15th Triennial World Congress, Barcelona, Spain

NONLINEAR BACKSTEPPING CONTROL FOR THE FUEL DIFFUSION LAG IN FUSION REACTORS [1]

Eugenio Schuster * **Miroslav Krstić** * **George Tynan** *

* *Department of Mechanical and Aerospace Engineering*
University of California at San Diego
La Jolla, CA 92093-0411
fax: (858) 822-3107

schuster@mae.ucsd.edu	*krstic@ucsd.edu*	*gtynan@ucsd.edu*
(858) 822-1936	*(858) 822-1374*	*(858) 534-9724*

Abstract: Control of plasma density and temperature magnitudes, as well as their profiles, are among the most fundamental problems in fusion reactors. Existing efforts use control techniques based on linearized models. In this work, a zero-dimensional nonlinear model involving approximate conservation equations for the energy and the densities of the species was used to synthesize a nonlinear feedback controller for stabilizing the burn condition of a fusion reactor. The model addresses the issue of the lag due to the finite time for the fresh fuel to diffuse into the plasma center. Nonlinear backstepping is used to deal with this imposed lag. In this way we make our control system independent of the fueling system and the reactor can be fed either by pellet injection or by gas puffing. The controller exhibits excellent properties of robustness and the boundness of the state variables is guaranteed for a large set of values of the lag constant. In addition the nonlinear controller proposed guarantees a much larger region of attraction than the previous linear controllers, it is capable of rejecting perturbations in initial conditions leading to both thermal excursion and quenching, and its effectiveness does not depend on whether the operating point is an ignition or a subignition point.

Keywords: Nuclear reactors, nonlinear control, Lyapunov stability.

1. INTRODUCTION

Economical and technological constraints sometimes require the fusion reactors to operate in a zone of low temperature and high density where the thermonuclear reaction is inherently thermally unstable. For low temperatures, the rate of thermonuclear reaction for a D-T mixture increases as the plasma temperature rises. In this thermally unstable zone, a small increase of temperature leads to an increase of power which results in *thermal excursion*. On the other hand, a small decrease of temperature leads to a decrease of power and *quenching*.

Active burn control is often required to maintain near-ignited or ignited conditions (Auxiliary power $\simeq 0$). The objective of the controller is to keep the plasma at a desired equilibrium or operating point; rejecting perturbations in initial conditions and forcing the plasma back to the equilibrium.

The common denominator of existing works is the approximation of the nonlinear model of the fusion reactor by a linearized one and generally the use of only one among the actuation concepts (single-input control). Schuster, Krstic and Tynan (Schuster *et al.*, 2001) have proposed recently a new approach based on a full nonlinear model that is able to stabilize the system against large perturbations in initial conditions, can work as well for suppressing thermal excursions as for preventing quenches and can operate

[1] This work was supported by a grant from NSF.

both at subignition or ignition points. Nevertheless, the issue of the lag introduced by the diffusion of the fresh fuel into the plasma was not considered. For burn control purposes we can mention mainly two types of fueling systems: pellet injection and gas puffing. Pellet injection is a better actuator in the sense that its neutral fuel transportation time is shorter. However, it is also technically more complex. This technical aspect of the burn control problem forces us to modify the model in order to introduce the effect of the lag due to the diffusion of the fresh fuel into the plasma. We introduce a new tool such as nonlinear backstepping that allows us to handle the lag imposed by the actuator without any kind of further approximation of the model.

Over the years, the physical and technological feasibility of different methods for controlling the burn condition have been studied (Mandrekas and Stacey, 1990; Haney *et al.*, 1990; Anderson *et al.*, 1993). As we want to work at a ignited point ($\bar{P}_{aux} = 0$) or to have the capability of rejecting a large set of perturbations in the initial conditions when working at a subignited point ($\bar{P}_{aux} > 0$), we consider the controlled injection of impurities as an actuator in addition to the auxiliary power and fueling rate modulations. Towards this end, the model used in (Schuster *et al.*, 2001) is modified here modeling the power losses due to the line and recombination radiations in addition to the bremsstrahlung radiation. In this way we are no longer restricted to work only with low Z impurities. In addition, in order to achieve stability with a less demanding injection of impurities, we combine a singular perturbation approach (Schuster *et al.*, 2001) with a passivity approach for the design of the control law for the injection of impurities.

The paper is organized as follows. In Section 2 the model of our plant is stated. In Section 3 the control objectives are presented and the control laws for the different actuators are synthesized in Section 4 for the stabilization of the deviation state variables. Section 5 makes a presentation of the simulation results. Finally, the conclusions and some suggestions about future work are presented in Section 6.

2. MODEL

In this work we use a zero-dimensional model for a fusion reactor which employs approximate particle and energy balance equations. The alpha-particle balance is given by

$$\frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + \left(\frac{n_{DT}}{2}\right)^2 < \sigma v > \tag{1}$$

where n_α and n_{DT} are the alpha and deuterium-tritium (DT) densities respectively, and τ_α is the confinement time for the alpha particles. This approximate model implies that the 3.52 MeV alpha particles slow down instantaneously, depositing their energy in the flux surface where they are born, which is a reasonable approximation for reactor-size tokamaks. A first order lag is introduced to take into account the diffusion time for neutral fuel atoms to transport into the tokamak core. This lag runs from the start of the fuel injection to the change in deuterium-tritium (DT) ion particle density. The set of equations governing the neutral atom balance and the deuterium-tritium (DT) fuel particle balance is given by

$$\frac{dn_{DT}}{dt} = -\frac{n_{DT}}{\tau_{DT}} - 2\left(\frac{n_{DT}}{2}\right)^2 < \sigma v > + \frac{n_n}{\tau_d} \tag{2}$$

$$\frac{dn_n}{dt} = -\frac{n_n}{\tau_d} + S \tag{3}$$

where n_n is the neutral density, S (input) is the refueling rate (50:50 D-T), τ_{DT} is the confinement time for the fuel particles and τ_d is the delay time. The impurity presence is determined by the balance equation

$$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I \tag{4}$$

where n_I is the impurity density, τ_I is the confinement time for the impurity particles and S_I (input) is the impurity injection rate. The energy balance is given by

$$\frac{dE}{dt} = -\frac{E}{\tau_E} + (\frac{n_{DT}}{2})^2 < \sigma v > Q_\alpha - P_{rad} + P_{aux} \tag{5}$$

where E is the plasma energy, τ_E is the energy confinement time, $Q_\alpha = 3.52$ MeV is the energy of the alpha particles, P_{aux} (input) is the auxiliary power and the radiation loss P_{rad} is given by

$$\begin{aligned} P_{rad} = & [A_b \left(n_{DT} + 4n_\alpha + Z_I^2 n_I\right) T^{\frac{1}{2}} \\ & + A_l \left(16n_\alpha + Z_I^4 n_I\right) T^{-\frac{1}{2}} \\ & + A_r \left(64n_\alpha + Z_I^6 n_I\right) T^{-\frac{3}{2}}] n_e \end{aligned} \tag{6}$$

where $A_b = 4.8\,10^{-37} Wm^3/\sqrt{KeV}$, $A_l = 1.8\,10^{-38}$ $Wm^3/\sqrt{KeV}$ and $A_r = 4.1\,10^{-40} Wm^3/\sqrt{KeV}$ are the bremsstrahlung, line and recombination radiation coefficients respectively. The DT reactivity $< \sigma v >$ is a highly nonlinear, positive and bounded function of the plasma temperature T given by

$$< \sigma v >= \exp\left(\frac{a_1}{T^r} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right)$$

and its constant parameters a_i and r are taken from (Hively, 1977). No explicit evolution equation is provided for the electron density n_e since we can obtain it from the neutrality condition

$$n_e = n_{DT} + 2n_\alpha + Z_I n_I$$

whereas the total density and the energy are written as

$$n = n_\alpha + n_{DT} + n_e + n_I = 2n_{DT} + 3n_\alpha + (Z_I + 1) n_I$$

$$E = \frac{3}{2} nT \Rightarrow T = \frac{2}{3} \frac{E}{2n_{DT} + 3n_\alpha + (Z_I + 1) n_I}$$

The energy confinement scaling used in this work is ITER90H-P (Uckan, 1994). Although newer scalings are available, this one, together with the ITER configuration used, allows the comparison with previous linear controllers based on it. However, it will be clear from the synthesis procedure that the results can be extended to the newer scalings. It scales with plasma parameters as

$$\tau_E = f\,0.082 I^{1.02} R^{1.6} B^{0.15} A_i^{0.5} \kappa_\chi^{-0.19} P^{-0.47} = k P^{-0.47} \quad (7)$$

where the isotopic number A_i is 2.5 for the 50:50 DT mixture, the ITER machine parameters I, R, B and κ_χ can be obtained from (Schuster *et al.*, 2001) and the factor scale f depends on the confinement mode. The isotopic number, factor scale and ITER machine parameters can be rewritten as a constant k. The net plasma heating power P is defined as

$$P = \left(\frac{n_{DT}}{2}\right)^2 < \sigma v > Q_\alpha - P_{rad} + P_{aux} \quad (8)$$

The confinement times for the different species are scaled with the energy confinement time τ_E as

$$\tau_\alpha = k_\alpha \tau_E, \qquad \tau_{DT} = k_{DT}\tau_E, \qquad \tau_I = k_I \tau_E,$$

where $k_\alpha = 7$ and $k_{DT} = 3$. Since τ_d is related to the finite time for the fresh fuel to diffuse into the plasma center, we also scale it with the energy confinement time τ_E as $\tau_d = k_d \tau_E$.

3. CONTROL OBJECTIVE

The possible operating points of the reactor are given by the equilibria of the dynamic equations. The values of the variables at the equilibrium are denoted by a bar. An ignition point is characterized by $\bar{P}_{aux} = 0$ while at a subignition point we have $\bar{P}_{aux} > 0$. In this case we look for those operating points where $\bar{S}_I = 0$ because we are interested in an operating condition free of impurities. The density state variables $\bar{n}_\alpha, \bar{n}_{DT}, \bar{n}_n, \bar{n}_I$, energy state variable $\bar{E}$ and inputs $\bar{P}_{aux}$, $\bar{S}$ at the equilibrium, are calculated as solutions of the nonlinear algebraic equations obtained by setting the left hand sides in equations (1) - (5) to zero when one of the plasma parameters such as β, for example, is chosen arbitrarily.

Taking into account that $\bar{S}_I = 0$ and $\bar{n}_I = 0$, we define the deviations from the desired equilibrium values as $\tilde{n}_\alpha = n_\alpha - \bar{n}_\alpha$, $\tilde{n}_{DT} = n_{DT} - \bar{n}_{DT}$, $\tilde{n}_n = n_n - \bar{n}_n$, $\tilde{n}_I = n_I - \bar{n}_I = n_I > 0$, $\tilde{E} = E - \bar{E}$, $\tilde{P}_{aux} = P_{aux} - \bar{P}_{aux}$, $\tilde{S} = S - \bar{S}$ and $\tilde{S}_I = S_I - \bar{S}_I = S_I > 0$, we write the dynamic equations for the deviations as

$$\frac{d\tilde{n}_\alpha}{dt} = -\frac{\tilde{n}_\alpha}{\tau_\alpha} + u_\alpha + \left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > + \frac{1}{2}\tilde{n}_{DT}\bar{n}_{DT} < \sigma v > \quad (9)$$

$$\frac{d\tilde{n}_{DT}}{dt} = -\frac{\tilde{n}_{DT}}{\tau_{DT}} + \frac{\tilde{n}_n}{\tau_d} + u_{DT} - 2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > - \tilde{n}_{DT}\bar{n}_{DT} < \sigma v > \quad (10)$$

$$\frac{d\tilde{n}_n}{dt} = S^* \quad (11)$$

$$\frac{d\tilde{n}_I}{dt} = -\frac{\tilde{n}_I}{\tau_I} + S_I \quad (12)$$

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} - \left[\frac{\bar{E}}{\tau_E} - \left[(\frac{n_{DT}}{2})^2 < \sigma v > Q_\alpha + u\right]\right] \quad (13)$$

where

$$u_\alpha = -\frac{\bar{n}_\alpha}{\tau_\alpha} + (\frac{\bar{n}_{DT}}{2})^2 < \sigma v > \quad (14)$$

$$u_{DT} = -\frac{\bar{n}_{DT}}{\tau_{DT}} - 2(\frac{\bar{n}_{DT}}{2})^2 < \sigma v > + \frac{\bar{n}_n}{\tau_d} \quad (15)$$

$$S^* = -\frac{\tilde{n}_n}{\tau_d} + \tilde{S} - \frac{\bar{n}_n}{\tau_d} + \bar{S} = -\frac{n_n}{\tau_d} + S \quad (16)$$

$$u = P_{aux} - P_{rad} \quad (17)$$

The control objective is to drive the initial perturbations in $\tilde{n}_\alpha, \tilde{n}_{DT}, \tilde{n}_n, \tilde{n}_I, \tilde{E}$ to zero using actuation through P_{aux}, S and $S_I = \tilde{S}_I > 0$. It is important to note that in the ignition case ($\bar{P}_{aux} = 0$) we have $\tilde{P}_{aux} > 0$ as a constraint, we do not have the possibility of modulating P_{aux} in both the positive and negative sense as we have in the subignition case ($\bar{P}_{aux} > 0$). However, the additional actuator $S_I = \tilde{S}_I > 0$, although constrained in sign by itself, helps us to overcome the constraint in P_{aux}.

All the states are assumed to be availabe for feedback, either by measurement or by estimation.

4. CONTROLLER DESIGN

We start by looking for a control which stabilizes $\tilde{E}$. We choose u such that

$$\frac{\bar{E}}{\tau_E} - \left[(\frac{n_{DT}}{2})^2 < \sigma v > Q_\alpha + u\right] = 0 \quad (18)$$

This means, after replacing u by its expression, that we choose P_{aux} and P_{rad} (n_I) such that

$$\frac{\bar{E}}{\tau_E} = \left(\frac{n_{DT}}{2}\right)^2 < \sigma v > Q_\alpha - P_{rad} + P_{aux} = P \quad (19)$$

From the equilibrium equation for the energy E, $0 = -\frac{\bar{E}}{\bar{\tau}_E} + \bar{P}$, and the correlation between the energy confinement scaling τ_E and the power P given by (7), we realize that the solution for equation (19) is $P = \bar{P}$. Therefore, the control strategy will be to adjust P_{aux} and n_I to make P constant and equal to $\bar{P}$ satisfying equation (19) and reducing equation (12) to

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E}$$

Since $\tau_E > 0$, the subsystem $\tilde{E}$ is exponential stable. The controller that implements (19) is synthesized now in two steps:

First Step: We compute

$$P_{aux} = \bar{P} - \left[\left(\frac{n_{DT}}{2}\right)^2 < \sigma v > Q_\alpha - P_{rad}\right] \quad (20)$$

If $P_{aux} \geq 0$ then we keep this value for P_{aux} and let $S_I = 0$.
If $P_{aux} < 0$ then we take $P_{aux} = 0$ and go to the Second Step,

Second Step: Remembering that P_{rad} is a function of n_I, we look for the least $n_I = n_I^* > 0$ such that

$$\frac{\bar{E}}{\tau_E} - \left[\left(\frac{n_{DT}}{2}\right)^2 < \sigma v > Q_\alpha - P_{rad}\right] = 0 \quad (21)$$

Defining

$$\hat{n}_I = \tilde{n}_I - n_I^*$$
$$f = -\left[\frac{\bar{E}}{\tau_E} - \left[\left(\frac{n_{DT}}{2}\right)^2 < \sigma v > Q_\alpha - P_{rad}\right]\right]$$
$$S_I = \frac{n_I^*}{\tau_I} + S_I^*$$

We can rewrite equations (12)–(13) as

$$\frac{d\hat{n}_I}{dt} = -\frac{\hat{n}_I}{\tau_I} + S_I^* \tag{22}$$
$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} + f(\hat{n}_I, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_{DT}). \tag{23}$$

We take $V = \frac{\hat{n}_I^2 + \tilde{E}^2}{2}$ as the Lyapunov function candidate, write $f = \hat{n}_I \phi$, where ϕ is a continuos function because $f(0, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_{DT}) = 0$, and compute

$$\dot{V} = -\frac{\hat{n}_I^2}{\tau_I} - \frac{\tilde{E}^2}{\tau_E} + \hat{n}_I \left[S_I^* + \tilde{E}\phi(\hat{n}_I, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_{DT})\right] \tag{24}$$

where $(\dot{\ }) = \frac{d}{dt}(\ )$. We take

$$S_I^* = -\tilde{E}\phi(\hat{n}_I, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_{DT}) - K_I \hat{n}_I, \qquad K_I \geq 0$$
$$S_I = \frac{n_I^*}{\tau_I} - \tilde{E}\phi(\hat{n}_I, \tilde{E}, \tilde{n}_\alpha, \tilde{n}_{DT}) - K_I \hat{n}_I, \tag{25}$$

which gives $\dot{V} = -\left(\frac{1}{\tau_I} + K_I\right)\hat{n}_I^2 - \frac{\tilde{E}^2}{\tau_E} < 0$.

If the potential perturbations in initial conditions are such that they can be rejected only by the modulation of the auxiliary power P_{aux} according to the control law (20), we are in the case where impurities are not needed and $S_I = 0$. In this case, P is always equal to $\bar{P}$, equation (19) is always satisfied and consequently $\tau_E = \bar{\tau}_E$, $\tau_\alpha = \bar{\tau}_\alpha$, $\tau_{DT} = \bar{\tau}_{DT}$, $\tau_I = \bar{\tau}_I$ and $\tau_d = \bar{\tau}_d$.

We note from eq. (12) that $\tilde{n}_I$ is input-state stable (ISS) (See (Khalil, 1996), section 5.3) with respect to S_I. This ensures that $\tilde{n}_I$ will be bounded as long as S_I is bounded, and it will be exponentially stable once S_I becomes zero.

After stabilizing $\tilde{E}$ using P_{aux} and S_I as controllers, we must focus on equations (9) and (10) to achieve stability for $\tilde{n}_{DT}$ and $\tilde{n}_\alpha$. We apply a backstepping procedure to achieve stability of $\tilde{n}_{DT}$. Toward this goal, we start taking $\tilde{n}_n$ as the virtual control v,

$$\frac{d\tilde{n}_{DT}}{dt} = -\frac{\tilde{n}_{DT}}{\tau_{DT}} - 2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > -\tilde{n}_{DT}\bar{n}_{DT} < \sigma v > + u_{DT} + \frac{v}{\tau_d}$$

since $[\frac{1}{\tau_{DT}} + \bar{n}_{DT} < \sigma v >]$ is positive, we exponentially stabilize $\tilde{n}_{DT}$ taking

$$\begin{aligned} v &= \alpha(n_\alpha, n_{DT}, E) \\ &= \tau_d\left[2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > - u_{DT}\right] \\ &= \tau_d\left[2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > + \frac{\bar{n}_{DT}}{\tau_{DT}} + 2\left(\frac{\bar{n}_{DT}}{2}\right)^2 < \sigma v > - \frac{\bar{n}_n}{\tau_d}\right] \end{aligned} \tag{26}$$

reducing in this way equation (10) to:

$$\frac{d\tilde{n}_{DT}}{dt} = -\left[\frac{1}{\tau_{DT}} + \bar{n}_{DT} < \sigma v >\right]\tilde{n}_{DT}$$

Defining $z = \tilde{n}_n - \alpha \iff \tilde{n}_n = z + \alpha$, we can write

$$\dot{z} = \dot{\tilde{n}}_n - \dot{\alpha} = S^* - \dot{\alpha} \tag{27}$$

Taking $V = \frac{\tilde{n}_{DT}^2}{2} + \frac{z^2}{2}$ as the Lyapunov function candidate, from equations (10) and (27) and taking into account our definition (26) for α we can compute

$$\begin{aligned} \dot{V} &= \tilde{n}_{DT}\dot{\tilde{n}}_{DT} + z\dot{z} \\ &= \tilde{n}_{DT}\left[-\left(\frac{1}{\tau_{DT}} + \bar{n}_{DT} < \sigma v >\right)\tilde{n}_{DT}\right. \\ &\quad \left. -2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > + u_{DT} + \frac{\alpha + z}{\tau_d}\right] + z[S^* - \dot{\alpha}] \\ &= -\left[\frac{1}{\tau_{DT}} + \bar{n}_{DT} < \sigma v >\right]\tilde{n}_{DT}^2 + z\left[S^* - \dot{\alpha} + \frac{\tilde{n}_{DT}}{\tau_d}\right] \end{aligned}$$

Taking

$$S^* = -K_S z + \dot{\alpha} - \frac{\tilde{n}_{DT}}{\tau_d} = -K_S(\tilde{n}_n - \alpha) + \dot{\alpha} - \frac{\tilde{n}_{DT}}{\tau_d},$$
$$S = -K_S(\tilde{n}_n - \alpha) + \dot{\alpha} - \frac{\tilde{n}_{DT}}{\tau_d} + \frac{n_n}{\tau_d} \tag{28}$$

with $K_S > 0$, we have

$$\dot{V} = -\left[\frac{1}{\tau_{DT}} + \bar{n}_{DT} < \sigma v >\right]\tilde{n}_{DT}^2 - K_S z^2 < 0$$

and we achieve exponential stability for the subsystem $\tilde{n}_{DT}$-$\tilde{n}_n$.

To have a closed expression for the control law (28) for S, we compute $\dot{\alpha}$ as

$$\begin{aligned} \dot{\alpha} &= \dot{\tau}_d\left[2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \sigma v > + \frac{\bar{n}_{DT}}{\tau_{DT}} + 2\left(\frac{\bar{n}_{DT}}{2}\right)^2 < \sigma v >\right] \\ &\quad + \tau_d\left[< \sigma v > \tilde{n}_{DT}\dot{\tilde{n}}_{DT} + 2\left(\frac{\tilde{n}_{DT}}{2}\right)^2 < \dot{\sigma v} >\right. \\ &\quad \left. - \frac{\bar{n}_{DT}}{\tau_{DT}^2}\dot{\tau}_{DT} + 2\left(\frac{\bar{n}_{DT}}{2}\right)^2 < \dot{\sigma v} >\right] \end{aligned} \tag{29}$$

where the time derivatives are propagated and written in terms of $\dot{n}_\alpha$, $\dot{n}_{DT}$, $\dot{n}_I$ and $\dot{E}$ which can be obtained from (9), (10), (12) and (13).

In order to finish our stability analysis, we note from equation (9) that $\tilde{n}_\alpha$ is ISS with respect to $\tilde{n}_{DT}$ and u_α. Therefore, since $\tilde{n}_{DT}$ is bounded (because it is exponentially stable), and u_α is bounded (because $< \sigma v >$ is a bounded function and $\tilde{E}$ is exponentially stable), $\tilde{n}_\alpha$ will be bounded for all time. In addition, once E converges to $\bar{E}$ ($\tilde{E} \to 0$), n_{DT} converges to $\bar{n}_{DT}$ ($\tilde{n}_{DT} \to 0$) and n_I converges to $\bar{n}_I = 0$ this equation reduces to

$$\frac{d\tilde{n}_\alpha}{dt} = -\frac{\tilde{n}_\alpha}{\bar{\tau}_\alpha} + u_\alpha^*, \quad u_\alpha^* = -\frac{\bar{n}_\alpha}{\bar{\tau}_\alpha} + \left(\frac{\bar{n}_{DT}}{2}\right)^2 < \sigma v > \tag{30}$$

The function $< \sigma v >$ is a function of $T = \frac{2}{3}\frac{\bar{E}}{2n_{DT} + 3n_\alpha}$, once $n_I = \tilde{n}_I$ converges to zero, and has a positive derivative in the region of interest. Consequently u_α^* has the same sign as $-\frac{\tilde{n}_\alpha}{\bar{\tau}_\alpha}$ and vanishes when $\tilde{n}_\alpha$ vanishes ($< \sigma v > = < \bar{\sigma v} >$) because $0 = -\frac{\bar{n}_\alpha}{\bar{\tau}_\alpha} + \left(\frac{\bar{n}_{DT}}{2}\right)^2 < \bar{\sigma v} >$ is the equilibrium equation for n_α. This implies exponential stability for $\tilde{n}_\alpha$.

5. SIMULATION RESULTS

In this section the performance of the controller stabilizing the equilibrium (ignition) point characterized by those values given in table 1 is studied through computer simulations. For all the simulations presented here, a scale factor $f = 0.85$ for the energy confinement time (7) have been used. It should be noted that our controller can be independent of k_I with a sufficiently large K_I, consequently it tolerates any size of uncertainty in this parameter. Therefore the choice of $k_I = 10$ can be considered completely arbitrary and with the only purpose of the simulation.

T	Temperature	8.04 KeV
$\bar{n}_e$	Electron Density	9.93 10^{19} m^{-3}
$\bar{f}_\alpha$	Alpha Fraction	6.09 %
β	Beta	2.65 %
$\bar{n}_\alpha$	Alpha Density	6.05 10^{18} m^{-3}
$\bar{n}_{DT}$	DT Density	8.72 10^{19} m^{-3}
$\bar{E}$	Energy	3.72 10^5 $J.m^{-3}$
$\bar{P}_{aux}$	Auxiliary Power	0 $W.m^{-3}$
$\bar{S}$	Fuel Rate	4.05 10^{18} $m^{-3}.sec^{-1}$

Table 1. ITER Equilibrium point

The controller designed shows capability of rejecting different types of large perturbations in initial conditions. A study was carried out generating initial perturbations around the equilibrium for T and n_e and keeping the alpha-particle fraction $f_\alpha := n_\alpha / n_e$ equal to that of the equilibrium. Figure 1 compares its performance with other two controllers synthesized by linear pole placement (Hui and Miley, 1992) and linear robust (Bamieh *et al.*, 1994) techniques, for a linearization point very close to our equilibrium point, which use mainly the same dynamical model presented here but considering only the fueling rate as actuator. While the boundaries shown for the linear controllers are absolute, for the nonlinear controller they only indicate the limits within which we performed our tests.

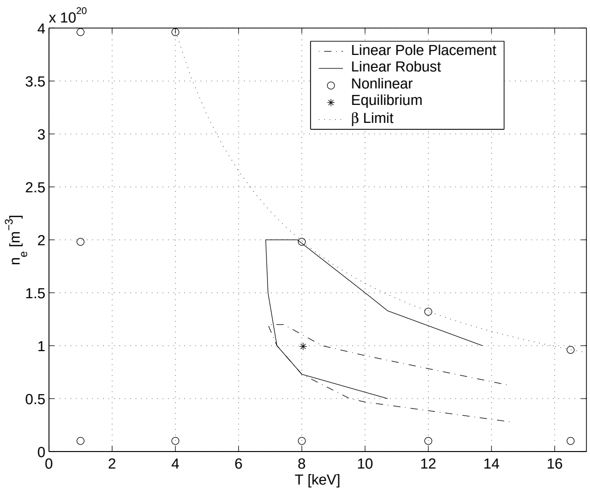

Fig. 1. Stability Domain Comparison.

The good robustness properties of the controller against uncertainties in the parameters of the confinement times of the different species were already presented in (Schuster *et al.*, 2001). However, we now need to study the robustness against uncertainties in the parameter k_d. Figure 2 shows the regions of stability against uncertainty in the parameter k_d, whose nominal value is equal to 1, when the system suffers perturbations in the initial temperature. Again, the region shown for the nonlinear controller is not a limit. With the sole objective to show its performance we tested it against uncertainties up to 400% and perturbations for initial T between -90% and 100%. The plot compares the robustness of the nonlinear controller with other controller synthesized by linear robust techniques (Hui *et al.*, 1994) for a linearization point very close to the equilibrium point considered here. However, we have to mention a difference between both controllers; while the nonlinear controller was synthesized here for a nominal value of 1, the robust linear controller was synthesized for the nominal no-lag case.

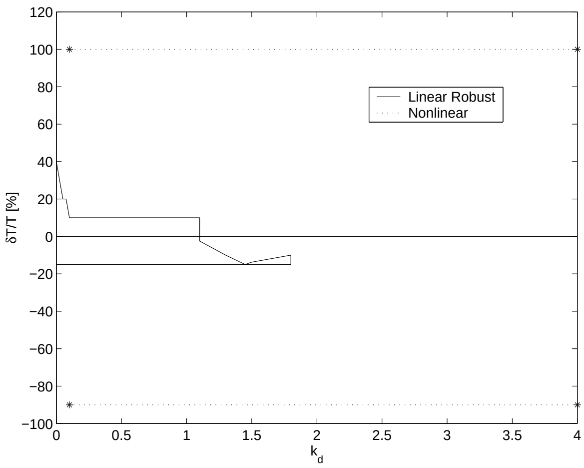

Fig. 2. Robustness comparison against k_d uncertainties.

Figures 3 and 4 show the response of the system against a 50% perturbation in the temperature initial condition. The introduction of the controlled impurity injection as actuator allows us to deal succesfully with such a large positive perturbation in T. We note that the control effort in S_I can be reduced decreasing K_I. However, the energy and consequently the beta excursions increase accordingly.

6. CONCLUSIONS

Through the use of nonlinear backstepping it was possible to synthesize a controller which is independent of the reactor fueling system allowing either pellet injection or gas puffing. It must be mentioned that the control law (28) for S is remarkably simplified when the controller does not need to inject impurities in the reactor. In this case the control law (20) for P_{aux} ensures $P = \bar{P}$ and consequently $\tau_d = \bar{\tau}_d$. Therefore, the expression for $\dot{\alpha}$ in (29) adopts a much simpler form. In addition, the combination of a singular perturbation and a passivity approach allows us to regulate, through the gain K_I, the compromise between the actuation force in control law (25) for S_I and the energy excursion.

This new approach to the problem of burn control allows us to deal with perturbations in initial conditions

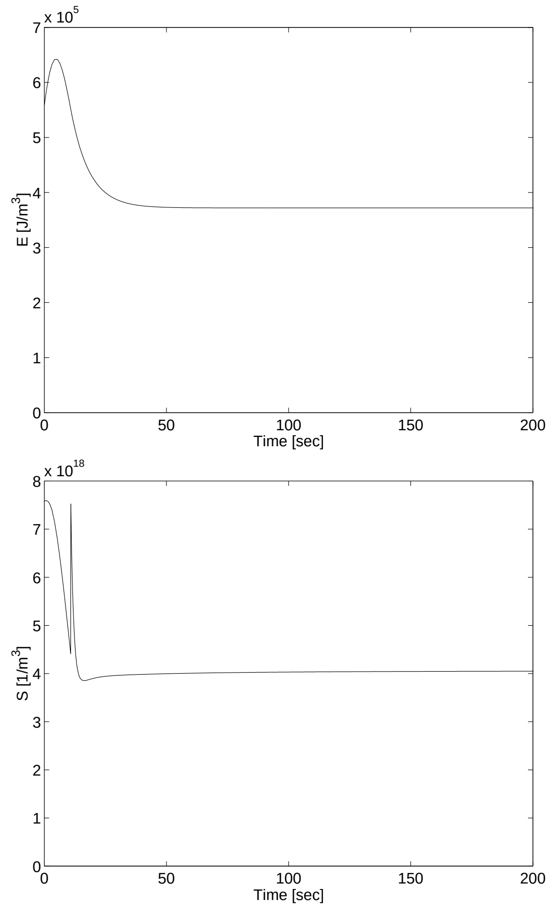

Fig. 3. With control, under initial perturbation of 50% in T, ($Z_I = 8, K_I = 0.1, K_S = 1$).

that were unmanageable until now. The information taken into account by the controller when it is synthesized using the full nonlinear model makes it capable of dealing with a larger set of perturbations in initial conditions. On the other hand, the multi-input nature of the controller allows it to reject large perturbations in initial conditions leading to both thermal excursion and quenching. In addition, the effectiveness of the controller does not depend on whether the operating point is an ignition or a subignition point. Since the nonlinear controller depends parametrically on the equilibrium point, it can drive the system from one equilibrium point to another allowing in this way the change of power, other plasma parameters and ignition conditions.

The controller exhibits excellent properties of robustness and the boundness of the state variables is guaranteed for a large set of values of k_d.

7. REFERENCES

Anderson, D., T. Elevant, H. Hamen, M. Lisak and H. Persson (1993). Studies of fusion burn control. *Fusion Technology* **23**(1), 5–41.

Bamieh, B. A., W. Hui and G. H. Miley (1994). Robust burn control of a fusion reactor by modulation of the refueling rate. *Fusion Technology* **25**(3), 318–25.

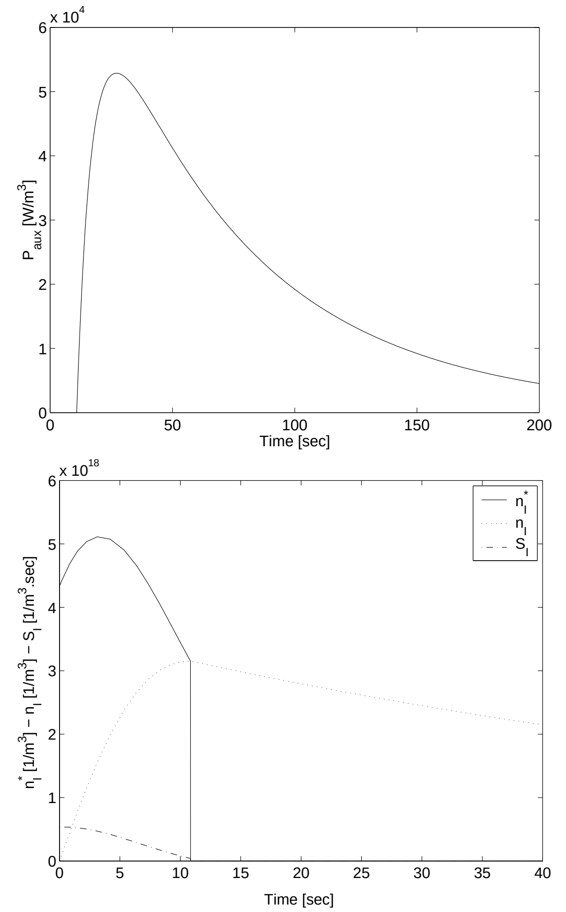

Fig. 4. The system returns to the equilibrium. Note how P_{aux} and S_I work in tandem.

Haney, S. W., L. J. Perkins, J. Mandrekas and W. M. Stacey, Jr. (1990). Active control of burn conditions for the international thermonuclear experimental reactor. *Fusion Technology* **18**(4), 606–17.

Hively, L. M. (1977). Convenient computational forms for maxwellian reactivities. *Nuclear Fusion* **17**(4), 873.

Hui, W. and G. H. Miley (1992). Burn control by refueling. *Bull. Am. Phys. Soc.* **37**(6), 1399.

Hui, W., K. Fischbach, B. Bamieh and G. H. Miley (1994). Effectiveness and constraints of using the refueling system to control fusion reactor burn. *Proceedings of the 15th IEEE/NPSS Symposium on Fusion Engineering* **2**, 562–4.

Khalil, H. K. (1996). *Nonlinear Systems*. Prentice Hall.

Mandrekas, J. and W. M. Stacey (1990). Evaluation of different burn control methods for the international thermonuclear experimental reactor. *Proceedings of the 13th IEEE/NPSS Symposium on Fusion Engineering* **1**, 404–7.

Schuster, E., M. Krstic and G. Tynan (2001). Nonlinear control of burn instability in fusion reactors. *Proceedings of the 40th IEEE Conference on Decision and Control*.

Uckan, N. A. (1994). Confinement capability of itereda design. *Proceedings of the 15th IEEE/NPSS Symposium on Fusion Engineering* **1**, 183–6.