$$\dot{V} = \dot{e}^T e + e^T \dot{e} \tag{26}$$

This function can be bounded as follows:

$$\dot{V} \leq -g||e||^2 + ||\hat{\mu}_0 - \mu||\,||e|| + g||r||\,||e|| \tag{27}$$

If variables evolve in a compact set so that

$$||\hat{\mu}_0 - \mu|| \leq \lambda||e|| + M \tag{28}$$

$$||r|| \leq \rho \tag{29}$$

then the following inequality is obtained from (27):

$$\dot{V} \leq -g||e||^2 + g\rho||e|| + \lambda||e||^2 + M||e|| \tag{30}$$

so that,

$$\frac{\dot{V}}{2\sqrt{V}} \leq \left(\frac{-g+\lambda}{2}\right)\sqrt{V} + \frac{g\rho + M}{2} \tag{31}$$

Denoting $\gamma = \frac{g-\lambda}{2} > 0$, recalling that g is an arbitrary value and given that $\sigma = \frac{g\rho+M}{2} > 0$,

$$\frac{d\sqrt{V}}{dt} \leq -\gamma\sqrt{V} + \sigma \tag{32}$$

then

$$\sqrt{V} \leq \varepsilon^{-\gamma t}\sqrt{V(0)} + \int_0^t \varepsilon^{-\gamma(t-\tau)}\sigma d\tau \tag{33}$$

Integrating the right-hand side from (33) the following bound is obtained:

$$||e|| \leq \varepsilon^{-\gamma t}||e(0)|| + \frac{\sigma}{\gamma} = z(t) \leq \eta \tag{34}$$

where η is the maximum of $z(t)$.

Observers based on simplified models have been presented in other works (Shouse and Taylor, 1998; Tomita *et al.*, 1998; Orlowska-Kowalska, 1998; Harnefors and Nee, 2000). The model simplifications can be modeled as uncertainties such as remarked in section 2. Then, an estimation error bound can be obtained in each case. In addition, it must be remarked that when uncertainty in the electrical submodel is equal to zero (*i.e.* $\rho = 0$), a big value of g can be chosen for diminishing the bound of the EMF estimation error. However, if the uncertainty in the electrical submodel is not equal to zero, then g value will be chosen taking into account the uncertainty in both the electrical submodel and the mechanical one.

EMF errors are propagated to mechanical variables as follows:

$$e_{\omega_{re}} = -\omega_{re} + \omega_{re}\left[\frac{K_E}{K_{E_o}}\sqrt{1+\frac{2(f_\alpha\, e_\alpha + f_\beta\, e_\beta) + e_\alpha^2 + e_\beta^2}{K_E^2\,\omega_{re}^2}}\right] \quad (35)$$

$$e_{\theta_{re}} = tg^{-1}\left[\frac{e_\beta\, f_\alpha - e_\alpha\, f_\beta}{(K_E\,\omega_{re})^2 + f_\alpha e_\alpha + f_\beta\, e_\beta}\right] \quad (36)$$

Bounds for rotor position and speed estimates can be obtained from (35) and (36) as function of EMF estimation bound. They result in:

$$|e_{\omega_{re}}| \leq |\omega_{re}|\left[\frac{K_E}{K_{E_o}}\sqrt{1+\frac{2||f||\eta + \eta^2}{K_E^2\omega_{re}^2}} - 1\right] \quad (37)$$

$$|tg(e_{re})| \leq \frac{||f||\eta}{K_E^2\omega_{re}^2 - ||f||\eta} \quad (38)$$

4. IMPLEMENTATION ASPECTS

In the implementation of the observer, mechanical variables in mechanical degrees are considered. For this reason, those in electrical degrees are divided by the number of pole pairs (p). In addition, in order to avoid taking derivatives of the measurements, the following equations are actually implemented for the observer:

$$\dot{\nu}_1 = \frac{p^2 K_{T_0}}{J_0\, K_{E_0}}\,\frac{\hat{f}_\alpha\left(i_\alpha\,\hat{f}_\alpha + i_\beta \hat{f}_\beta\right)}{\hat{\omega}^2} - \frac{B_0}{J_0}\hat{f}_\alpha - \frac{\hat{f}_\beta\,\hat{\omega}}{p} + g\left(-R_0\, i_\alpha - \hat{f}_\alpha + v_\alpha\right) \quad (39)$$

$$\dot{\nu}_2 = \frac{p^2 K_{T_0}}{J_0\, K_{E_0}}\,\frac{\hat{f}_\beta\left(i_\alpha\,\hat{f}_\alpha + i_\beta \hat{f}_\beta\right)}{\hat{\omega}^2} - \frac{B_0}{J_0}\hat{f}_\beta + \frac{\hat{f}_\alpha\,\hat{\omega}}{p} + g\left(-R_0\, i_\beta - \hat{f}_\beta + v_\beta\right) \quad (40)$$

$$\hat{f}_\alpha = \nu_1 - L_0\, g\, i_\alpha \quad (41)$$

$$\hat{f}_\beta = \nu_2 - L_0\, g\, i_\beta \quad (42)$$

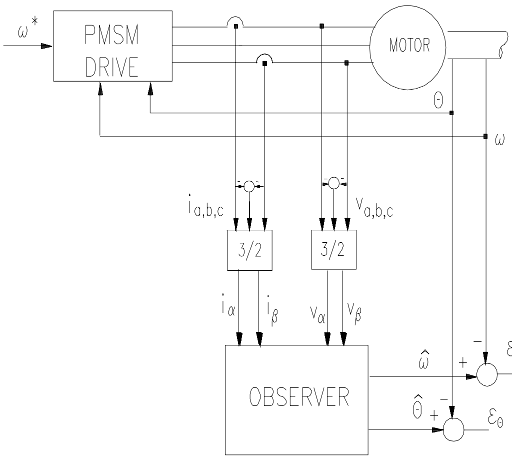

Fig. 1. Block diagram

$$\hat{\omega} = \frac{\sqrt{\hat{f}_\alpha^2 + \hat{f}_\beta^2}}{pK_{E_0}} \quad (43)$$

$$\hat{\theta} = \frac{1}{p}\; tg^{-1}\left(\frac{-\hat{f}_\alpha}{\hat{f}_\beta}\right) \quad (44)$$

5. SIMULATION RESULTS

A model of the PMSM is built and simulated. The observer measures voltages and currents of this model and performs the estimation in open loop. The rotor position and speed errors are built comparing the estimated values of the observer against the rotor position and speed of the simulated drive as shown in the block diagram of Fig. 1. The data and parameters of the motor are $P_N = 0.75\,kW$, $\Omega_N = 2000\,rpm$, $Pole\ pairs = 3$, $L = 4.5\,mHy$, $R = 2.63\,\Omega$, $K_E = 0.156\,V.s/rad$, $K_T = 0.81\,Nm/A$, $J = 28.5\,10^{-4}\,kg.m^2$, $B = 0.01\,kg.m^2/sec$. A gain g equal to 400 is used.

First, only mechanical uncertainties are considered. The parameters of the observer were set to $J_o = 5.7\,10^{-4}\,kg.m^2$, $B_o = 0.0005\,kg.m^2/sec$. In this way, big uncetainties are considered since J_o is

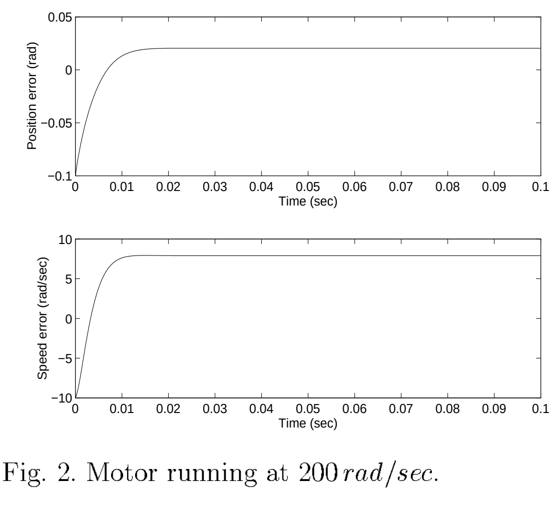

Fig. 2. Motor running at $200\,rad/sec$.

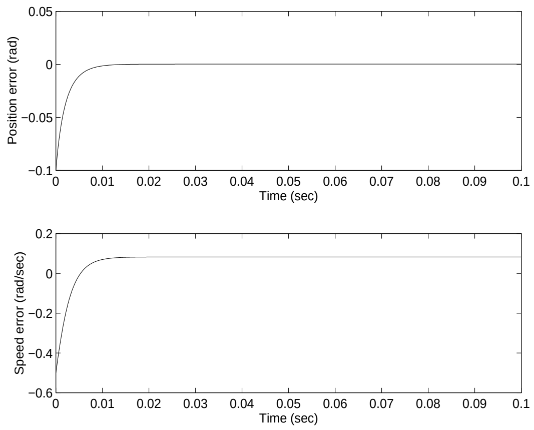

Fig. 3. Motor running at $2\,rad/sec$.

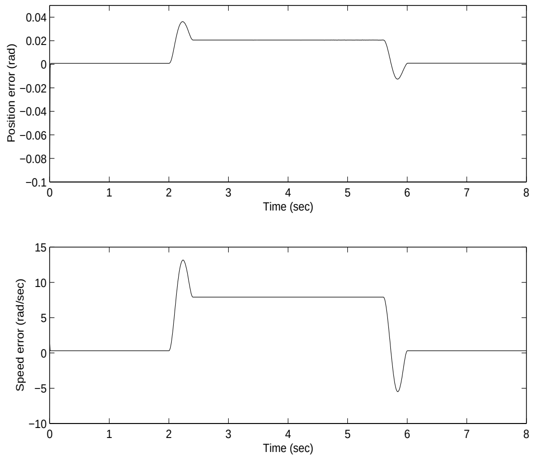

Fig. 4. Performance for varying speed.

5 times smaller than the actual value and B_o is 20 times smaller than the actual B. Besides, there is a big difference between the actual and modelled mechanical time constants. Figs. 2 and 3 show the transient behavior for the motor running at two constant speeds; nominal speed ($200\,rad/sec$), and low speed ($2\,rad/sec$), respectively. The observer presents an error smaller than $5\,\%$ of the running speed in the whole speed range. Besides, the position error is smaller than 0.02 rad. These results show that the proposed observer have an acceptable performance even when the mechanical submodel is practically unknown.

Next, the observer performance during transient operation of the drive is shown in Fig.4, using the same parameters as in the previous test. A reference speed profile is applied to the drive. The motor is accelerated from 10 $rad/sec.$ to 200 $rad/sec.$ (nominal speed) and then is decreased again to 10 $rad/sec.$ Fig. 4a depicts rotor position error, while Fig. 4b shows the speed error. The errors present some overshoot during acceleration and braking, mainly due to mismatch in the inertia coefficient.

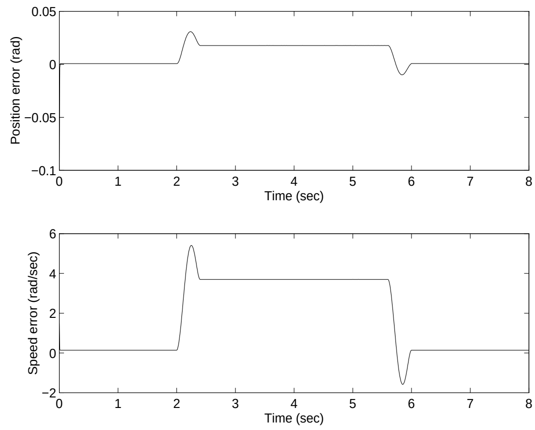

Fig. 5. Performance for varying speed.

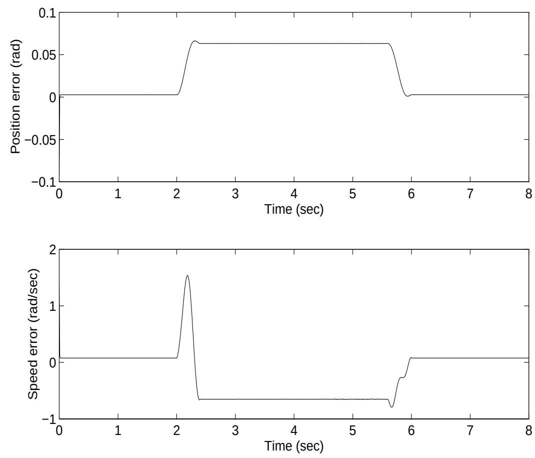

Fig. 6. Performance for varying speed.

Then, mechanical and electrical uncertainties are considered together. The observer parameters were set to $J_o = 5.7\,10^{-4}\,kg.m^2$, $B_o = 0.0005\,kg.m^2/sec$, $L_o = 3\,mHy$, $R_o = 3.5\,\Omega$. Fig. 5a and Fig. 5b show rotor position and speed errors when the previous speed profile is applied to the motor. Comparing Fig. 4 and Fig. 5, it is evident that the observer is rather insensitive to uncertainties in R and L.

Finally, uncertainties in the motor torque constant is also considered and the results are shown in Fig. 6. The observer parameters are given by: $J_o = 5.7\,10^{-4}\,kg.m^2$, $B_o = 0.0005\,kg.m^2/sec$, $L_o = 3\,mHy$, $R_o = 3.5\,\Omega$, $K_{E_0} = 0.14\,V.s/rad$, $K_{T_0} = 0.73\,Nm/A$.

6. CONCLUSIONS

A nonlinear observer for estimating rotor position and speed in PMSMs has been analyzed. Uncertainties in mechanical and electrical parameters have been considered. In absence of uncertainties, the EMF estimation error converges to zero semiglobally in exponential way. By assuming bounded uncertainties in the model describing the motor, a bound for estimation errors has been calculated. The observer behavior was tested through simulations showing a good performance even with big mismatches in the mechanical submodel. In addition, the bound calculated in section 3 can be extended to several observers based on simplified models.

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