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OPTIMAL FUZZY CONTROL OF NONLINEAR SYSTEMS WITH APPLICATION TO AN UNDERACTUATED ROBOT

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Abstract: To deal with complex nonlinear systems such as underactuated mechanisms, in this paper a global stable as well as optimal fuzzy controller is presented to achieve output tracking control of nonlinear systems by combining the linear optimal control theory and linear regulator theory with the Takagi-Sugeno fuzzy methodology. The stability of the entire closed-loop fuzzy system is ensured by the Lyapunov stability analysis. This paper also includes a hardware description and the real-time results for the application to a benchmark underactuated robot: Pendubot. *Copyright © 2002 IFAC*

Keywords: Fuzzy system, nonlinear system, trajectory tracking, linear optimal control, linear regulator, global optimization.

1. INTRODUCTION

Recently, the Takagi-Sugeno fuzzy model (Takagi and Sugeno, 1985) has been applied for the control of nonlinear systems (see, for example, Taniguchi *et al*., 1999; Begovich, *et al*., 2000). Using this fuzzy description, a nonlinear plant is represented by a set of linear models interpolated by membership functions and then a model-based fuzzy controller was developed. This technique seems particularly suitable for the control of complex nonlinear systems since the dynamics of a nonlinear system are easily obtained by linearization of nonlinear models near different operation points or by input /output identification around these points. In Taniguchi, *et al*. (1999), output tracking is achieved by minimizing the error between the nonlinear system and a nonlinear reference model, both of them are modeled by T-S methodology. In the work (Begovich, *et al*., 2000), a fuzzy control scheme combining linear regulatory theory (Isidori, 1995) with Takagi-Sugeno fuzzy control methodology, which could allow the external perturbation rejection, is proposed.

In this paper we will also use the Takagi-Sugeno fuzzy methodology to model a class of nonlinear systems, i.e., each fuzzy local model represents a linearized model of the operation point of the controlled nonlinear system. The control algorithm employs the fuzzy control that is designed by an aggregation of the fuzzy local controllers. However, different from the previous approaches, the local controller for each local model consists of an optimal feedback plus a term for perturbation rejection, which is design based on the optimal control and the linear regulatory theory. The advantage for such a design is only a simple fuzzy controller is used in the approach and the proposed control law ensures global stability of the closed-loop system and guarantees the asymptotic optimal output trajectory tracking.

To verify the proposed scheme, experiments are conducted in an under-actuated robot: Pendubot (Spong and Block, 1995). Pendubot, as a benchmark nonlinear system, is a planar two-link underactuated robotic mechanism. This system has extensively been used in education and research to evaluate the performance of different control algorithms. Linear and nonlinear control algorithms have been frequently used to stabilize the Pendubot in vertical position (see, for example, Block, 1996; Haro and Begovich, 1998). Fuzzy control has also been used in Sanchez, *et al.* (1998). For trajectory tracking, there are few works in literature. This paper illustrates the application of the proposed algorithm, by forcing the Pendubot to track a sinusoid signal of significative amplitude in real time, around an unstable equilibrium point. The contribution of this paper can, therefore, be summarized as 1): a optimal control law, which can realize the system optimal output tracking control as well as allow the external perturbation rejection, is proposed by using the Takagi-Sugeno fuzzy model; 2): this law is implemented in a real time application.

2. OPTIMAL T-S FUZZY ALGORITHM

In this section, an optimal fuzzy control scheme for nonlinear systems is presented. This algorithm uses T-S fuzzy rules, whose design for local control laws is based on the linear optimal control theory and linear regulatory theory. For the sake of completeness, we present some well-known preliminaries about T-S model, linear regulator theory and linear optimal control theory. Based on these preliminaries, the novel control algorithm is proposed.

2.1. T-S fuzzy model

In this methodology, nonlinear systems are approximated by a set of linear local models. A dynamic Takagi-Sugeno fuzzy model (Takagi and Sugeno, 1985) is described by a set of fuzzy "IF-THEN" rules, with fuzzy sets in the antecedents and local linear time invariant systems in the consequents. Every *i*th rule of a T-S fuzzy model has the following form:

Plant Rule *i*.
IF $z_1(t)$ is M_{1l}… ,$z_j(t)$ is M_{ji}, …, $z_q(t)$ is M_{qi}
THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases}$$

where i=1,…r with r is the number of rules; the z_j (j=1,…q) are the premise variables, which may be functions of the states of an another variables; M_{ji} are the fuzzy sets; x∈ R^n is the state vector, u∈ R^m is the input vector; y∈ R^p is the state output vector; A_i, B_i, C_i, are the matrices of adequate dimensions. z(t) means the vector containing all the z_j.

The final state and output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} \lambda_i(z(t))(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} \lambda_i(z(t))}$$

$$y(t) = \frac{\sum_{i=1}^{r} \lambda_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} \lambda_i(t)}$$

where $\lambda_i : R^q \to [0,1]$, *i*=1, …r , are the membership function of the system belonging to plant rule *i*.

The above fuzzy model is a general nonlinear time-varying equation and has been used to model the behaviours of complex nonlinear dynamic systems.

2.2. Linear Regulator and Optimal Control theory

Let us consider a linear system

$$\dot{x}(t) = Ax(t) + Bu(t) + P\omega(t)$$
$$\dot{\omega}(t) = S\omega(t)$$
$$e(t) = Cx(t) + Q\omega(t)$$

where x is the internal state vector of the plant; u is the control input vector; ω is a vector containing external disturbances and or references and e is the tracking error if ω is treated as a reference signal.

In linear optimal control theory, if $\omega(t) = 0$, the control goal is to obtain the optimal gain matrix K such that the feedback law *u=Kx* minimizes the cost function:

$$J = \int_0^\infty \left(e^T \Lambda e + u^T R u\right) dt$$

where Λ and *R* are positive definite matrices.
Assuming that the pair (*A, B*) is stabilizable. Then, let *F* be the unique positive definite solution to the associated matrix Riccati equation:

$$0 = FA + A^T F - FBR^{-1}B^T F + C^T \Lambda C$$

The optimal control gain can be defined as $K=B^TF$.

When the disturbances and or references $\omega(t) \neq 0$, the linear regulator theory (Francis, 1977; Isidori, 1995), can be utilized to ensure the asymptotic output tracking. In the linear regulator theory the control goal is to obtain a stable close loop system and asymptotic tracking error, for every possible exogenous input in a prescribed family of functions of time.
To ensure $e(t) \to 0$, the following is assumed:

A1. The exosystem is antistable, i.e, all the eigenvalues of *S* have nonnegative real part.

With Assumption A1, the problem of output regulation via state feedback can be determined if and only if there exist matrices Π and Γ which solve the following linear matrix equation.

$$\Pi S = A\Pi + B\Gamma + P$$
$$0 = C\Pi + Q \qquad (1)$$

The optimal output control law with the external perturbation rejection can be specified as

$$u(t) = Kx(t) + (\Gamma - K\Pi)\omega(t)$$
$$= Kx(t) + L\omega(t)$$

where $K=B^TF$ and $L = \Gamma - K\Pi$. (2)

2.3. Proposed Algorithm

At this stage, a new T-S fuzzy control for output tracking control of nonlinear systems can be presented. To design the control rules, the same fuzzy sets as antecendents are used in the above plant rules. As consequents, local laws, based on the linear regulator theory and linear optimal control theory for each local linear model, can be designed. The rules for the plant and controller are

i^{th} **Plant Rule**:

IF $z_i(t)$ is M_{ij}, … , $z_j(t)$ is M_{ji},…, $z_q(t)$ is M_{qi}.

$$\text{Then}\begin{cases}\dot{x}(t)=A_i x(t)+B_i u(t)+P_i\omega(t)\\ \dot{\omega}(t)=S\omega(t)\\ e(t)=C_i x(t)+Q_i\omega(t)\end{cases}\qquad (3)$$

i^{th} **Controller Rule:**

IF $z_i(t)$ is M_{ij}, … , $z_j(t)$ is M_{ji},…, $z_q(t)$ is M_{qi}

Then u= $K_i x(t)+L_i\omega(t)$ (4)

where $K_i=B_i^T F_i$, $L_i=\Gamma_i-B_i^T F_i\Pi_i$, and $A_i+B_iK_i$ is stable; Γ_i and Π_i satisfy (1) for each (A_i, B_i, C_i, P_i, S, Q_i), $i=1...r$.

Hence the overall optimal fuzzy controller is given by

$$u(t)=\frac{\sum_{i=1}^{r}\lambda_i(z(t))[K_i x(t)+L_i\omega(t)]}{\sum_{i=1}^{r}\lambda_i(z(t))}\qquad (5)$$

The design purpose in this study is to specify the fuzzy control in (4) to achieve a global stable and optimal output control. Then, we obtain the following result:

Theorem: If the local linear systems in (3) satisfy Assumption A1 and the local output regulators (4) via state feedback can asymptotically stabilize the local linear systems in (3) and ensure the asymptotic output tracking for every possible initial state and every possible exogeneous input. Then, the fuzzy control (5) can globally stabilize the Takagi-Sugeno system (3) and ensures the asymptotic output tracking. Furthermore, its output is optimal.

Proof: Connecting plant rules with controller rules, the closed-loop system is obtained:

$$\dot{x}(t)=\frac{\sum_{i=1}^{r}\lambda_i(z(t))A_i x(t)}{\sum_{i=1}^{r}\lambda_i(z(t))}+\frac{\sum_{i=1}^{r}\lambda_i(z(t))B_i}{\sum_{i=1}^{r}\lambda_i(z(t))}\left(\frac{\sum_{i=1}^{r}\lambda_i(z(t))(K_i x(t)+L_i\omega(t))}{\sum_{i=1}^{r}\lambda_i(z(t))}\right)+\frac{\sum_{i=1}^{r}\lambda_i(z(t))P_i\omega(t)}{\sum_{i=1}^{r}\lambda_i(z(t))}$$

$$\dot{\omega}(t)=\frac{\sum_{i=1}^{r}\lambda_i(z(t))S\omega(t)}{\sum_{i=1}^{r}\lambda_i(z(t))}$$

First, let $\omega(t)=0$ and prove that the equilibrium x=0 is globally asymptotically stable in the first approximation. Denote the sum over all possible combinations of $\left(\sum_{i=1}^{r}\lambda_i(z(t))\right)\left(\sum_{i=1}^{r}\lambda_i(z(t))\right)$ as $\sum_{i,j}$, the first approximation can be written as:

$$\dot{x}(t)=\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))A_i x(t)}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}+\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))B_i(K_i x(t))}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}$$

$$=\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))(A_i+B_iK_i)}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}x(t)$$

For stability analysis, a Lyapunov function is chosen:

$$V(x)=x^T Px$$

where P is a symmetric positive definite matrix. The derivative of $V(x)$ can be written as

$$\dot{V}(x)=x^T P\dot{x}+\dot{x}^T Px$$

$$\dot{V}(x)=x^T P\left(\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))(A_i+B_iK_i)}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}\right)x+x^T\left(\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))(A_i+B_iK_i)}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}\right)^T Px$$

$$\dot{V}(x)=x^T\left[P\left(\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))(A_i+B_iK_i)}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}\right)+\left(\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))(A_i+B_iK_i)}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}\right)^T P\right]x$$

$$=x^T\left(\frac{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))\begin{pmatrix}P(A_i+B_iK_i)+\\(A_i+B_iK_i)^T P\end{pmatrix}}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}\right)x$$

Since

$$0\le\frac{\lambda_i(z(t))\lambda_j(z(t))}{\sum_{i,j=1}^{r}\lambda_i(z(t))\lambda_j(z(t))}\le 1$$

one has:

$$\dot{V}\le\sum_{i,j=1}^{r}x^T\left(P(A_i+B_iK_i)+(A_i+B_iK_i)^T P\right)x$$

$$\le x^T P\left(\sum_{i=1}^{r}(A_i+B_iK_j)+\left(\sum_{i=1}^{r}(A_i+B_iK_j)\right)^T\right)Px$$

Because $A_i+B_iK_i$ is stable, *i.e.*, $A_i+B_iK_i<0$, which implies

$$\sum_{i=1}^{r}(A_i+B_iK_j)<0$$

The linear matrix inequalities (LMI):

$P\sum_{i=1}^{r}\left(A_i + B_i K_j\right) + \left(\sum_{i=1}^{r}\left(A_i + B_i K_j\right)\right)^T P < 0$ is, therefore, established.

From the above, we have proved that $\dot{V}(x) < 0$ and the equilibrium x=0 is globally asymptotically stable in the first approximation.

Secondly, we shall show that the system output is also optimal when $\omega(t) = 0$. Let consider the quadratic cost function for the whole system:

$$J = \int_0^\infty \left(e^T \Lambda e + u^T R u\right) dt$$

According to the "additive property of energy" (Wu and Lin, 2000) the whole cost J is the combination of the local cost: J_i.

$$J_i = \int_0^\infty \left(e^T \Lambda e + u_i^T R u_i\right)$$

where the subscript i can be any element of set $\{1,...,r\}$. Based on this additive property of energy, we know that, at any time instant, because we can find the optimal control law for each subsystem to minimize the local cost, J_i, the fuzzy "blended" control law (4) is the global minimizer of the total cost, *J*.

When $\omega(t) \neq 0$, it can be shown that $\underset{t\to\infty}{Lim}\, e(t) = 0$.

$$e(t) = \frac{\sum_{i=1}^{r}\lambda_i(z(t))C_i x(t)}{\sum_{i=1}^{r}\lambda_i(z(t))} + \frac{\sum_{i=1}^{r}\lambda_i(z(t))Q_i\omega(t)}{\sum_{i=1}^{r}\lambda_i(z(t))}$$

$$= \frac{\sum_{i=1}^{r}\lambda_i(z(t))(C_i x(t) + Q_i\omega(t))}{\sum_{i=1}^{r}\lambda_i(z(t))}$$

Because $0 \le \dfrac{\lambda_i(z(t))}{\sum_{i=1}^{r}\lambda_i(z(t))} \le 1$,

$$0 \le e(t) \le \sum_{i=1}^{r}(C_i x(t) + Q_i\omega(t)) = \sum_{i=1}^{r}(e_i(t))$$

Because the equation $\Pi_i S = A_i\Pi_i + B_i\Gamma + P_i$ is satisfied, the graph of the mapping $x = \Pi_i\omega$ is a center manifold of the linear system (Isidori, 1995). By the equation ($0 = C_i\Pi_i + Q_i$), we get:

$$e(t) = C_i x(t) + Q_i\omega(t) - (C_i\Pi_i + Q_i)\omega(t)$$
$$= C_i x(t) - C_i\Pi_i\omega(t)$$

The point $(x,\omega) = (0,0)$ is a stable equilibrium of the linear system. Then, for sufficiently small $(x(0),\omega(0))$, the solution $(x(t),\omega(t))$ of the linear system remains in any arbitrarily small neighborhood of (o,o) for all t ≥ 0. Using a property of center manifold (Isidori, 1995), it is deduced that there exist real numbers *M>0* and *a>0* such that

$$\|x(t) - \Pi_i\omega(t)\| \le Me^{-at}\|x(0) - \Pi_i\omega(0)\|$$

for all t>0. By continuity of $e_i(t)$, one has $\underset{t\to\infty}{Lim}\, e_i(t) = 0$.

Therefore, one can conclude that $\underset{t\to\infty}{Lim}\sum_{i=1}^{r} e_i(t) = 0$ and $\underset{t\to\infty}{Lim}\, e(t) = 0$. So there exists a neighbourhood of (0,0), for each initial state and each possible exogenous input $(x(0),\omega(0))$, $\lim_{t\to\infty} e(t) = 0$.

3. REAL-TIME IMPLEMENTATION

3.1 Description and Model of the Pendubot.

To demonstrate the validity of the proposed fuzzy output control law, a real-time implementation of the control strategy was developed for a benchmark underactuated robot: Pendubot. The Pendubot consists of two rigid aluminum links. The first link is coupled to a DC motor mounted to a base. Link 2 is coupled to the end of link 1 and in the joint there are no actuator. The angular positions of link 1 and 2 are measured using two high resolution optical encoders. The design gives both links full 360 degrees of rotational motion (Spong and Block, 1995).

The schematic diagram of the Pendubot is illustrated in Figure 1.

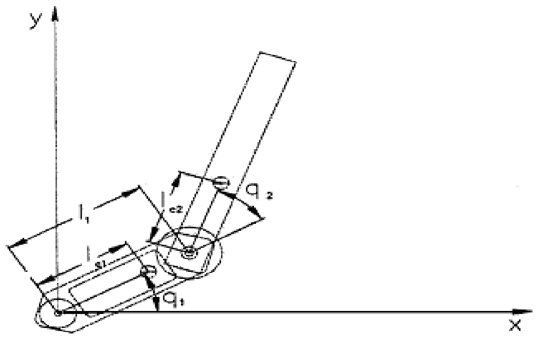

Fig. 1. Pendubot Scheme

Nonlinear system models: The dynamic matrix equation of motion for the Pendubot is given by (Spong and Block, 1995);

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau \qquad (5)$$

where τ is the vector of torque applied to the links and q is the vector of joint angle positions with

$$D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$d_{12} = d_{21} = m_2\left(l_{c2}^2 + l_1 l_{c2}\cos q_2\right) + I_2$$
$$d_{22} = m_2 l_{c2}^2 + I_2$$

$$C(q,\dot{q}) = \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}$$
$$h = -m_2 l_1 l_{c2}\sin q_2$$

$$g(q) = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

where l_1 is the length of link 1, l_{c1} is the distance of the center of mass of link 1, l_{c2} is the distance of the center of mass of link 2, m_1 is the total mass of link 1, m_2 is the total mass of link 2, I_2 is the moment of

$$\phi_1 = (m_1 l_{c1} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g$$
$$\cos(q_1 + q_2)$$
$$\phi_2 = m_2 g l_{c2} \cos(q_1 + q_2)$$
$$F(\dot{q}) = \begin{bmatrix} F_1(\dot{q}) \\ F_2(\dot{q}) \end{bmatrix} = \begin{bmatrix} k_{\mu 1}\dot{q}_1 \\ k_{\mu 2}\dot{q}_2 \end{bmatrix}$$

inertia of link one, I_1 is the moment of inertia of link two about its centroid. $k_{\mu 1}$ is the friction constant of link 1. $k_{\mu 2}$ is the friction constant of link 2, q_1 and q_2 are angular positions of the respective links.

Using the invertible property of the mass matrix D(q), the state equations are given by:

$$x_1 = q_1,\ x_2 = \dot{q}_1,\ x_3 = q_2,\ x_4 = \dot{q}_2$$

The state representation for equation (5) is

$$\dot{x}(t) = f(x) + g(x)u(t) = F(x(t), u(t), t) \qquad (6)$$

where $u(t) = [\tau_1 \quad 0]^T$ is the input vector and the output is selected as y(t)=q_2.

Linear local model: A local linear model for a specific equilibrium point can be obtaining using Taylor series. For the Pendubot every equilibrium point should satisfy: $q_2 + q_1 = 90^0$

Linearization of equation (6) by Taylor series is given as:

$$F_x(t) = \frac{\partial F(x(t), u(t), t)}{\partial x}\Big|_{x^0, u^0}$$
$$F_u(t) = \frac{\partial F(x(t), u(t), t)}{\partial u}\Big|_{x^0, u^0}$$

where

$x^0 = [q_1{}^0, \dot{q}_1{}^0, q_2{}^0, \dot{q}_2{}^0]^T$ and $u^0 = [\tau_1^0 \quad 0]^T$ are the values of x and u for the specific equilibrium point.

Defining $x := \delta x$; $A := F_x(t)$; $B := F_u(t)$, the linear local model is:

$$\dot{x} = Ax(t) + Bu(t)$$

3.2 The experimental application:

In this section the real-time implementation of the proposed algorithm to the Pendubot is discussed. The objective is to force the angular position of link 2 (q_2) to follow a sinusoidal signal of 60^0 amplitude. In order to track this signal, it is required that at every trajectory point the equality $q_1 + q_2 = 90^0$ has to be filled.

Fuzzy plant: To model the Pendubot, the five fuzzy sets are proposed in Figure 2. The notation is BP: Big Positive, MP: Medium Positive; Z: Zero; MN: Medium Negative; BN: Big Negative. To obtain the linear models for the consequents, the nonlinear model of Pendubot is linearized around the following equilibrium points

$$x_i^0 = [q_{1i}^0, 0, q_{2i}^0, 0],\ i = 1,...5$$

Where $q_{21}^0 = 70^0$, $q_{22}^0 = 35^0$, $q_{23}^0 = 0^0$, $q_{24}^0 = -35^0$, $q_{25}^0 = -70^0$ and $q_{1i}^0 = 90^0 - q_{2i}^0$, $i = 1,...5$

For each equilibrium point a linear system (A_i, B_i), $i = 1,...,5$ is obtained. The values of (A_i, B_i) are omitted here due to the limitation of space. Since the output y is q_2, thus C=[0 0 1 0] is valid for all local systems. The external perturbation is not considered for this system and hence the matrix P=0.

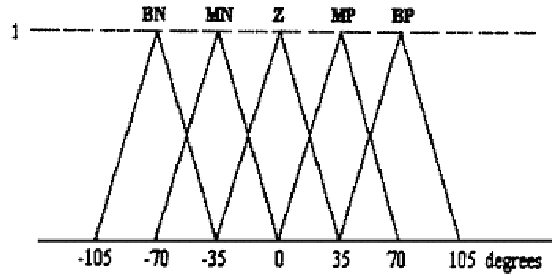

Fig.2. Fuzzy Sets

The control goal is to track a sinusoidal signal. In order to obtain it, for each equilibrium point an additional signal (offset) has be added to the sinusoidal signal; hence the following reference has to be generated.

$$y_r = u_{ofsi} + k \sin \alpha t; \qquad i = 1,...,5$$

where u_{ofsi} is the required offset for each equilibrium point and it is equal to q_{2i}^0, k is the amplitude of the sinusoidal signal and α is its angular frequency.

The exosystem for each linear region is given as:

$$\dot{\omega}(t) = S\omega(t);$$
$$y_r(t) = Q_i \omega(t)$$

where:

$$\omega^T = [\omega_1(t) \quad \omega_2(t) \quad \omega_3(t)]$$
$$\omega_0{}^T = [1 \quad 0 \quad 1]$$
$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha \\ 0 & -\alpha & 0 \end{bmatrix} \qquad \forall i = 1,...,5$$
$$Q_i = [-u_{ofsi} \quad -k \quad 0]; \qquad i = 1,...,5$$

Fuzzy controller: The fuzzy sets for the controller are the same as for the plants. For each $(A_i, B_i, C_i, P_i, S, Q_i)$ the matrices Γ_i and Π_i are determined from (1). The L_i is calculated using (2), each K_i is selected according to linear optimal control theory. The local control signals are obtained from (4). Finally the control signal applied to the system is given by (5).

3.3 Experiments Results

The schematic diagram of the experimental setup is shown in Fig. 3. The motor is a high torque 90VDC permanent magnet one. Link position is measured with 1024 encorder and a servo amplifer 25A8PWM is used to drive the motor. It was operated in torque mode. The DAC card provides analogic signal between –10 and +10 volts. The control algorithm was implemented in C language, on a Pentium IBM PC 125 MHZ. The sampling time is 5ms.

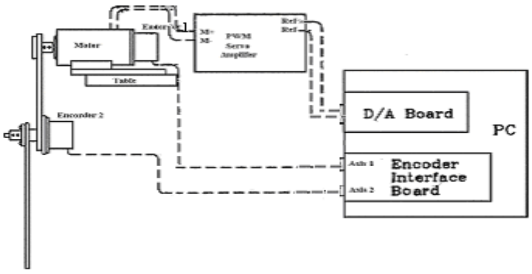

Fig. 3. Experiment setup configuration

Figure 4 shows results when the sinusoidal signal to be tracked is 60 sin0.5t. In this case, the close loop system presents good tracking and there are not control signal saturations.

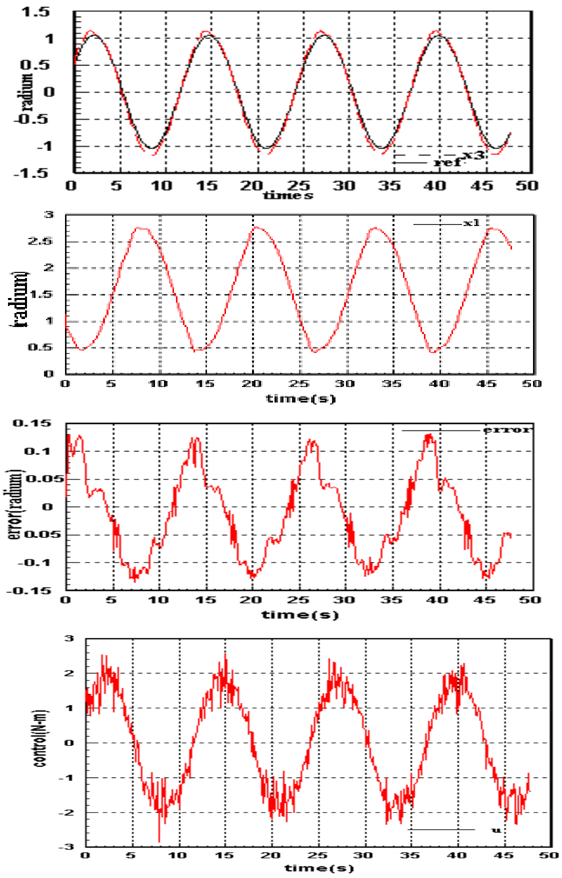

Fig.4. Starting from top: Link 2 angular position; link 1 angular position, tracking error and control signal.

4. CONCLUSIONS

Based on the T-S fuzzy model, an optimal fuzzy control algorithm is proposed in this paper to deal with the output control of complex nonlinear systems via state feedback. The advantage of the proposed control design is that only a simple fuzzy controller is used as an alternative to the complexity design. The proposed algorithm was implemented on an underactuated robot, Pendubot. The experimental results confirm the validity of the accurate output tracking capability and the robust performance.

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