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GEAR FAULT DIAGNOSIS USING CYCLIC BISPECTRUM

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Abstract: This paper introduces the theory of cyclic statistics as a powerful tool for the diagnosis of gear faults. More precisely, a new method based on the cyclic bispectrum, a third order cyclic statistical function, is used for monitoring. This estimator furnishes a valuable means of detecting and characterising non-linear coupling effects as well as any periodic correlation between different components in machines. Therefore, the cyclic bispectrum provides much more combined information than classical methods such as spectrum and cepstrum analysis. Application to the diagnosis of spalling of the gear teeth of the U.S. Navy helicopter demonstrates the effectiveness of this new parameter for a good diagnosis. *Copyright © 2002 IFAC*

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1. INTRODUCTION

This paper is concerned with the development of signal processing methods to perform an early diagnosis of gears faults using vibration signals. Until now, in signal processing, most established methods often rely on a fundamental assumption of stationarity and ergodicity of the processes involved. These notions are appealing because they give the possibility of estimating parameters from a single realisation. However, this assumption is a mathematical idealisation which, in some case, may be valid only as an approximation to the real situation. Thus, it can exclude many real-life non stationary signals. More particularly, there is a subclass of non stationary signals called cyclostationary signals. These signals are characterised by a periodic variation of their statistical parameters. The importance of this class is that it matches the physical behaviour of gear vibration signals (Cadpessus, *et al.,* 2000). For analysing such signals, several techniques were applied such as spectral analysis, cepstrum (Randall, 1975), time frequency analysis (Wang and McFadden, 1993) and higher order statistics (Nikias and Raghuveer, 1987; Hinish, 1994).

The theory of estimation of periodically correlated processes, i.e. second order cyclostationary, was introduced first by Hurd (1970) and exploited with success in several domains especially in the diagnosis of gears faults (Capdessus, 1992; Capdessus, 2000; Bouillaut, 2000). For higher orders, the general theory of cyclic statistics has been developed in both the stochastic and fraction of time (FOT) probability frameworks. An important statistical parameter in the study of cyclostationarity properties is the kth-order cyclic polyspectrum. Estimators for this cyclic polyspectrum have been proposed by Gardner (1994a, b) and by Giannakis and Dandawate (1994) for continuous time and discrete time signals respectively. In Gardner (1994a), the whole study is based within a deterministic framework by using the fraction of time probability. Alternatively, Giannakis and Dandawate (1994) built estimators for real signals in a stochastic framework. Their estimation depends primarily on the generalisation of the kth-order periodogram suggested by Brillinger and Rosenblatt (1967a, b). Applications of higher order cyclic statistical signals are limited to a few areas. Spooner and Napolitano (2001) recently proposed an application of cyclic analysis to the estimation of parameters of modulated signals.

This paper addresses an application of cyclic statistics for the diagnosis of tooth spalling in a gearbox. This application is based on the estimation of the third order cyclic polyspectrum, often called the cyclic bispectrum. The paper is organised as follows. After presenting the major definitions and properties of cyclic statistics in section 2, methods of estimation of the cyclic bispectrum are proposed in section 3 To illustrate these methods, simulated signals are used in order to compare the results to the theory. Finally, an application to industrial vibration signals recorded on a U.S. Navy helicopter is provided and discussed in section 4. Conclusions are drawn in section 5.

2. HIGHER ORDER CYCLIC STATISTICS

In this paper, the theory of Giannakis and Dandawate (1994) will be used and reviewed in the next two paragraphs.

2.1 Main definitions

Let us consider a cyclostationary process $x(t)$ where $C_{kx}(t,\boldsymbol{\tau})$ and $m_{kx}(t,\boldsymbol{\tau})$ represent respectively its kth-order cumulant and moment with $\boldsymbol{\tau}=(\tau_1,...,\tau_{k-1})$. More details about cumulants and cyclostationarity are given in (Lacoume, *et al.*,1997).

If $C_{kx}(t,\boldsymbol{\tau})$ has a Fourier series representation with respect to t , then

$$C_{kx}(t,\boldsymbol{\tau}) = \sum_{\alpha\in\chi_k} c_{kx}(\alpha,\boldsymbol{\tau})e^{2\pi j\alpha t} \;;$$

$$c_{kx}(\alpha,\boldsymbol{\tau}) = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} C_{kx}(t,\boldsymbol{\tau})e^{-2\pi j\alpha t} \quad (1)$$

with $\chi_k = \{\ \alpha / c_{kx}(\alpha,\boldsymbol{\tau})\neq 0, 0\leq\alpha\leq 1\ \}$.

The Fourier coefficients $c_{kx}(\alpha,\boldsymbol{\tau})$ are called the kth-order cyclic cumulants of $x(t)$ and α is known as cyclic frequency. These coefficients are time-invariant, and thus can be estimated by using only one realisation. Single record estimation of statistical parameters is very significant in practice.

Kth-order cyclic cumulant spectra can be obtained by using kth-order cyclic cumulants as follows :

$$S_{kx}(\alpha,\mathbf{f}) = \sum_{\tau} c_{kx}(\alpha,\boldsymbol{\tau})e^{-2\pi j\,\mathbf{f}\,\boldsymbol{\tau}} \quad (2)$$

with $\mathbf{f}=(f_1,\dots,f_{k-1})$. Cyclic cumulant spectra can also be called spectral multicorrelations (Lacoume, *et al.* , 1997) or kth-order cyclic polyspectra. The same procedure is applicable to the kth-order moment $m_{kx}(\alpha,\boldsymbol{\tau})$.

2.2 Estimation of kth-order cyclic cumulant spectra

The Kth-order cyclic periodogram of $x(t)$ is defined as:

$$I_{kx}^{(T)}(\alpha; f_0,\dots,f_{k-1}) = \frac{1}{T}X_T(f_0)\dots X_T(f_{k-1}) \quad (3)$$

with $X_T(f)=\sum_{t=0}^{T-1}x(t)e^{-2\pi jf\,t}$ is its Fourier transform. This cyclic periodogram $I_{kx}^{(T)}$ is non null when $f_0+\dots+f_{k-1}=\alpha$. For $\alpha=0$, it denotes the conventional multiperiodogram for stationary processes. It is shown in (Giannakis and Dandawate, 1994) that the kth-order cyclic periodogram is a sample estimator of cyclic moment spectra. Such estimators are unbiased and inconsistent but can be made consistent by a suitable averaging. This study will be limited for the third order, therefore cumulants and moments are equal if the process is centred. As a consequence, third-order cyclic periodogram is a pure potential estimator of the cyclic cumulant bispectrum.

3. ESTIMATION OF CYCLIC BISPECTRUM

The cyclic biperiodogram is defined as

$$I_{3x}^{(T)}(\alpha; f_0,f_1,f_2) = \frac{1}{T}X_T(f_0)X_T(f_1)X_T(f_2) \quad (4)$$

If $x(t)$ is a discrete time real process, $X_T(f_0)=X_T(\alpha-f_1-f_2)=X^*{}_T(f_1+f_2-\alpha)$ and $I_{3x}^{(T)}$ can be expressed as follows :

$$I_{3x}^{(T)}(\alpha; f_1,f_2) = \frac{1}{T}X_T(f_1)X_T(f_2)X^*{}_T(f_1+f_2-\alpha) \quad (5)$$

The cyclic bispectrum is no more than a product of three versions of the Fourier transform of $x(t)$. In a similar manner to the classical spectrum estimation methods, the cyclic bispectrum can be then estimated by two families of methods: the averaged cyclic biperiodogram (temporal averaging) and the smoothed cyclic biperiodogram (frequency smoothing). These new estimation algorithms are detailed in a recent paper (Raad and Sidahmed, 2001)

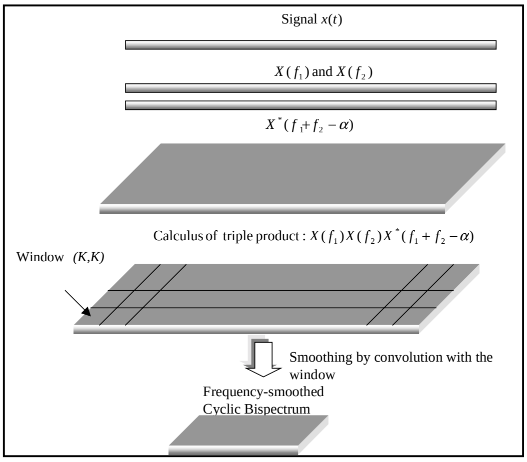


Fig. 1. Cyclic bispectrum estimation by frequency-smoothing cyclic biperiodogram.

- In the time-averaged cyclic biperiodogram, the triple product is computed on many successive lags of the data and averaged on all these lags. It is apparent that in this method it is necessary to correct the phase shift between lags. Indeed, for the ith lag, the triple product has to be multiplied by $e^{2j\pi\alpha iT_r}$ where T_r is the duration of the lag.

- For the frequency-smoothed cyclic biperiodogram, illustrated in Fig. 1., the triple product is calculated on the totality of the signal and then smoothed with an appropriate frequency smoothing window. In this paper, this technique will be used.

To illustrate this algorithm, let us consider a bilinear and cyclic coupled phase signal, given by :

$$x(t) = a(t)e^{2j\pi(f_1t+\phi_1)} + b(t)e^{2j\pi(f_2t+\phi_2)} + c(t)e^{2j\pi(f_3t+\phi_3)} \quad (6)$$

with $a(t)$, $b(t)$ and $c(t)$ independent and random narrow band amplitude modulations. Phases ϕ_i are random and stationary. Moreover, $f_3 = f_1 + f_2$ and $\phi_3 = \phi_1 + \phi_2$.

For the simulations, $f_1 = 0.15$ Hz and $f_2 = 0.25$ Hz. The sampling frequency is equal to one. The simulated signal consists of 512 samples. The size of FFTs is equal to the length of the signal.

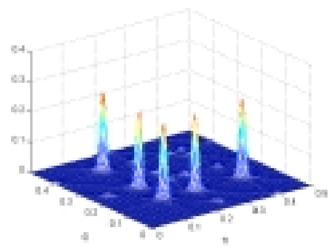
The cyclic bispectrum of $x(t)$ can be computed by taking the triple Fourier transformation of the third order cumulant $C_{3x}(t,\tau)$ with respect to t and τ. The detailed calculation is very long and can be reviewed in (Bouillaut, 2000).

For $\alpha = 0$, the cyclic bispectrum is reduced to bispectrum, a peak appears for the pairs of frequencies $(f_2; f_1)$ and its symmetric partner $(f_1; f_2)$.

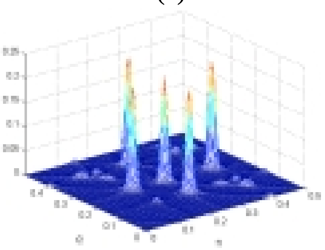
The non null cyclic frequencies of cyclic bispectrum for positive frequencies are:

- For $\alpha = f_1$, peaks are found at $(f_1; f_1)$, $(f_2; f_1)$ and $(f_1; f_2)$. Furthermore, we find a peak at $(f_3; f_1)$ and $(f_1; f_3)$.
- For $\alpha = f_2$, peaks are present for $(f_2; f_2)$, $(f_2; f_1)$ and $(f_1; f_2)$. Furthermore, two peaks appear for $(f_3; f_2)$ and $(f_2; f_3)$.
- For $\alpha = f_3$, peaks are present for $(f_3; f_3)$, $(f_3; f_1)$, $(f_1; f_3)$, $(f_3; f_2)$ and $(f_2; f_3)$.
- For $\alpha = 2f_1$, peaks appear for $(f_3; f_1)$ and $(f_1; f_3)$.
- For $\alpha = 2f_2$, peaks are found at $(f_3; f_2)$ and $(f_2; f_3)$.
- For $\alpha = 2f_2 - f_1$, one peak is present for the pair of frequencies $(f_2; f_2)$.
- For $\alpha = 2f_1 - f_2$, one peak is present for $(f_2; f_2)$.
- For $\alpha = 2f_3 - f_1$, one peak appears for $(f_3; f_3)$.
- For $\alpha = 2f_3 - f_2$, one peak appears for $(f_3; f_3)$.
- For $\alpha = f_2 - f_1$, one peak is found at $(f_2; f_2)$.

Fig. 2(a) shows the magnitude of the cyclic bispectrum for the cyclic frequency $\alpha = f_1$. Obviously, peaks appear at the same positions as indicated by the results of the theoretical calculus presented above. Fig. 2(b) represents the magnitude of the cyclic bispectrum for $\alpha = f_2$.



(a)



(b)

Fig. 2. Magnitude of cyclic bispectrum for (a) $\alpha=f_1=0.15$ Hz, (axis of f_1 and f_2 are normalised). (b) $\alpha=f_2=0.25$ Hz.

4. APPLICATION TO REAL VIBRATION SIGNALS

4.1 Presentation of the system

In this section, applications of the cyclic bispectrum to an industrial system are presented. The signals consist of vibration data recorded from the aft main power transmission of a U.S. Navy CH-46E helicopter. These signals, when used, are supposed to be either cyclic or bilinear. In fact, they combines these two properties and this is the focus of this study. Fig.3 presents a general view of the system The CH-46E is a twin-rotor, fore/aft transmission aircraft powered by two turbine engines. A single mixbox and aft main transmission were installed on a test stand and run at nine different torque levels.

Vibration data was collected using eight accelerometers and a tachometer. Seven types of faults were included, along with a base test with a fault-free transmission. Faulty components were sequentially installed in the mixbox and transmission Only one faulty component was present in the assembly during any of the data collections. Some tests involved two different instances of the same type of fault at different levels of severity. The data were digitised at a sample rate of 103,116.08Hz with 16-bit quantization using a 10-channel data acquisition system. Each signal is 412464 samples long.

The aim of this campaign of measurement was to evaluate diagnostic tools in general, including pattern recognition techniques. Several methods were applied to this data base and this gave rise to a special issue of the MSSP (Mechanical Systems & Signal Processing) in July 2000. For this paper, the study is limited to fault 4, which is spiral bevel input pinion spalling.

A simplified version of the Westland Helicopter transmission, including the spur pinion and collector gear, the common quill shaft, and the spiral bevel pinion/gear is presented in Fig. 4. In this figure, meshing frequencies f_{m_i} for i=1 to 2 and shaft rotation frequency f_{r_j} for j=1 to 3 are indicated.

Fig. 3. Photo of the helicopter

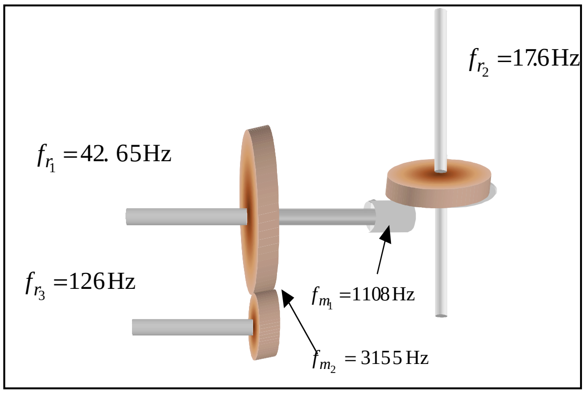


Fig. 4. Simplified schema of Westland Helicopter transmission.

4.2 Application of cyclic bispectrum and results

Signals were filtered around the two meshing frequencies with a band of 600 Hz, then transformed to analytic signals by applying a Hilbert transform. In the end, signals were decimated by 8. The sampling frequency f_s is then equal to 12890 Hz. Only 8192 samples of the resulting signals and frequency smoothed cyclic biperiodogram are used for computation. The size of FFTs is equal to 1024.

Fig. 5. represents the magnitude of the cyclic bispectrum of a vibration signal resulting from fault 4 (level 2 i.e. established damage) with a torque level of 100% for a cyclic frequency $\alpha = f_{r_1}$. Several peaks indicating non linear or cyclostationary links appear in the magnitude of the cyclic bispectrum. 4 peaks appear for the pairs of frequencies (f_{m_1}, f_3), (f_3, f_{m_1}) and (f_{m_2}, f_3), (f_3, f_{m_2}) where $f_3 = (f_{m_1} + f_{m_2})/2$.

Bouillaut (2000) has shown that when a fault appears in a industrial gearbox, the meshing frequency f_{m_1} modulates the meshing frequency f_{m_2}. Moreover, spectral analysis shows the appearance of a frequency f_3 which was the consequence of the link between the meshing phenomena. The appearance of this frequency gives rise in the cyclic bispectrum to all these bilinear and cyclostationary links between $f_{m_1}; f_{m_2}$ and f_3. The third order cyclic analysis therefore allows us to detect the presence of faults in helicopter gearbox. Two other peaks appear for the pairs of frequencies (f_{r_1}, f_3) and (f_3, f_{r_1}). These peaks underline links between the rotating frequency and the f_3 characteristic of the fault. So, these peaks can also be used to make a good diagnostic of the gearbox.

Peaks are present also for the two pairs of frequencies $(f_{m_2}, f_{r_1}), (f_{r_1}, f_{m_2})$. These peaks indicate a correlation between these two frequencies.

However, it is normal that the vibration produced by the pinion will be transmitted to the collector gear and therefore, a correlation of lower order exists between the meshing frequency of the collector gear and f_{r_1} .

In this analysis, tests were also performed on signals issuing from the same fault 4, but for level 1. When observing Fig. 6., many peaks that denoted the presence of the fault 4 level 2 are absent when the signal results from a fault with a lower degree of damage. Only three peaks exist for the pairs of frequencies $(f_{m_2}, f_{r_1}), (f_{r_1}, f_{m_2})$ and for (f_{m_2}, f_{m_2}) . Therefore the third order cyclic statistics is not able to predict the evolution of the fault at maximum torque.

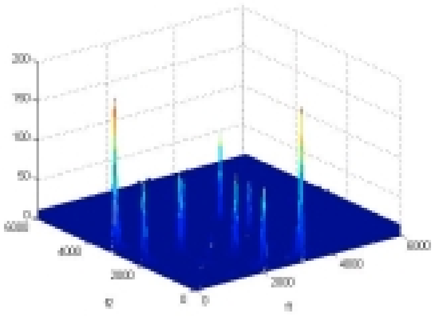


Fig. 5. Magnitude of cyclic bispectrum of Helicopter signal (default 4, level 2, torque level 100%) for a cyclic frequency $\alpha = f_{r_1}$, the axis f_1 and f_2 are in the range [0.. $f_s / 2$].

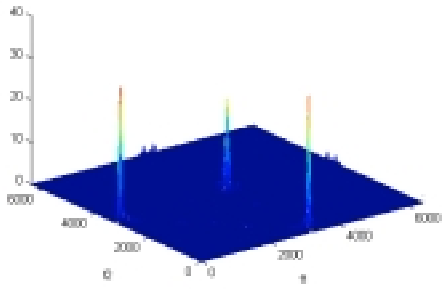


Fig. 6. Magnitude of cyclic bispectrum of Helicopter signal (default 4, level 1, torque level 100%) for a cyclic frequency $\alpha = f_{r_1}$.

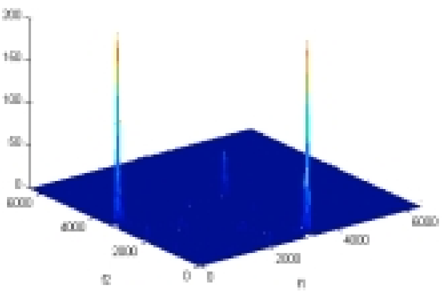


Fig. 7. Magnitude of cyclic bispectrum of Helicopter signal (No fault) for a cyclic frequency $\alpha = f_{r_1}$.

To complete the analysis, a comparison with a vibration signal with fault-free components is necessary. In Fig. 7., only three peaks appear for the pairs of frequencies (3155.5,42.6) and (42.6, 3155.5). These same peaks were found in the case of the fault 4 level 1 but with an increase in the amplitude of the peaks. This means that several new peaks do appear for the severly damaged gearbox. The multitude of these peaks show that the cyclic bispectrum can provide a precise and rich information about the state of gearbox. This information is related to bilinear and cyclostationary links between the characteristic frequencies shown in Fig. 4. Bilinear links are obtained generally from the analysis of the bispectrum (Nikias and Raghuveer, 1987) and cyclostationary links are provided by the analysis of second-order cyclic analysis and especially by the spectral correlation (Capdessus, *et al.,* 2000). Therefore, the cyclic bispectrum could be used as a combined tool for diagnosis, joining the properties of the bispectrum and the spectral correlation.

As a conclusion for this section, this study underlines that third order cyclic statistics will be a useful and powerful diagnostic tool for complex industrial systems such as the U.S. Navy helicopter gearbox. For an established fault, at a maximum torque, a diagnosis based on higher order cyclic statistics is easy to obtain, because of the multitude of peaks that appear due to the fault. The cyclic bispectrum provides also useful information about bilinearities and cyclostationarities in the system. A complete study of the effectiveness of third order cyclic statistics when applied to gearboxes and the influence of torque on the results will be the subject of a future research.

5. CONCLUSION

This paper has introduced the theory of higher order cyclic statistics as a new tool for the analysis and diagnostics of gears. Until now, higher order cyclic statistics has only been the subject of a few studies. Gardner (1994a); Giannakis and Dandawate (1994) were the pioneers in extensively studying the theory of cyclostationnarity and in proposing consistent estimators to compute kth-order cyclic polyspectra. For the second order cyclic statistics, spectral correlation was demonstrated to be a powerful tool for the diagnosis of gear faults in rotating machines. For higher orders, Spooner and Napolitano (2001) proposed the exploitation of higher order cyclic statistics as detectors, estimators of parameters, separators of signals and finally classifiers of modulation.

In this paper, the main definitions of the theory of higher order cyclic statistics have been recalled as well as estimators for kth-order cyclic spectra. More particularly, methods of estimation of the cyclic bispectrum were reviewed. These new methods were developed in a previous paper, see (Raad and Sidahmed, 2001). A simulation example was given to demonstrate the performance of these new algorithms. The originality of this paper is the application of the cyclic bispectrum to the diagnosis of gear faults in a complex industrial system. It appears that the results obtained are promising, from which, it can be concluded that third-order cyclic statistics is appropriate for making a good diagnosis. Furthermore, the use of third-order cyclic statistics can make it possible to characterise the fault, offering insight into bilinear and cyclostationary coupling between frequencies where traditional linear (i.e. power spectral) analysis provides insufficient information.

Consequently, it is not difficult to imagine a future complete industrial application of the cyclic bispectrum. This new tool could be used for the diagnosis of gears faults in rotating machinery more generally and as a complementary tool to existing and currently available techniques.

REFERENCES

Bouillaut, L. (2000). *Approches cyclostationnaire et non-linéaire pour l'analyse vibratoire de machines tournantes : Aspects théoriques et Application au diagnostic.* PH.D dissertation, Université de Technologie de Compiègne, France.

Brillinger, D and M. Rosenblatt (1967a). *Computation and Interpretation of kth-order spectra, Spectral analysis of Time series*. B.Harris, Ed. New York, Wiley.

Brilllinger, D and M. Rosenblatt (1967b). *Asymptotic theory of estimates of kth-order sepctra, Spectral analysis of Time series.* B.Harris, Ed. New York, Wiley .

Capdessus, C (1992). *Aide au diagnostic des machines tournantes par traitement de signal. PH.D dissertation*, Institut National Polytechnique de Grenoble, France.

Capdessus, C, M. Sidahmed and J.L. Lacoume (2000). Cyclostationary Processes : Application in Gear Faults Early Diagnosis. *Mechanical Systems and Signal Processing*, **14(3)** , 371-385.

Gardner, W.A (1994a). The Cumulant Theory of Cyclostationary Time-Series, Part I : Foundation. *IEEE Transaction on Signal Processing,* **42(12),** 3387-3409.

Gardner, W.A (1994b). The Cumulant Theory of Cyclostationary Time-Series, Part II : development and Applications. *IEEE Transaction on Signal Processing*, **42(12),** 3409-3429.

Giannakis, G and A.V. Dandawate (1994). Nonparametric Polyspectral Estimators for kth-order (Almost) Cyclostationary Processes. *IEEE transactions on Information Theory,* **40(1)**, 67-84.

Hinish, M.J (1994). Higher order cumulant and cumulant spectra. *Circuit System and Signal Processing,* **13(4),** 391**.**

Hurd (1970). *An investigation of periodically correlated processes*. PH.D dissertation, Duke University.

Lacoume, J.L, P.O. Amblard and P. Common (1997). *Statistiques d'ordre supérieur pour le traitement du signal*. Masson, Paris.

Nikias, C.L and M.R. Raghuveer (1987). Bispectrum estimation : A digital signal processing framework. *Proceedings IEEE*, **75**, 869-891.

Priestly, M.B (1991). *Non-Linear and Non-Stationary Time Series Analysis.* Academic Press, London .

Raad, A and M. Sidahmed (2001). Estimation of Cyclic Bispectrum : Algorithms and Applications. *4th International Conference Acoustical and Vibratory Surveillance Methods and Diagnostic Techniques*, Compiegne, France, 255-269.

Randall, R.B (1975). Gearbox fault diagnosis using cepstrum analysis. *Proceedings of Fourth World Congress on T. of M. and M.,* **1,**169-171.

Spooner, C.M and A. Napolitano (2001). Cyclic Spectral Analysis of Continuous-Phase Modulated Signals. *IEEE Transactions On Signal Processing*, **49(1)**, 30-44.

Wang, W.J. and P.D. McFadden (1993). Early detection of gear failure by vibration analysis : calculation of the time-frequency distribution. *Mechanical Systems and Signal Processing,* **7**, 193-203.