

## ASYMPTOTIC STABLE TRACKING FOR ROBOT MANIPULATORS VIA SECTORIAL FUZZY CONTROL<sup>1</sup>

Victor Santibañez,\* Rafael Kelly\*\* and Miguel Llama\*

\* *Instituto Tecnológico de la Laguna*  
*Apdo. Postal 49, Adm. 1*  
*Torreón, Coahuila, 27001, MEXICO*  
*e-mail: vsantiba@itlalaguna.edu.mx,*  
*Fax: + 52 (871) 7-13-09-70*

\*\* *División de Física Aplicada, CICESE*  
*Apdo. Postal 2615, Adm. 1,*  
*Carretera Tijuana-Ensenada Km. 107*  
*Ensenada, B. C., 22800, MEXICO*  
*e-mail: rkelly@cicese.mx, Fax: + 52 (646) 1-75-05-54*

**Abstract:** This paper shows that fuzzy control systems satisfying sectorial properties are effective for motion tracking control of robot manipulators. We propose a controller whose structure is composed by a sectorial fuzzy controller plus a full non-linear robot dynamics compensation, in such a way that this structure leads to a very simple closed-loop system represented by an autonomous non-linear differential equation. We demonstrate via Lyapunov theory, that the closed-loop system is globally asymptotically stable. Experimental results show the performance of the proposed controller.

**Keywords:** Fuzzy control, stability analysis, robot control.

### 1. INTRODUCTION

Most of fuzzy control applications use a class of fuzzy controllers which have specific sectorial properties of their input-output mappings. This class of fuzzy controllers, so-called Sectorial Fuzzy Controllers (SFC), has very interesting sectorial properties, which have been presented in Calcev (1998) and Calcev *et al* (1998). A SFC has two inputs and one output and it can be characterized from an input-output point of view as a nonlinear static mapping.

The sectorial properties of the SFC allow us to face up one of the most controversial issues in

fuzzy control: the stability analysis of the fuzzy closed-loop systems.

On the other hand, the application of fuzzy techniques to control the motion of robot manipulators has grown in recent years. Some of these works can be found in Fukuda and Shibata (1992), Meslin *et al* (1993), Begon *et al* (1995), Commuri and Lewis (1996), Hsu *et al* (1997), Kelly *et al* (1999), Yoo and Ham (2000), Llama *et al* (2000), Santibañez *et al* (2000), Llama *et al* (2001). The structure of fuzzy controllers used in Hsu *et al* (1997), Kelly *et al* (1999), Llama *et al* (2000) and Llama *et al* (2001) is based on fuzzy tuning algorithms to select the Proportional and Derivative (PD) gains according to the actual position error. For these fuzzy controllers, global asymptotic stability of the closed-loop system has been proven.

---

<sup>1</sup> Work partially supported by COSNET and CONACYT grants No. 31948-A and 32613-A.

In this paper, inspired by Calcev (1998), instead of using fuzzy techniques for tuning PD gains, we propose a motion tracking controller for robot manipulators based on a sectorial fuzzy scheme. Its structure is composed by a sectorial fuzzy controller plus a full non-linear robot dynamics compensation, in such a way that this structure leads to a very simple closed-loop system, which is represented by an autonomous non-linear differential equation. We prove via Lyapunov theory, that the closed-loop system—in absence of friction—is globally asymptotically stable. The performance of the proposed control scheme is illustrated via real-time control experiments on a two degrees of freedom direct-drive vertical robot arm.

## 2. ROBOT DYNAMICS

In the absence of friction, the dynamics of a serial  $n$ -link robot can be written as (Spong and Vidyasagar, 1989):

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \quad (1)$$

where  $\mathbf{q}$  is the  $n \times 1$  vector of joint displacements,  $\dot{\mathbf{q}}$  is the  $n \times 1$  vector of joint velocities,  $\boldsymbol{\tau}$  is the  $n \times 1$  vector of applied torque inputs,  $M(\mathbf{q})$  is the  $n \times n$  symmetric positive definite manipulator inertia matrix,  $C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$  is the  $n \times 1$  vector of centripetal and Coriolis torques, and  $\mathbf{g}(\mathbf{q})$  is the  $n \times 1$  vector of gravitational torques.

## 3. PROPOSED FUZZY MOTION TRACKING CONTROLLER

The motion control problem of manipulators in joint space can be stated in the following terms. Assume that joint position  $\mathbf{q}$  and joint velocity  $\dot{\mathbf{q}}$  are available for measurement. Let the desired joint position  $\mathbf{q}_d$  be a twice differentiable vector function. We define a motion controller as a controller to determine the actuator torques  $\boldsymbol{\tau}$  in such a way that the following control aim be achieved

$$\lim_{t \rightarrow \infty} \mathbf{q}(t) = \mathbf{q}_d(t).$$

To solve this problem we resort in this paper to fuzzy control strategies. The structure of the proposed sectorial fuzzy motion control strategy is captured by the following control law

$$\boldsymbol{\tau} = M(\mathbf{q})[\ddot{\mathbf{q}}_d + \Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})] + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) \quad (2)$$

where  $\mathbf{q}_d$ ,  $\dot{\mathbf{q}}_d$  and  $\ddot{\mathbf{q}}_d$  are the  $n \times 1$  vectors of desired position, desired velocity and desired acceleration, respectively. The joint position error is denoted by the  $n \times 1$  vector  $\tilde{\mathbf{q}} = \mathbf{q}_d - \mathbf{q}$  while  $\dot{\tilde{\mathbf{q}}} = \dot{\mathbf{q}}_d - \dot{\mathbf{q}}$  stands

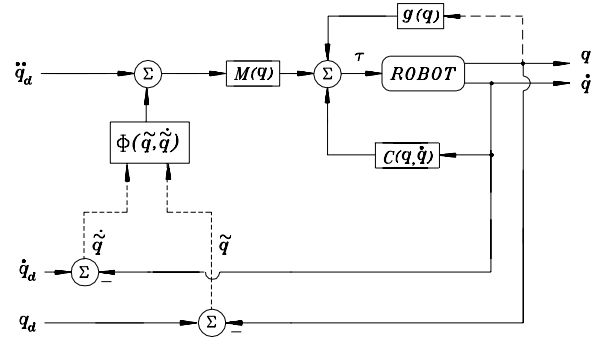


Fig. 1. Closed-loop system.

for the  $n \times 1$  vector of velocity error.  $\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a  $n \times 1$  vector whose entries  $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$ , with  $i = 1, 2, \dots, n$  are the real input-output mappings of the sectorial fuzzy controller (SFC), whose properties were established in Calcev (1998) and that we list later on.  $\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a decoupled nonlinear mapping (in the sense that  $\phi_i(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  depends only on  $\tilde{q}_i, \dot{\tilde{q}}_i$ ) of the form

$$\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) = \begin{bmatrix} \phi_1(\tilde{q}_1, \dot{\tilde{q}}_1) \\ \phi_2(\tilde{q}_2, \dot{\tilde{q}}_2) \\ \vdots \\ \phi_n(\tilde{q}_n, \dot{\tilde{q}}_n) \end{bmatrix}. \quad (3)$$

The proposed control law (2) is composed by a sectorial fuzzy controller plus a full non-linear robot dynamics compensation.

In order to analyse the stability of the closed-loop system we recall some interesting properties of the SFC, whose proofs were presented in Calcev (1998).

- **Property 1.**  $\phi_i(0, 0) = 0$
- **Property 2.**  $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i) = -\phi_i(-\tilde{q}_i, -\dot{\tilde{q}}_i)$
- **Property 3.** There exist  $\lambda, \gamma > 0$  such that

$$0 < \tilde{q}_i[\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i) - \phi_i(0, \dot{\tilde{q}}_i)] \leq \lambda \tilde{q}_i^2 \quad \forall \tilde{q}_i \neq 0.$$

$$0 \leq \dot{\tilde{q}}_i[\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i) - \phi_i(\tilde{q}_i, 0)] \leq \gamma \dot{\tilde{q}}_i^2$$

- **Property 4.**  $\phi_i(\tilde{q}_i, 0) = 0 \Rightarrow \tilde{q}_i = 0$ .

## 4. STABILITY ANALYSIS

The closed-loop system, whose block diagram is shown in Figure 1, is obtained by combining the robot dynamic model (1) with the control law (2). This can be written as:

$$\frac{d}{dt} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} = \begin{bmatrix} \dot{\tilde{\mathbf{q}}} \\ -\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) \end{bmatrix} \quad (4)$$

which is an autonomous nonlinear differential equation and, due to property 4 of  $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$ , the origin of the state space is an equilibrium point.

To carry out the stability analysis we propose the following Lyapunov function candidate:

$$V(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) = \frac{1}{2} \dot{\tilde{\mathbf{q}}}^T \dot{\tilde{\mathbf{q}}} + \sum_{i=1}^n \int_0^{\tilde{q}_i} \phi_i(\xi_i, 0) d\xi_i. \quad (5)$$

The first term of  $V(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a positive definite function with respect to  $\dot{\tilde{\mathbf{q}}}$ . For the second term of (5), notice that, from properties 1 and 3 of  $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$ , it results that  $0 < \tilde{q}_i \phi_i(\tilde{q}_i, 0) \leq \lambda \tilde{q}_i^2$ , for all  $\tilde{q}_i \neq 0$ . This means that  $\phi(\tilde{q}_i, 0)$  belongs to the sector  $(0, \lambda]$  and hence  $\int_0^{\tilde{q}_i} \phi_i(\xi_i, 0) d\xi_i > 0 \quad \forall \tilde{q}_i \neq 0$ , and  $\int_0^{\tilde{q}_i} \phi_i(\xi_i, 0) d\xi_i \rightarrow \infty$  as  $\tilde{q}_i \rightarrow \infty$ , so that,  $V(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a globally positive definite and radially unbounded function, therefore (5) qualifies as a Lyapunov function candidate.

The time derivative of the Lyapunov function candidate is

$$\begin{aligned} \dot{V}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) &= \dot{\tilde{\mathbf{q}}}^T \ddot{\tilde{\mathbf{q}}} + \sum_{i=1}^n \frac{\partial}{\partial \tilde{q}_i} \left[ \int_0^{\tilde{q}_i} \phi_i(\xi_i, 0) d\xi_i \right] \dot{\tilde{q}}_i \\ &= \dot{\tilde{\mathbf{q}}}^T \ddot{\tilde{\mathbf{q}}} + \dot{\tilde{\mathbf{q}}}^T \Phi(\tilde{\mathbf{q}}, \mathbf{0}) \end{aligned} \quad (6)$$

where we have used the Leibnitz' rule for differentiation of integrals. By using (4), the time derivative of the Lyapunov function candidate along of the closed-loop system trajectories yields

$$\dot{V}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) = -\dot{\tilde{\mathbf{q}}}^T \left[ \Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) - \Phi(\tilde{\mathbf{q}}, \mathbf{0}) \right].$$

Since  $\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a decoupled nonlinearity of the form (3), we can use property 3 of  $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$  to conclude that  $\dot{V}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a globally negative semidefinite function. Thus by invoking the Lyapunov's direct method (Vidyasagar, 1993) we conclude stability of the closed-loop system.

In order to prove global asymptotic stability we exploit the autonomous nature of the closed-loop system (4) to apply the Krasovskii-LaSalle's theorem (Vidyasagar, 1993). In the region

$$\begin{aligned} \Omega &= \left\{ \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} : \dot{V}(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) = 0 \right\} \\ &= \left\{ \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{q}} \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^{2n} \right\} \end{aligned}$$

the unique invariant is  $[\tilde{\mathbf{q}}^T \quad \dot{\tilde{\mathbf{q}}}^T]^T = \mathbf{0}$ , because, from the closed-loop equation (4), we have  $\dot{\tilde{\mathbf{q}}} = \mathbf{0} \Rightarrow \Phi(\tilde{\mathbf{q}}, \mathbf{0}) = \mathbf{0}$  and, from properties 1 and 4,  $\Phi(\tilde{\mathbf{q}}, \mathbf{0}) = \mathbf{0} \Leftrightarrow \tilde{\mathbf{q}} = \mathbf{0}$ . Therefore invoking the Krasovskii-LaSalle's theorem we conclude that the origin of the state space is a globally

asymptotically stable equilibrium of the closed-loop system (4).

## 5. DESIGN OF THE SECTORIAL FUZZY CONTROLLER

The following SFC was designed following the steps given in Calcev (1989) to satisfy the sectorial properties of this mapping.

A fuzzy rule base, for the two input case, consists of a set of fuzzy IF-THEN rules comprising the following rules (named  $R^{l_1 l_2}$ )

$$IF \ x_1 \text{ is } A_1^{l_1} \text{ AND } x_2 \text{ is } A_2^{l_2} \text{ THEN } y \text{ is } B^{l_1 l_2}, (7)$$

where  $l_i = -\frac{N_i-1}{2}, \dots, -1, 0, 1, \dots, \frac{N_i-1}{2}$ , and, for two inputs  $i = 1, 2$ .  $N_1$  is the number of fuzzy sets of input 1, and  $N_2$  is the number of fuzzy sets of input 2. The designed fuzzy rule base is summarized in the look-up table (Table 1 shown ahead).

This fuzzy rule base is chosen in such a way that sectorial properties be satisfied, namely (Calcev, 1989); the control rules are symmetric with respect to the inputs  $x_1$  and  $x_2$ ; the output is null for null inputs (the central area of the look-up table is zero) and; within a row the control action increases gradually from left to right, and within a column this increases from top to bottom. The total number of rules is  $M = N_1 N_2$ .

Since according to (3)  $\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$  is a decoupled system, we can focus on designing a single fuzzy system; i.e.  $\phi_1(\tilde{q}_1, \dot{\tilde{q}}_1)$ , and for facility we rename it as  $\phi(x_1, x_2)$ , where  $x_1 = \tilde{q}_1$  and  $x_2 = \dot{\tilde{q}}_1$ . In order to build this fuzzy system we select singleton fuzzification, product premises connective, product inference, and center average defuzzifier. Then  $\phi(x_1, x_2)$  can be carried out by the well known fuzzy system with center average defuzzifier (Wang, 1997) whose equation for two inputs is given by

$$\begin{aligned} \phi(x_1, x_2) &= \\ &= \frac{\sum_{l_1=-\frac{N_1-1}{2}}^{\frac{N_1-1}{2}} \sum_{l_2=-\frac{N_2-1}{2}}^{\frac{N_2-1}{2}} \bar{y}^{l_1 l_2} \left( \prod_{i=1}^2 \mu_{A_i^{l_i}}(x_i) \right)}{\sum_{l_1=-\frac{N_1-1}{2}}^{\frac{N_1-1}{2}} \sum_{l_2=-\frac{N_2-1}{2}}^{\frac{N_2-1}{2}} \left( \prod_{i=1}^2 \mu_{A_i^{l_i}}(x_i) \right)} \end{aligned} \quad (8)$$

where  $\mu_{A_i^{l_i}}(x_i)$  represents the membership function number  $l_i$  associated to the fuzzy set  $A_i^{l_i}$  for the  $i$ -th input, and  $\bar{y}^{l_1 l_2}$  is the center of the output membership function corresponding to the consequent of rule  $R^{l_1 l_2}$ .

$x_1 \backslash x_2$	$l_2 = -2$ NB	$l_2 = -1$ NS	$l_2 = 0$ Z	$l_2 = 1$ PS	$l_2 = 2$ PB
$l_1 = -2$ NB	NB	NB	NS	Z	Z
$l_1 = -1$ NS	NB	NB	NS	Z	PS
$l_1 = 0$ Z	NS	NS	Z	PS	PS
$l_1 = 1$ PS	NS	Z	PS	PB	PB
$l_1 = 2$ PB	Z	Z	PS	PB	PB

Table 1. Look-up table for the fuzzy rule base.

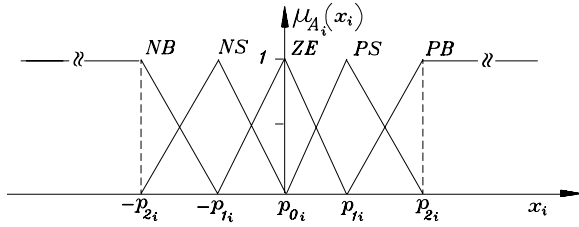


Fig. 2. Input membership functions.

### 5.1 Design parameters

The universe of discourse for both inputs  $x_1$  and  $x_2$  were partitioned in five fuzzy sets:  $A_i^{-2} = \text{NB}$  (Negative Big),  $A_i^{-1} = \text{NS}$  (Negative Small),  $A_i^0 = \text{Z}$  (Zero),  $A_i^1 = \text{PS}$  (Positive Small), and  $A_i^2 = \text{PB}$  (Positive Big). We selected triangular membership functions  $\mu_{A_i^{l_2}}(x_i)$  for both inputs, and singleton membership functions  $\mu_{B^{\bar{y}}}(y)$  for the output, where  $\bar{y} = \{-\bar{y}_2, -\bar{y}_1, \bar{y}_0, \bar{y}_1, \bar{y}_2\}$ , is chosen in agreement with the consequent of the fired rule  $R^{l_1 l_2}$  in the look-up table. As well, the output  $y$  was partitioned in five fuzzy sets:  $B^{\bar{y}_2} = \text{NB}$  (Negative Big),  $B^{\bar{y}_1} = \text{NS}$  (Negative Small),  $B^{\bar{y}_0} = \text{Z}$  (Zero),  $B^{\bar{y}_1} = \text{PS}$  (Positive Small), and  $B^{\bar{y}_2} = \text{PB}$  (Positive Big).

In all cases the input and output membership functions are symmetrical with respect to zero. This is shown in Figures 2 and 3, where  $\mathbf{p}_{A_i} = \{-p_{2i}, -p_{1i}, p_{0i}, p_{1i}, p_{2i}\}$  is the set of the support parameters bounds (also called fuzzy partition of the universe of discourse) which defines the input membership functions and  $\mathbf{p}_B = \{-\bar{y}_2, -\bar{y}_1, \bar{y}_0, \bar{y}_1, \bar{y}_2\}$  is the set of the support parameters bounds for the associated output membership functions. Also, for any input, the sum of the membership values of two adjacent fuzzy sets is one, and the membership value for any  $x_i > p_{2i}, x_i < -p_{2i}$  is equal to one. Since both inputs have 5 fuzzy sets ( $N_1 = 5$  and  $N_2 = 5$ ), then the number of rules is  $M = 25$ . These rules are shown in Table 1.

## 6. EXPERIMENTAL EVALUATION

Real-time control experiments on a well identified direct-drive robot arm have been carried out to

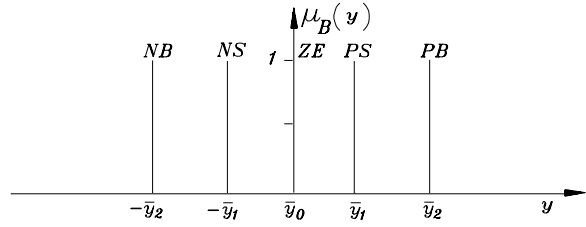


Fig. 3. Output membership functions.

evaluate the performance of the proposed controller. The two degrees of freedom direct-drive robot arm model is given in Reyes and Kelly (1997). It is worth mentioning that Coulomb and static friction are also present in the robot joints, however, we have decided to consider them as unmodeled dynamics. Two types of experiments were conducted: The first one was made with the well known computed torque control with fixed gains (Spong and Vidyasagar, 1989), and the second one with the proposed sectorial fuzzy tracking controller (2).

Special attention was paid in all cases to avoid torque saturation; that is, we take into account the actuator torque limits provided by the manufacturer:  $\tau_1^{\max} = 150$  [Nm] and  $\tau_2^{\max} = 15$  [Nm].

The desired position trajectory  $\mathbf{q}_d$  given by

$$\mathbf{q}_d = \begin{bmatrix} 1.57 \\ 1.57 \end{bmatrix} + \begin{bmatrix} 0.78[1 - e^{-2.0 t^3}] + 0.17[1 - e^{-2.0 t^3}] \sin(\omega_1 t) \\ 1.04[1 - e^{-1.8 t^3}] + 2.18[1 - e^{-1.8 t^3}] \sin(\omega_2 t) \end{bmatrix} \quad [\text{rad}] \quad (9)$$

was inspired from the structure of desired trajectories used by other authors for experimental evaluation of control algorithms (Dawson *et al*, 1994; De Queiroz *et al*, 1996). In our application, the second term of (9) was chosen in such a way to exploit the arm in its fastest motion but without invading the actuators saturating zone, and the first one was chosen to add a step reference to demand an initial big torque.

In expression (9),  $\omega_1$  and  $\omega_2$  represent the frequency of desired trajectory for the shoulder and elbow joints respectively. In our simulation tests, we use  $\omega_1 = 15$  rad/sec and  $\omega_2 = 3.5$  rad/sec.

The final partitions of the universes of discourse, for implementation of the proposed controller, were: For the input  $\tilde{\mathbf{q}}$ :  $\mathbf{p}_{\tilde{q}_1} = \{-180, -7, 0, 7, 180\}$  [degrees] and  $\mathbf{p}_{\tilde{q}_2} = \{-90, -0.5, 0, 0.5, 90\}$  [degrees]; for the input  $\dot{\tilde{\mathbf{q}}}$ :  $\mathbf{p}_{\dot{\tilde{q}}_1} = \{-180, -45, 0, 45, 180\}$  [degrees/sec] and  $\mathbf{p}_{\dot{\tilde{q}}_2} = \{-180, -90, 0, 90, 180\}$  [degrees/sec]; for the output  $\Phi(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$ :  $\mathbf{p}_{\phi_1(\tilde{q}_1, \dot{\tilde{q}}_1)} = \{-7000, -2800, 0, 2800, 7000\}$  [degrees/sec<sup>2</sup>] and

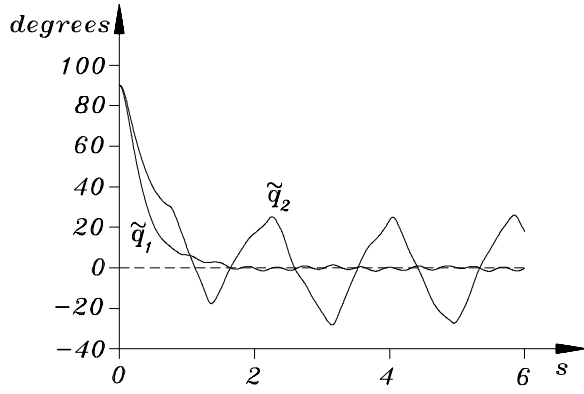


Fig. 4. Position errors for the classical computed torque control.

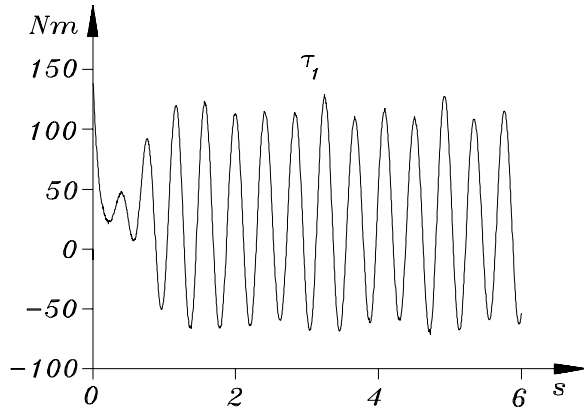


Fig. 5. Applied torque to link 1 for the classical computed torque control.

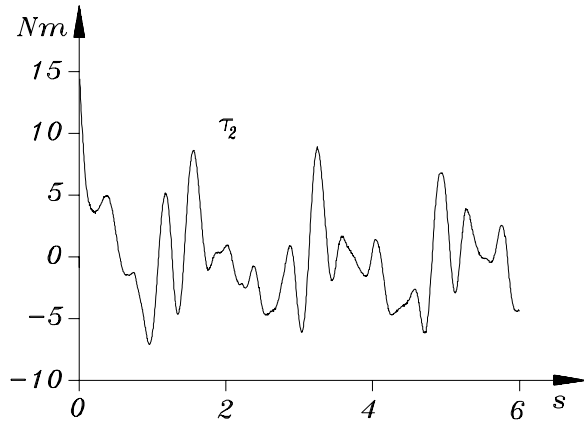


Fig. 6. Applied torque to link 2 for the classical computed torque control.

$\mathbf{p}_{\phi_2(\tilde{q}_2, \dot{\tilde{q}}_2)} = \{-6000, -3000, 0, 3000, 6000\}$  [degrees/sec<sup>2</sup>], where each parameters set denotes the support bounds of corresponding membership functions.

The experimental results are depicted in Figures 4 to 9. Figures 4, 5 and 6 show position errors and applied torques for the fixed gains computed torque controller; Figures 7, 8 and 9 show the same variables for the proposed sectorial fuzzy tracking controller (2).

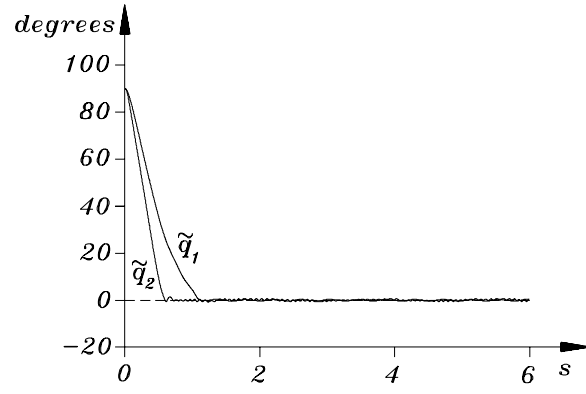


Fig. 7. Position errors for the proposed sectorial fuzzy control.

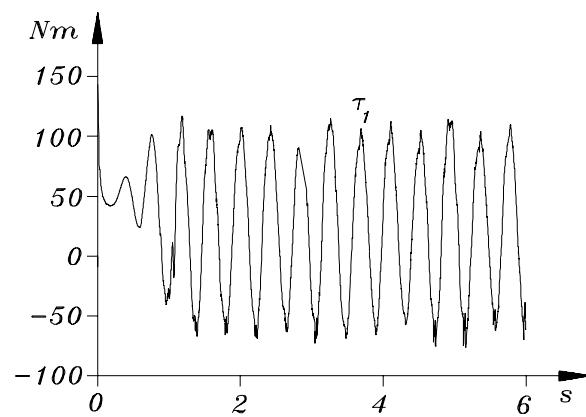


Fig. 8. Applied torque to link 1 for the proposed sectorial fuzzy control.

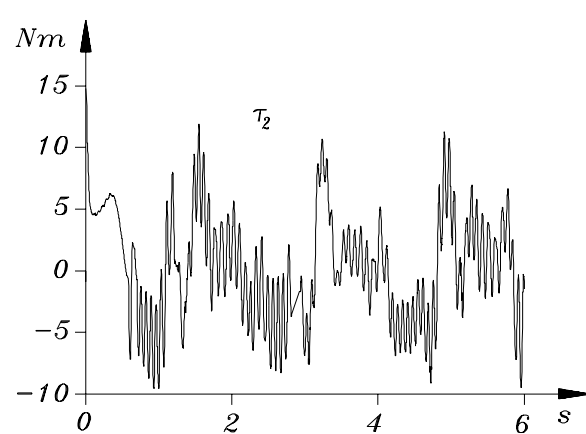


Fig. 9. Applied torque to link 2 for the proposed sectorial fuzzy control.

As one can see, from Figure 4 the fixed gains computed torque control can not reduce, near to zero, the position errors due mainly to the effect of the unmodeled friction on the joints. In this case, the experimental results are really disappointing because the components of the position error have unacceptable large oscillatory behavior. There was no way to reduce the position tracking error by increasing the proportional gains without saturating the actuators.

In contrast, the proposed sectorial fuzzy tracking controller (2) present considerably smaller position errors—see Figure 7—and its torques remain inside the prescribed allowable maximum torque for each actuator—see Figures 8 and 9—.

We may conclude that this kind of desired position trajectory given by equation (9) (an step plus a fast periodical signal) is too severe to be tracked by the classical Computed-torque controller, however, in spite of the trajectory severity, the fuzzy proposed controller presents a very good performance.

## 7. CONCLUSIONS

We have proposed a sectorial fuzzy based motion controller to cope with the tracking control of robot manipulators. Using some properties of the sectorial fuzzy controllers, Lyapunov theory as well as LaSalle's Theorem, we have proven—in absence of friction—that the closed-loop system is globally asymptotically stable. Experimental results show the advantages of the proposed controller.

## REFERENCES

- Begon G., F. Pierrot and P. Dauchez (1995), Fuzzy sliding mode control of a fast parallel robot, in *Proc. of the IEEE International Conference on Robotics and Automation*, Nagoya, Japan, 1178–1183.
- Calcev G. (1998), Some remarks on the stability of Mamdani fuzzy control systems, *IEEE Transactions on Fuzzy Systems*, **6**, No. 3, 436–442.
- Calcev G., R. Gorez and M. de Neyer (1998), Passivity approach to fuzzy control systems, *Automatica*, **34**, No. 3, 339–344.
- Commuri S. and F.L. Lewis (1996), Adaptive-fuzzy logic control of robots manipulators, in: *Proc. of the IEEE International Conference on Robotics and Automation*, Minneapolis, Minnesota, 2604–2609.
- Dawson D. M., J. J. Carroll and M. Schneider (1994), Integrator backstepping control of a brush DC motor turning a robotic load, *IEEE Transactions on Control System Technology*, **2**, 233–244.
- De Queiroz M. S., D. Dawson and T. Burg (1996), Reexamination of the DCAL controller for rigid link robots, *Robotica*, **14**, 41–49.
- Driankov D., H. Hellendoorn and M. Reinfrank M. (1996), *An Introduction to Fuzzy Control*, Springer Verlag, New York.
- Fukuda T. and T. Shibata (1992), Hierarchical intelligent control for robotic motion by using fuzzy, artificial intelligence, and neural networks, in *Proc. Int. J. Conference on Neural Networks*, Baltimore, MD, **1**, 269–274.
- Hsu Y. Ch., G. Chen and E. Sanchez (1997), A fuzzy PD controller for multi-link robot control: stability analysis, *Proc. of the IEEE International Conference on Robotics and Automation*, Albuquerque, New Mexico, 1412–1417.
- Kelly R., R. Haber, R. E. Haber and F. Reyes (1999), Lyapunov stable control of robot manipulators: A fuzzy self-tuning procedure, *Intelligent Automation and Soft Computing*, **5**, No. 4, 313–326.
- Llama M. A., R. Kelly, and V. Santibañez (2000), Stable computed-torque control of robot manipulators via fuzzy self-tuning, *IEEE Transactions on Systems, Man, and Cybernetics-Part B*, **30**, 143–150.
- Llama M. A., R. Kelly, and V. Santibañez (2001), A stable motion control system for manipulators via fuzzy self-tuning, *Fuzzy Sets and Systems*, **124**, 133–154.
- Meslin J. M., J. Zhou and P. Coiffet (1993), Fuzzy dynamic control of robot manipulators: A scheduling approach, in *Proc. IEEE Int. Conf. on Syst. Man and Cybern.*, Le Tourquet, France, 69–73.
- Reyes F. and R. Kelly (1997), Experimental evaluation of identification schemes on a direct-drive robot, *Robotica*, **15**, 563–571.
- Santibañez V., R. Kelly and M. A. Llama (2000), Fuzzy PD+ control for robot manipulators, *Proc. of the IEEE International Conference on Robotics and Automation*, San Francisco, CA, USA. 2112–2117.
- Spong M. and M. Vidyasagar (1989), *Robot Dynamics and Control*, John Wiley and Sons, NY.
- Vidyasagar M. (1993), *Nonlinear Systems Analysis*, Prentice-Hall, Englewood Cliffs, NJ.
- Wang L. X. (1997), *A Course in Fuzzy Systems and Control*, Prentice Hall PTR, Upper Saddle River N.J.
- Yoo B. K. and W. C. Ham (2000), Adaptive control of robot manipulators using fuzzy compensator, *IEEE Trans. on Fuzzy Systems*, **8**, No. 2, 186–199.