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PRECISION-LIMIT POSITIONING OF DIRECT DRIVE SYSTEMS WITH THE EXISTENCE OF FRICTION

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Abstract: This paper discusses the issue of precision-limit positioning of a direct drive system with the existence of friction. By precision-limit positioning we mean that, in addition to the common sense of sub-micro, nano, or pico precision, the precision error equals to the resolution of the position sensor used in the feedback loop. This means a zero-count precision error when a digital type position sensor is used. Precision-limit positioning also enforces 100% repeatability as precision is concerned. Of course, this is the limit of the precision that a real control system can achieve. Traditionally, this kind of performance was not considered possible mainly for lacking accurate and enough information about friction, especially the static friction. However, as more and more knowledge of the pre-sliding motion has been revealed, it is shown in this research that such kind of performance can be achieved. *Copyright ©2002 IFAC*

Keywords: Positioning systems, Position accuracy, Position control, Friction

1. INTRODUCTION

High precision positioning is an important issue in the fields of manufacturing, assembly and measurement. In this research, the precision-limit positioning of direct drive systems under no or constant load is studied. By precision-limit positioning (PLP) we mean that, in addition to the common sense of high precision requirement, the precision achieved is the limit that the particular system hardware can achieve. Of course, there are lots of factors that influence the precision limit, such as the resolution of the input force, the resolution of the position sensor, and so on. In this research, the precision limit is defined to be the resolution of the sensor that is used for position feedback. It is under the assumption that all other parts of the system are capable of achieving the goal of PLP. We further restrict ourselves to the case that a digital type position sensor is used. In this case the precision error should be zero-count error. It is also noted that to make zero-count precision error a meaningful statement, the precision should be 100% repeatable. This is quite different from what is encountered in existing commercial systems. The reason we ask for PLP is two folds: (i) academic curiosity and (ii) cutting down the cost of an ultra high precision positioning system.

Of course, PLP was not deemed possible in a common sense. This was mainly because we lacked enough knowledge about friction and the induced non-linearity. This ignorance is even worse for the case of static friction. To overcome the problem caused by friction, the industry uses brushless motors, air-bearings, magnetic bearings and other special designed bearings to get rid of friction. Despite the high cost of the special equipment, the theoretical precision limit for such kind of systems is ±1 count. This implies 3 times the precision error of ±0 count case. To get same level of precision, a sensor with triple precision should be used. This, of course, increases the cost tremendously for a high precision system.

Instead of using expensive hardware, researchers in the control field have been working on complicated control laws to compensate the influence of friction. However, the very fundamental purposes of these efforts are to improve the precision performance. None of them tried to claim to achieve zero-count precision error with 100% repeatability. However, some works do obtain zero-count precision error in the reports they published (Futami, et al., 1990; Yamaguchi, 1990; Jeon, et al., 1997; Huang, et al., 1999; Yen, et al., 1997). A common point of these controls is that no matter what kind of algorithms they claim, there is always an integral feedback embedded in the design. In fact, it is a common practice to include an integral part in a positioning system. However, the importance and contribution of this integral control is ignored in all these works. As a matter of fact, based on the traditional Coulomb friction model, it can be proved theoretically that a single integral control alone cannot stabilize a direct drive system.

In this research, it is proved that, with the new knowledge of pre-sliding dynamics revealed, the integral control or equivalent is necessary for obtaining PLP, and an integral control alone can stabilize the system in the pre-sliding phase. Some practical issues of PLP are also addressed. This includes how to select an appropriate D/A converter. The derivation presented in this paper is mainly based on the authors' earlier work on pre-sliding behavior (Hsieh and Pan, 2000). Readers interested in static friction and positioning problems are encouraged to read the paper.

The block diagram of the system used in this research to conduct experiments is shown in Fig. 1. The plant is a direct drive DC torque motor mounted on ordinary ball bearings. The motor is driven by a linear amplifier. Two optical encoders with different resolutions are mounted to measure the rotor position. The resolution of the coarse one is 86,400 count/rev or, equivalently, 15 arcsec/count, while that of the fine one is 1,620,000 count/rev or, equivalently, 0.8 arcsec/count. The rotor angle is measured simultaneously by these two encoders. The outputs are compared to guarantee that the data obtained is correct. The data acquisition and control is accomplished by a PC-486/66 together with a D/A converter and an A/D converter. Both D/A and A/D converters have a resolution of 12 bits.

In the next section, the pre-sliding friction behavior will be described. Its influence on the position control will also be discussed. A simple and explicit stability criterion for fine positioning using PID control is given in section 3. Some practical issues are also addressed in this section. Section 4 contains the experimental results and is followed by the conclusion.

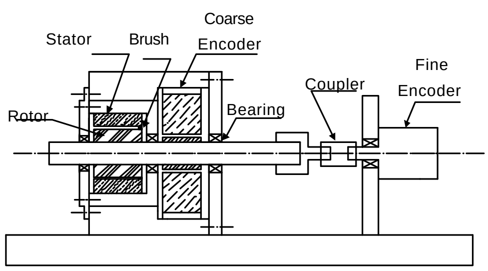

Fig. 1. The test platform

2. PRE-SLIDING BEHAVIOR AND ITS INFLUENCE ON THE POSITION CONTROL

The traditional friction model is the well-known Amonton-Coulomb friction model. According to this model there is no displacement in the static friction range. Hence, it is usually modelled as a dead-zone non-linearity in control design. This assumption is appropriate from the macroscopic point of view. However, in 1899, Stevens, with the help of an interferometer, found that elastic displacement does exist before the normal slip occurs. This small amount of displacement can indeed be ignored if the precision requirement is not high. However, it becomes the vital factor when we are talking about high precision positioning. Since, in this case, the range of pre-sliding motion is usually greater than the precision requirement by an order of at least 2. The pre-sliding dynamics can be summarized as follows.

Quantitative properties:

(QN1) *The parameters of any friction model are both time varying and position varying*. It is well known in the field of tribology that it is hard to get repeated friction data. This is true even for well-prepared metal-to-metal friction experiments (Courtney-Pratt, 1957), and not to mention what happen in commercial machines. This property is one of the main reasons that people believed that it is impossible to get zero-count precision error with 100% repeatability.

(QN2) *It is temporarily time-invariant and locally position-invariant.* By temporarily time-invariant we mean that the parameters remain constant during a short interval of time. This short interval is, however, long enough for ordinary control purposes. By locally position-invariant we mean that the parameters remain unchanged if the system stays in the pre-sliding range of any individual point. This is a very important property. Just because of this property, we can discover the following consistent and repeatable qualitative properties. Furthermore, when we make up a model to match these properties, the parameters of this model can be treated as constants with "time-invariant" uncertainties. From the control point of view, it is then possible to obtain zero-count precision error with 100% repeatability. It should be noted, however, this does not mean that we can achieve 100% repeatable transient response.

Qualitative properties:

The pre-sliding motion consists of two kinds of motion.

(QL1) *Non-linear spring deformation.* This motion demonstrates a special Preisach hysteresis. This hysteresis demonstrates memory and wiping-out effect of reverse points and congruency in both input- and output-wise.

(QL2) *Plastic deformation.* This motion demonstrates creep and work hardening.

Based on these properties, a friction model is introduced by Hsieh and Pan (2000) and is shown in Fig. 2(a). This model consists of 4 elements - a plastic module, a non-linear spring module, a viscous damper and a hook. These elements are massless and, hence, do not really exist. They are phenomenological elements. Each element exhibits a special mechanical property and is described by a simple mathematical expression. The combined result is proved to match all the pre-sliding behaviour. The hook at the left end sticks at the position where it is if the internal force is less than the breakaway force, i.e., the maximum static friction. Once the hook starts to move the tangential contacting force between the hook and ground can be described as a function of velocity. This kind of function describes the dynamic friction and has been proposed by many investigators. As pre-sliding motion is concerned, the model is reduced to Fig. 2(b). The damper c is a standard linear damper and the non-linear spring module and plastic module are described below.

Non-linear spring module: Consider the non-linear spring shown in Fig. 3. Let σ_s be the applied external force and x_s be the elongation. The x_s - σ_s relation consists of a special Preisach hysteresis as described in property QL1. The constitutive equation of each branch of the hysteretic motion can be expressed as

$$\frac{d\sigma_s}{dx_s} = k_1 + k_2 e^{-\beta|x_s - x_r|} \qquad (1)$$

where k_1, k_2 and β are positive scalars and x_r is the reversal point of the associated branch of motion. It is noted that to get complete information of non-linear spring, we have to know the history of x_r. The discussion of x_r is beyond the scope of this paper and is omitted here. A typical response of the non-linear spring is shown in Fig. 4. If we treat the hysteresis as a spring then the right-hand-side of (1) can be considered as the spring constant. It varies from k_1 to $k_1 + k_2$. For very small displacement, the non-linear spring can be linearized as a linear spring.

Plastic module: Consider the plastic module shown in Fig. 5. Let σ be the applied force and x_p be the extension, then the relation between σ and x_p can be expressed as

$$\begin{cases} \dot{x}_h = \begin{cases} \alpha(\dfrac{|\sigma^n|}{\lambda} - x_h), & \text{if } \dfrac{|\sigma^n|}{\lambda} > x_h \\ 0 & , \text{ otherwise} \end{cases} \\ \dot{x}_p = \mathrm{sgn}(\sigma)\dot{x}_h \end{cases} \qquad (2)$$

where α and λ are positive scalars and $n > 1$. In this model, x_h is monotonically non-decreasing and stands for the accumulated work hardening and x_p is the final plastic deformation. It is noted that when $|\sigma^n|/\lambda > x_h$, new plastic deformation occurs. This deformation is slow as compared with that of the non-linear spring module and is called creep motion. On the other hand, if $|\sigma^n|/\lambda \le x_h$, there is no new deformation. The module is work hardened. A typical response of the plastic module is shown in Fig. 6.

The pre-sliding motion is a combined result of these modules according to the relation shown in Fig. 2(b). In a further study, it is shown that the parameters of these models are force-rate independent.

We now consider the position control of a typical direct drive system. The governing equation is

$$m\ddot{x} + c_s\dot{x} = u - \tau_f \qquad (3)$$

where τ_f is the friction force, u is the control input and c_s is the damping coefficient in the slip phase. We note that, based on the Amonton-Coulomb friction, when u is less than the maximum static

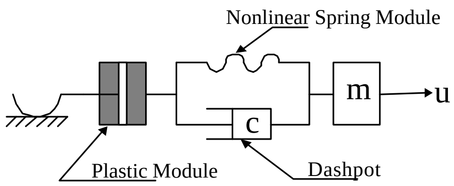

Fig. 2(a). Model of friction

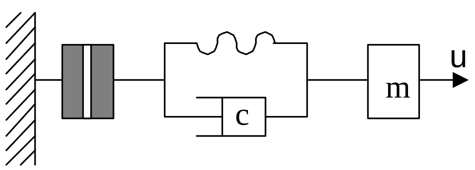

Fig. 2(b). Static friction model

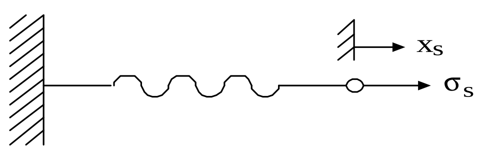

Fig. 3. Non-linear spring module

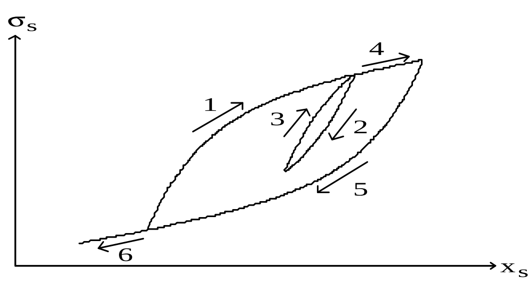

Fig. 4 Typical response of the hysteresis non-linearity of the non-linear spring module

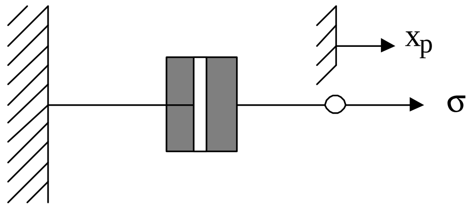

Fig. 5. Plastic module

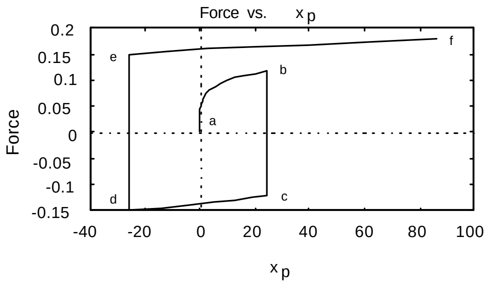

Fig. 6. Typical response of the plastic module

friction then $u = \tau_f$ and any point $(x, \dot{x} = 0)$ in the state space is an equilibrium point. Therefore, the control u can be eliminated as the desired position is reached. However, this is not the case in a real system. The real scenario is as follows. The control force u becomes small as mass m gets close to the desired position. The system finally gets stuck due to static friction. The point it sticks is usually not the desired point. The system behaviour switches from that shown in Fig. 2(a) to 2(b). As the plastic module gets work hardened, the system's behaviour is very close to that of a mass-spring-damper system. Therefore, to stay at the desired position, say x_d, a final holding force to compensate the spring force is required. If this force is removed, the mass bounces back and leaves the desired position due to the retraction of non-linear spring. This fact justifies the necessity of an integral type control in PLP systems since a nonzero holding force is a typical property of this kind of control. This also explains why in those works mentioned earlier showing zero-count precision errors in their experiments always have an integral part embedded in their feedback loop.

It is clear now that system (3), in fact, consists of two different systems: a type-1 slip system and a type-0 stick system. Since the two systems are so different, it is natural to take different control strategies for different phases. This dual mode control philosophy has been adopted in many research works. The control design for coarse positioning in the slip phase has been well developed. In the next section we will concentrate on the fine positioning in the stick phase.

3. STABILITY CRITERION OF PID CONTROL IN THE PRE-SLIDING RANGE

As discussed in the last section, an integral type control is necessary for a PLP system. Therefore, in this research we restrict the control structure to be a simple PID control. Though the concept of pre-sliding motion as represented by Fig. 2(b) is simple, the combined dynamics and non-linearity embedded in (1) and (2) still make it difficult to analyze the complete system mathematically. As a first try, we neglect the plastic module in the friction model to simplify the analysis. This is a mild assumption because of two reasons. First, the creep motion is much slower than the non-linear spring deformation. As will be discussed later, the main influence of it is to postpone the time required to achieve 0-count error; the system may stay with 1-count error for a period of time due to creep effect. Secondly, as long as the control remains in the pre-sliding phase, the plastic deformation will finally disappear due to work hardening.

With this assumption, the governing equation for each branch of motion in the fine positioning phase becomes

$$m\ddot{x} + c\dot{x} + \sigma_s(x) = -k_P e - k_D \dot{e} - k_I \int_0^t e\, d\tau \qquad (4)$$

where $e = x - x_d$, and the relation between σ_s and x follows (1). Differentiate (4) and express it in the state space form we have

$$\dot{z} = g(z)$$

$$P: \quad = \begin{bmatrix} z_2 \\ z_3 \\ -\dfrac{k_D + c}{m} z_3 - \dfrac{k_1 + k_2 e^{-\beta\left|z_1 + x_d - x_r\right|} + k_P}{m} z_2 - \dfrac{k_I}{m} z_1 \end{bmatrix} \qquad (5)$$

where $z \equiv [z_1 \quad z_2 \quad z_3]^T \equiv [x - x_d \quad \dot{x} \quad \ddot{x}]^T$. It is noted that the hysteresis non-linearity of the non-linear spring is a system with varying state dimension. (Hsieh and Pan, 2000) More specifically, those reversal points x_rs that have not been wiped out form part of the system state. This is quite different from other existing friction models. Standard Liapunov theorem cannot be applied to this system. To analyze the stability of such kind of systems, we treat system (5) as a switched system, a system that switches among an infinite set F of subsystems, $F = \{P | \dot{z} = g(z), x_r \in R\}$. It is obvious that $z = 0$ is the only equilibrium point for all $P \in F$. This is an autonomous continuous switched system. Since the spring constant is lower and upper bounded by two constants k_1 and $k_1 + k_2$, Pan et al. (2001) have proved that the system (5) is globally asymptotically stable about the desired point x_d if the following condition is satisfied.

$$k_I < \underline{k_I} = \frac{(c + k_D)(k_1 + k_p)}{m} \qquad (6)$$

The proof is based on the Liapunov direct method and LaSalle theorem modified for switched systems that have an infinite set of subsystems.

From Liapunov indirect method we can further prove that the system is unstable if

$$k_I > \overline{k_I} = \frac{(c + k_D)(k_1 + k_2 + k_p)}{m} \qquad (7)$$

If $\underline{k_I} < k_I < \overline{k_I}$, the system has higher and higher possibility to become unstable as it approaches $\overline{k_I}$. From the design point of view, condition (6) should be satisfied. It is interesting to note that a pure integral control cannot stabilize system (3) in the slip phase. However, it can stabilize the system in the pre-sliding phase as long as $k_I < \frac{ck_1}{m}$.

To obtain PLP, the following practical issues should be noticed.

Practical issues in a PLP system:

1. k_1, same as other friction parameters, is position varying. To get a robust design and hence 100% repeatability of precision, the k_1 used in (6) must be the lowest non-linear spring constant over the region of working area.

2. Though k_1 is the lower bound of the non-linear spring constant, it is still very large. For example, it is 110Nm/rad for the system used in this research. Therefore, to get fast response, the integral gain k_I used in the fine positioning phase can be quite large. However, if the same gain is used in the slip range, it will destabilize the system.

3. The hard non-linear spring also implies that the bandwidth in the pre-sliding phase is much larger than that of the slip phase. For example, the bandwidth of the current system is about 198 Hz in the pre-sliding phase. This is unusual in an ordinary mechanical system.

4. Suppose Δx is the resolution of the digital sensor used in the loop for position feedback, and Δu is the resolution of the control input. Then, a change of one unit of the input force will make m to move a distance bounded by $\Delta u / k_1$. To get 0-count precision error we must have $\Delta u < k_1 \Delta x$.

5. When a digital sensor is used in feedback loop, it is not feasible, at the final stage of positioning, to obtain the velocity using difference method. The derivative part of the PID control should be avoided in the very final stage.

6. In the above analysis the creep effect is omitted mainly because of the assumption that creep motion is much slower than the non-linear spring deformation. From real-time tests, this assumption is acceptable. In fact, the main consequence of creep motion is to cause jerks with a magnitude of 1 count in the final stage of positioning. As discussed before, we have to include an integral control to get zero-count precision error. In most cases, when the desired position is achieved, a final holding force exists to get balance with the non-linear spring force. This holding force will then induce creep motion until the work hardening process is completed. The digital sensor cannot detect this creep until it is large enough to get into the range of next count. When this happens, the magnitude of the integral control is decreased by an amount of $2k_{ID}$ in two consecutive sampling instants. The non-linear spring retracts immediately and the sensor shows the count x_d again. This process is repeated until the work hardening is completed. According to the friction dynamics described in section 2, the jerks can happen in only one direction and coincide with the direction of the final holding force. Furthermore, since creep slows down gradually, the frequency of jerks slows down gradually too.

4. EXPERIMENTAL RESULTS

In this section, the results demonstrated is to verify the theories derived for stability and steady state error of the fine positioning. The coarse positioning and transient response are not the issue. Only fine encoder is used in the feedback loop. The whole control process is confined to the pre-sliding phase, and the control is a single integral control. The integral feedback is realized using trapezoidal rule. Interested readers are referred to Lin, et al., (2001) for more control cases.

First of all, the bounds of the non-linear spring constant are estimated according to the process described in Hsieh and Pan (2000). The lower bound k_1 is estimated to be 110Nm/rad, and the upper bound is $k_1 + k_2 = 860\text{Nm/rad}$.

With the fine encoder being considered, we have $k_1 \Delta x = 4.27 \cdot 10^{-4}\,\text{Nm}$. The resolution of input torque of our system is $\Delta u = 2.4 \cdot 10^{-4}\,\text{Nm}$ which is smaller than $k_1 \Delta x$. Therefore, this torque resolution is fine enough for sensor-accuracy positioning. The moment of inertia M of the rotor is $M = 5.54 \cdot 10^{-4}\,\text{Kg} \cdot \text{m}^2$. The damping coefficient is estimated to be c=0.2Nm/rad/sec. The highest natural frequency of the system is approximately $\omega_n = ((k_1 + k_2)/M)^{1/2}$ =198Hz. The sampling rate of our control system is 2k Hz, ten times ω_n.

From (6) and (7) the system is stable if $k_I \le \underline{k_I} = ck_1 / M = 0.15\text{Nm/count}$ and unstable if $k_I \ge \overline{k_I} = c(k_1 + k_2)/M = 1.2\,\text{Nm/count}$. The corresponding gain for digital control is $\underline{k_{ID}} = 3.85 \cdot 10^{-5}\,\text{Nm/count}$ and $\overline{k_{ID}} = 3.01 \cdot 10^{-4}\,\text{Nm/count}$, respectively.

Three different gains are tested for stability.

(a) $k_{ID} = 3.85 \cdot 10^{-5}\,\mathrm{Nm/count}$. In this case, $k_{ID} = \underline{k_{ID}}$. The system is stable in all tests as predicted by the theory. Fig. 7 shows the results of 10 successive experiments with the desired position starting from 1 count to 10 count. They all achieve zero-count error. Fig. 8 shows the results when a desired position of 30 count is commanded. The jerks created by creep motion appear. The direction of jerks coincides with the direction of final holding force. It is clear that the frequency of jerks slows down gradually.

(b) $k_{ID} = 9.45 \cdot 10^{-5}\,\mathrm{Nm/count}$. In this case, $\underline{k_{ID}} < k_{ID} < \overline{k_{ID}}$. It is stable in some cases, but is unstable in most cases. To stabilize the system, we add a proportional control with $k_p = 6.21 \cdot 10^{-4}\,\mathrm{Nm/count}$. In this case, stability criterion (6) is satisfied again. The system becomes stable in all the cases we tested. The responses are much faster as compared with those in case (a). But overshoot occurs.

(c) $k_{ID} = 2.08 \cdot 10^{-4}\,\mathrm{Nm/count}$, which is about 70% of $\overline{k_{ID}}$. It only remains stable once in all our tests. When $k_{ID} > \overline{k_{ID}}$ the system is unstable in all the tests.

5. CONCLUSION

In this paper, it has been shown that it is possible to achieve zero-count precision error positioning with 100% repeatability. This possibility is mainly based on the new knowledge of the pre-sliding dynamics revealed recently. It turns out that to get precision-limit positioning (PLP) an integral type control is necessary. A simple and explicit stability criterion is given. Some practical issues concerning PLP are also addressed. This research is partly supported by the grant NSC86-2213-E-006-045.

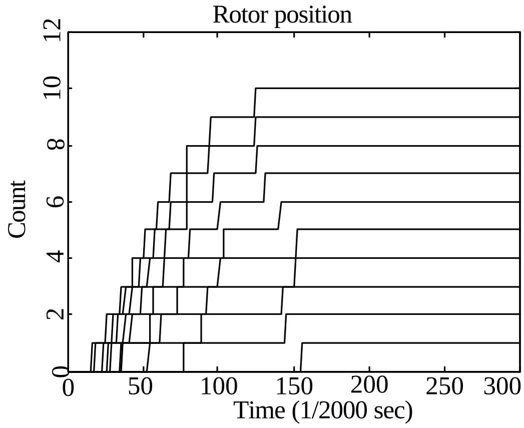

Fig. 7. Positioning with fine encoder $k_{ID} = 3.85 \cdot 10^{-5}\,\mathrm{Nm/count}$

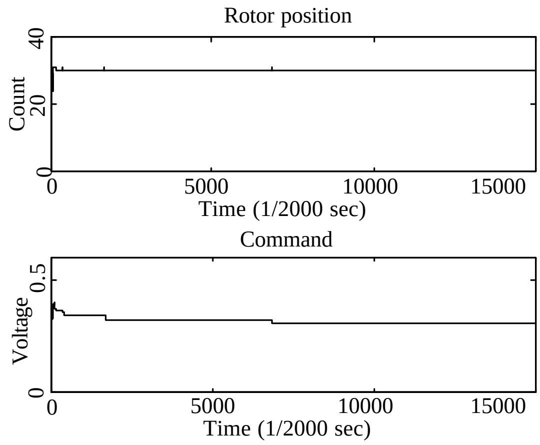

Fig. 8. Fine positioning - jerks appear due to creep motion

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