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THE PRODUCTION SCHEDULING OF MULTI-ITEMS IN A STOCHASTIC MANUFACTURING SYSTEM WITH PREEMPTIVE DECISION MAKING

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Abstract: A scheduling problem in a manufacturing system that produces different types of items is presented in this paper. The demands for the items are Poisson processes, and simultaneous production of two or more items is not allowed. The decisions on stopping the production can be made at any time, even if the item being produced is unfinished. An instantaneous setup cost is paid when the production is stopped or initialized, or when the production is switched from one item to another. The scheduling problem is formulated as an Impulse Control of Piecewise Deterministic Markov Processes. The value function is characterized and aproximation methods for the solution of the scheduling problem are proposed.

Keywords: Manufacturing Systems, Scheduling Algorithms, Production Control, Stochastic Control, Jump Process, Markovain Decision Processes.

1. INTRODUCTION

This work studies a stochastic control problem in a manufacturing system that produces different types of items. The demands for the items arrive during the running of production and they form Poisson Processes. Simultaneous production of two or more types of items is not allowed and the type of item to be produced is choosen at the end of the production interval of the previuous item. During the production interval, the interruption in favour of the production of another type of item is not allowed, however, one can decide to bring the production to a half at any time, in a preemptive decision policy. In returning to production, the unfinished item should be completed. An instantaneous setup cost is paid when the production is stopped or initialized, or when the production is switched from one item to another. The decision maker has to decide between producing or stopping, and when the plant is active, the type of items and production scheduling, in order to minimize the expect cost associated with setups, production and storage/shortage of the items.

The optimization problem is formulated as an Impulse Control of Piecewise Deterministic Markov

Processes (PMDP) as presented in (Davis, 1993). There are many interesting works dealing with the application of PDMP in the context of production and storage planning, e.g. see, (Akella and Kumar, 1986), (Boukas and Haurie, 1990) or (Jiang and Lou, 1994), where the production planning and the preventive maintenance in a flexible manufacturing system is studied as a continuous control problem. In (Yan and Zhang, 1997), the production planning and setup scheduling in a failure-prone manufacturing system is considered as a combination of continuous and intervention control problems. This scheduling model has some resemblances with the model we propose; however, the major difference is that it presents deterministic demand and instantaneous production. In those works the type of preemptive policy mentioned is never considered, although it is of clear interest when the production of each item is not instantaneous, and the time elapsed has to be taken account. Previous results on the problem of production & storage that assembles identical items with preemptive policy appears in (do Val and Salles, 1999) and (Salles and do Val, 2001).

This paper is organized as follows: in section 2 the production scheduling problem is formulated as a stochastic control problem of PMDP and in section 3 is presented two methods of solution for this problem. The first (see Proposition 3.2) is adapted from the general results presented in (Davis, 1993) for the Impulse Control of PMDP, and the second method (see Theorem 3.1) is derived from the results of (Salles and do Val, 2001), which explores the state space structure of the production scheduling model. Finally, the conclusions of this work are shown in section 4.

2. THE PRODUCTION SCHEDULING PROBLEM

The manufacturing system presented here produces J different types of item, with the restriction that only one item is produced at each time. The demand of each type of item forms a Poisson process with rate λ^j, $j = 1, 2, \ldots J$ and the item of type j requested by the demand until time t is indicated by d_t^j. Let ω_t^j the number of items of type j produced until time t; thus, the number of items of type j on storage or in backlog is indicated by the process:

$$n_t^j := \omega_t^j - d_t^j, \ t \geq 0,$$

where $n_t^j \in \mathbb{Z}$.

The production rate of each type of item is a constant given by $v \in \mathbb{R}^+$; the progression on production of each item is indicated by the process $t \to \xi_t$ and it satisfies $v = \frac{d}{dt}\xi_t$. The process ξ_t assumes values on interval $[0, \Gamma_t^j]$; thus the production of an item j is initialized if $\xi_t = 0$ and it is completed when $\xi_t = \Gamma^j$.

Let $z_t := (n_t^1, \ldots, n_t^J, \xi_t, j)$ the state of the production process. When the item j is being produced, $t \to z_t$ lies in the subset $S_j' = \mathbb{Z}^J \times [0, \Gamma^j]$. Let $\partial_{S_j'} = \mathbb{Z}^J \times \{\Gamma^j\}$ the boundary of S_j'. When the production of item j is stopped and no one item is produced, $t \to z_t$ lies in the subset $S_j'' = \mathbb{Z}^J \times [0, \Gamma^j]$. We denote $\partial_{S'} = \cup_j \partial S_j'$, $S' = \cup_j S_j'$, $S'' = \cup S_j''$ and $S = S' \cup S''$. For any $z = z_0 \in S$, let us define the drift function by :

$$\varphi(t, z) =$$

$$\begin{cases} (n^1, \ldots, n^J, \xi + v \cdot t, j), & 0 \leq t \leq t^*(z), \ z \in S_j', \\ (n^1, \ldots, n^J, \xi, j), & t \geq 0, \ z \in S_j''. \end{cases}$$

where

$$t^*(z) := \inf\{t \geq 0 : \varphi(t, z) \in \partial_{S'}^*\} \qquad (1)$$

with the convention inf $\emptyset = +\infty$, where $\partial_{S'}^*$ is a subset in the active boundary $\partial_{S'}$, defined by:

$$\partial_{S'}^* := \{z \in \partial_{S'} : t > 0, z_0 \in S \text{ such that } \varphi(s, z_0) \to z \text{ when } s \to t.\} \qquad (2)$$

Observe that $t^* : \partial_S^* \to \mathbb{R}_+$ in (1) is continuous, and denote

$$\Lambda(t, z) := \int_0^t \lambda(\varphi(s, z)) ds \qquad (3)$$

for any $z \in S$ and $t \leq t^*(z)$. Let $u : \partial_{S'}^* \to U_\partial = \{1, \ldots, J\}$, a control in the boundary of the state space such that, for $z \in \partial_{S_j'}$, $j \in \{1, \ldots, J\}$, if $u(z) = i$, for $i \in \{1, \ldots, J\}$, then we must start the production of an item of type i after the production of an item j has been finished. The process z_t as defined belongs to the class of PDMP (Davis, 1993) with state space $S = S' \cup S''$, and is characterized by the following elements:

(i) A deterministic drift $t \to \varphi(t, z), t \geq 0, \varphi(0, z) = z$, which is bounded in S. For each $z \in S''$ it satisfies $\varphi(t, z) = z, t \geq 0$, i.e. φ has null velocity at S''. The deterministic drift defines the active boundary ∂_S^*, as in (2).

(ii) A jump rate $\lambda \in C_b(S \cup \partial_S^*)$, where $C_b(S \cup \partial_S^*)$ is the space of real continuous and bounded functions on $S \cup \partial_S^*$;

(iii) A probability of transiction $\mu(\cdot, z, u)$ inside the state space. In the boundary $\partial_{S'}^*$, the transiction is controled and is denoted by μ_∂. When $z = (n^1, \ldots, n^j, \ldots, n^J, \Gamma^j, j) \in \partial_{S_j'}^*$ and $u(z) = i$ then $\mu_\partial(y, z, u) = 1$ if $y =$

$(n^1, \ldots, n^j + 1, \ldots, n^J, 0, i) \in S_i'$; otherwise, if $y \notin S_i'$, then $\mu_\partial(y, z, u) = 0$.

The processes $t \to z_t$ is described as follows. Starting at $z_0 = z$ let T_1 be the first jump of the process, such that

$$P_z(T_1 > t) = \begin{cases} e^{-\Lambda(t,z)} & t < t^*(z) \\ 0 & t \geq t^*(z) \end{cases} \quad (4)$$

and $z_t = \varphi(t, z)$ for $t < T_1$, P_z-a.s. Let $\mathcal{F}_t := \sigma(x_s : s \leq t)$ be the filtration of the process, and consider $\mathcal{F}_{T_1^-}$. We have that:

$$P_z(z_{T_1} \in A | F_{T_1^-}) = \mu(A, \varphi(T_1, z)) 1_{\{T_1 < t^*(z)\}} \quad (5)$$
$$+ \mu_\partial(A, \varphi(T_1, z)) \; 1_{\{T_1 = t^*(z)\}}$$

for any Borel measurable set $A \in S$. After the jump time T_1 the process starts afresh at z_{T_1} following the drift $\varphi(t, z_{T_1})$ on the interval $T_1 \leq t < T_2$, and similar stochastic definitions in (4) and (5) apply to T_2 and z_{T_2}, respectively.

Let us consider the sequences of intervention times denoted by $\pi = \{\tau_1, \tau_2, \ldots\} \in \Pi$, and $\theta = \{\xi_1, \xi_2, \ldots\} \in \Theta$ where Π and Θ are classes of stopping times with respect to the filtration $\mathcal{F}_t$. These classes contains respectively, the intervention times associated with transitions between the subsets S_j' and S_j'' and vice versa, and the completion times of each finished item. Notice that the intervention control $\pi \in \Pi$ is associated to the decision on stopping or starting the production, whereas at the times $\theta \in \Theta$ a decision $u(z) \in U_\partial$, for $z \in \partial_{S_j'}$, is associated to the switching of the type of item to be produced.

It is assumed that this manufacturing system is characterized by the following costs by unit of time: $L_j : \mathbb{Z} \to \mathbb{R}^+$, the storage costs $(n_t^j > 0)$ or the backlog costs $(n_t^j < 0)$ of each item j; $p : S \to \mathbb{R}^+$, the cost associated to the production rate of each item; $h : \partial_{S'} \cup U_\partial \to \mathbb{R}^+$, that is the setup cost associated to the choise of the type of item to be produced; finally, $g : S \to \mathbb{R}^+$ is the cost associated to the stoppage or initialization of the production of each item.

Let us define the functions $L = \sum_j L_j$, and $f(z) = L(z) + p(z)$ and denote the descont rate by the constant α. Assume that $f(z), h(z)$ and $g(z)$ belong to the class of continuous function $C_b(S \cup \partial_S)$. For any control policies $\pi \in \Pi$ and $u \in U_\partial$, let us associate the following cost

$$V^{\pi,u}(z) = E_z^{\pi,u}\Big\{\int_0^\infty e^{-\alpha s} f(z_s)\, ds$$
$$+ \sum_{i=1}^{\infty} e^{-\alpha T_i} h(z_{\theta_i}), u(z_{\theta_i})) 1_{\{z_{\theta_i^-} \in \partial_{S'}\}}$$
$$+ \sum_{i=1}^{\infty} e^{-\alpha \tau_i} g(z_{\tau_i^-})\Big\} (6)$$

where above expectation is get with respect to the process $t \to z_t$ with $z_0 = z$. In this scheduling problem, one has to determine a sequence of setups in order to minimize the operational cost given in (6). When it exists, such strategy is optimal and furnishes the value for the problem, $V : S \cup \partial_S^* \to \mathbb{R}$, defined by:

$$V(z) = \inf_{\pi \in \Pi, \; u \in U_\partial} V^{\pi,u}(z) \quad (7)$$

3. SOLUTIONS FOR THE PRODUCTION SCHEDULING PROBLEM

The solution of problem (7) is obtained from sucessive aproximation methods similar to that present in (Bertsekas and Shereve, 1978). A dynamic programming operator, denoted "the one-jump-or-one-intervention operator", with control on the boundary, generate a sequence of functions that converge to the value function of the scheduling problem. This section presents two approximation squemes which differ in the way that the end cost of the one-jump-or-intervention operator is updated at each iteration. One method has a simple convergence and it requires a restriction in the initial function of the procedure, whereas the other presents an uniform convergence and is initialized by any continuous function.

3.1 *The one Jump-or-Intervention Operator*

For a point $z \in S'(\in S'')$, let us denote the correspondent point in the copy $S''(S')$ by $\bar{z}$. Similar notation applies for the process $t \to z_t$, and we inidicate by $\bar{z}_t$ the correspondent point in S' or S'' to which the process is transferred whenever an intervention occurs at time t.
Let us denote $a \wedge b$ for $\min\{a, b\}$ and $\widehat{\lambda} := \alpha + \lambda(z)$. For $z \in S$ and any $\phi, \psi \in C_b(S \cup \partial_S^*)$, let us define the following operators:

$$\mathcal{M}[\phi](z) := \phi(z) + h(z, u(z)) 1_{\{z \in \partial_S^*\}}$$

$$\mathcal{R}_t[\phi, \psi](z) := \inf_{u \in U_\partial} E_z^{\pi,u}\Big\{\int_0^{t \wedge T_1} e^{-\alpha s} f(z_s) ds$$
$$+ e^{-\alpha T_1} \mathcal{M}[\phi](z_{T_1}) 1_{\{t \geq T_1\}}$$
$$+ e^{-\alpha t} (\psi(\bar{z}_t)) + g(z_t)) 1_{\{t < T_1\}}\Big\},$$
(8)

$$\mathcal{R}[\phi,\psi](z) := \inf_{0\leq t\leq t^*(z)} \mathcal{R}_t[\phi,\psi](z) \quad (9)$$

$$\mathcal{Q}_S[\phi](z) := \int_S \phi(y)\mu(dy,z), \quad (10)$$

$$\mathcal{Q}^u_\partial[\phi](z) := \int_{\partial_{S'}} \phi(y)\mu_\partial(dy,z,u), \quad (11)$$

$$\mathcal{N}[\phi](z) := \frac{1}{\widehat{\lambda}(z)}(f(z)+\lambda(z)\mathcal{Q}_S[\phi](z)), \quad (12)$$

Remark 1. The operador (9), denoted the "one-jump-or-Intervention operator," characterizes an stopping time problem with end costs $\mathcal{M}[\phi]$ or $\psi+g$, depending on the stopping time be the first jump or the first intervention, respectively.

We assume that:

(H_1) $g(z) \geq g_0 > 0$ for $g_0 \in \mathbb{R}^+$.
(H_2) ϕ, ψ and h are in $C_b(S)$.

The assumptions (H_1) and (H_2) guarantee that there only exists isolated interventions epochs in any candidate to optimal policy in the scheduling problem (7). The details to comply with standard conditions as in (Davis, 1993) are presented in appendix A of (Salles and do Val, 2001).

The value function for the one-jump-or-intervention problem, $\nu : S\cup\partial S \to \mathbb{R}$, is defined by:

$$\nu(z) := \mathcal{R}[\phi,\psi](z) \quad (13)$$

Let us denote:

$$\widehat{\Lambda}(t,z) := \int_0^t (\alpha+\lambda(\varphi(s,z)))\,ds = \int_0^t \widehat{\lambda}(\varphi(s,z))\,ds$$

Lemma 2. If H_2 holds and $\phi,\psi \in C_b(S\cup\partial^*_S)$, then $\nu \in C_b(S\cup\partial^*_S)$.

Proof: Let us define the following operator:

$$\mathcal{R}_t{}'[\phi,\psi](z) := \inf_{u\in U_\partial} E^{\pi,u}_z\Big\{\int_0^{t\wedge T_1} e^{-\alpha s} f(z_s)ds$$
$$+e^{-\alpha T_1}\phi(z_{T_1})1_{\{t\geq T_1\}} + e^{-\alpha t}\,(\psi(\bar{z}_t)) + g(z_t))1_{\{t<T_1\}}\Big\},$$

thus

$$\mathcal{R}_t[\phi,\psi](z) = \mathcal{R}'_t[\phi,\psi](z) + \inf_u E^{\pi,u}_z\{e^{-\alpha T_1}h(z_{T_1},u(z_{T_1})1_{\{z_{T_1}\in\partial^*_{S'}\}}\} \quad (14)$$

In view of the first jump time T_1 has the exponential distribution given by (4), we have that:

$$\inf_u E^{\pi,u}_z\{e^{-\alpha T_1}\{h(z_{T_1},u(z_{T_1})1_{\{z_{T_1}\in\partial^*_{S'}\}}\}$$
$$=\inf_u\{e^{-\Lambda(\varphi(t^*,z))}h(\varphi(t^*,z),u(\varphi(t^*,z)))\} \quad (15)$$

Since $t^*(z) \in C_b(S\cup\partial^* S')$ we conclude that the above expression is in $C_b(S\cup\partial^*_S)$. In addition $\mathcal{R}'[\phi,\psi](z) \in C_b(S\cup\partial^*_S)$ from Lemma(53.38), p. 224 in (Davis, 1993). Therefore, from (14), we conclude that $\mathcal{R}[\phi,\psi](z) \in C_b(S\cup\partial^*_S)$. □

Let $\mathcal{B}$ denote the vector field associated to the deterministic drift φ. For $\phi \in C_b(S\cup\partial^*_S)$, and for points where $\phi(z)$ is differentiable, we denote

$$\mathcal{B}[\phi](z) := \lim_{t\to 0} t^{-1}[\phi(\varphi(t,z))) - \phi(z)].$$

Smoothness of the vector field $\mathcal{B}$ is required, as in (Gatarek, 1992):

(H_3) There exists $a = (a_1\cdots a_q)$ with $a_j \in C_b(S), j=1,\ldots,d$ such that $\mathcal{B}[\phi](z) = < a(z), \nabla\phi(z) >$, for any $\phi \in C_b(S)$ which is differentiable at $z\in S$.

Lemma 3. Assume that H_2 is valid. Then ν in (13) is a viscosity solution of the following variational equations

$$\{\mathcal{B}[\nu](z) - \widehat{\lambda}(z)\nu(z) + \lambda(z)\mathcal{Q}_S[\phi](z) + f(z)\}\wedge \{\psi(\bar{z}) + g(z) - \nu(z)\} = 0, \quad z\in S', \quad (16)$$

$$\nu(z) = \{\psi(\bar{z}) + g(z)\}\wedge\mathcal{N}[\phi](z), \quad z\in S'' \quad (17)$$

with the boundary condition for $z\in\partial^*_S$:

$$\nu(z) = \inf_{u\in U_\partial}\{\mathcal{Q}^u_\partial[\phi](z) + h(z,u)\}\wedge(\psi(\bar{z})+g(z)), \quad (18)$$

Conversely, if ν satisfies (16)–(18) for each $z\in S$, then $\nu(z) = \mathcal{R}[\phi,\psi](z)$.

Proof: From the arguments in Lemma 6 of (Salles and do Val, 2001), we conclude that

$$\mathcal{R}[\phi,\psi](z) = \nu(z) = \inf_u E^{\pi,u}_z\Big\{\int_0^{t\wedge T_1} e^{-\alpha s}f(z_s)\,ds$$
$$+ e^{-\alpha T_1}(\mathcal{M}[\phi](z(T_1))1_{\{T_1\leq t\}}$$
$$+ e^{-\alpha t}(\psi((\bar{z}_t) + g(z_{t^-}))1_{\{T_1>t\}}\Big\}$$
$$-\int_0^t e^{-\widehat{\Lambda}(s,z)}\Omega(\varphi(s,z))\,ds$$
$$+ e^{-\widehat{\Lambda}(t,z)}(\Gamma_1(\varphi(t,z))_{\{t<t^*(z)\}}$$
$$+ \inf_u \Gamma^u_2(\varphi(t,z))_{\{t=t^*(z)\}} \quad (19)$$

where, we set

$$\Omega(z) := f(z) + \lambda(z)\mathcal{Q}_S[\phi](z) - \widehat{\lambda}(z)\nu(z) + \mathcal{B}[\nu](z) \quad \text{for } z\in S$$
$$\Gamma_1(z) := \psi(z) - \nu(z), \quad \text{for } z\in S$$
$$\Gamma^u_2(z) := \mathcal{Q}^u_\partial[\mathcal{M}\phi]](z) - v(z) = \mathcal{Q}^u_\partial[\phi](z) + h(z,u) - v(z), \text{for } z\in\partial^*_S.$$

(19)

By optimality, $\Omega(z) \geq 0$, $\Gamma_1(z) \geq 0$, $\Gamma_2(z) \geq 0$, and the minimum is achieved if and only if in (19) $\Omega(\varphi(s,z)) = 0$, $0 \leq s \leq t$ and $\Gamma_1(\varphi(t,z)) = 0$, $t < t^*(z)$, $\inf_u \Gamma_2^u(\varphi(t,z)) = 0$, $t = t^*(z)$. Therefore, $t = \arg\min\{\mathcal{R}_s[\phi,\psi](z) : 0 \leq s \leq t^*(z)\}$, and $\tau_1 = t$ is an optimal stopping time, respectively. It follows from the above that $\forall z \in \partial^*_{S'}$,

$$\Omega(z) \wedge \Gamma_1(z) = 0, \forall z \in S \text{ and } \Gamma_1(z) \wedge \inf_u \Gamma_2^u(z), \tag{20}$$

holds, expressed for $z \in S'$ by (16) with the boundary condition (18). For $z \in S''$ notice that $\mathcal{B}[\nu] = 0$, and from (12), $\Omega(z) = f(z) + \lambda(z)\mathcal{Q}_S[\phi](z) - \widehat{\lambda}(z)\nu(z) = \widehat{\lambda}(z)(\mathcal{N}[\phi](z) - \nu(z))$. Writing (20) in this situation we get (17) equivalently.

Finally, we verify that ν is indeed a viscosity solution by following identical steps to those in the proof of Proposition 5 of (Gatarek, 1992). □

3.2 *The First Aproximation Method*

For $\phi \in C_b(S)$ let us define the following operator:

$$\mathcal{L}[\phi](z) := \mathcal{R}[\phi,\phi](z), \tag{21}$$

Let us define the sequence of functions $V_i, i = 0,1,2,\ldots$, by:

$$V_i(z) := \mathcal{L}[V_{i-1}](z), \quad \forall z \in S \tag{22}$$

Definition 3.1. Let the sequence of jump times $\Sigma = \{T_i : i = 0,1,\ldots\}$ and the sequence of intervention times $\Pi = \{\tau_i : i = 0,1,\ldots\}$. Let the sequence of intervention or jump times $\Upsilon = \{\varsigma_i : i = 0,1,\ldots\}$, defined by:

$$\varsigma_0 := 0, \qquad \varsigma_i := \min\{t > \varsigma_{i-1} : z_t \neq z_{t^-}\},$$

where $(z_{t^-} := \lim_{s\uparrow t} z_s)$.

These sequences are $\{F_t\}$-stopping times, and in view of assumptions H_1 e H_2 we guarantee that $\tau_i \to \infty$ P_z-q.c. and $T_i \to \infty$ P_z-q.c., when $i \to \infty$; thus $\varsigma_i \to \infty$ P_z-q.c.

Lemma 4. For each $z \in S$ and $i = 0,1,\ldots$

$$\begin{aligned} V_i(z) = \inf_{\pi\in\Pi, u\in U_\partial} E_z^{\pi,u}\Big\{\int_0^{\varsigma_i} e^{-\alpha s} f(z_s)\,dt \\ + \sum_{j=1}^{i} e^{-\alpha\tau_j} g(z_{\tau_j}) 1_{\{\tau_j < T_i\}} + \\ \sum_{j=1}^{i} e^{-\alpha\varsigma_j} h(z_{\varsigma_j}), u(z_{\varsigma_j})) 1_{\{z_{\varsigma_j}\in\partial_{S'}\}} \\ + e^{-\alpha\varsigma_i} V_0(z_{\varsigma_i})\Big\}, \end{aligned} \tag{23}$$

Proof: Using in (8) the fact that $\varsigma_1 = \tau_1 \wedge T_1$ for $\tau_1 \in [0,\infty]$, we obtain from (3.1) the following representation:

$$\begin{aligned} &\mathcal{R}_t[\phi,\psi](z) := \inf_u E_z^{u,\pi}\Big\{\int_0^{\varsigma_1} e^{-\alpha s} f(z_s) ds \\ &+ e^{-\alpha T_1}[\phi(z_{T_1}) + h(z_{T_1}, u(z_{T_1}))1_{\{z_{T_1}\in\partial^*_{S'}\}}]1_{\{\tau_1 \geq T_1\}} \\ &+ e^{-\alpha t}(\psi(\bar{z}_t) + g(z_t))1_{\{\tau_1 < T_1\}}\Big\}. \end{aligned} \tag{24}$$

By definition, $V_1 = \mathcal{L}[V_0] = \inf_{\tau_1} \mathcal{R}_{\tau_1}[V_0,V_0](z)$. Thus from (24) we show this result for $i = 1$. Suppose that (23) is satisfied for$i > 1$. Since $\{z_t : t \geq 0\}$ is a Strong Markov process, we conclude the following result from the dynamic programming arguments:

$$\begin{aligned} &\mathrm{V}_{i+1}(z) = \mathcal{L}[V_i](z) = \inf_{\tau_1} \mathcal{R}_{\tau_1}[V_i,V_i](z) = \\ &\inf_{\tau_1,u} E_z^{\pi,u}\Big\{\int_0^{\varsigma_1} e^{-\alpha s} f(z_s)\,ds + e^{-\alpha\tau_1} g(z_{\tau_1}) 1_{\{\tau_1 < T_1\}} \\ &+ e^{-\alpha\varsigma_1} h(z_{\varsigma_1}), u(z_{\varsigma_1})) 1_{\{z_{\varsigma_1}\in\partial_{S'}\}} + e^{-\alpha\varsigma_1} V_i(z_{\varsigma_1})\Big\} \\ &= \inf_{\tau_1,u} E_z^{\pi,u}\Big\{\int_0^{\varsigma_1} e^{-\alpha s} f(z_s)\,ds + e^{-\alpha\tau_1} g(z_{\tau_1}) 1_{\{\tau_1 < T_1\}} \\ &+ e^{-\alpha\varsigma_1} h(z_{\varsigma_1}), u(z_{\varsigma_1})) 1_{\{z_{\varsigma_1}\in\partial_{S'}\}} \\ &+ e^{-\alpha\varsigma_1} \inf_{\pi,u} E^{\pi,u}\Big\{\int_{\varsigma_1}^{\varsigma_{i+1}} e^{-\alpha s} f(z_s)\,ds \\ &+ \sum_{j=2}^{i+1} e^{-\alpha\tau_j} g(z_{\tau_j}) 1_{\{\tau_j < T_{i+1}\}} \\ &+ \sum_{j=2}^{i+1} e^{-\alpha\varsigma_j}[h(z_{\varsigma_j}), u(z_{\varsigma_j})) 1_{\{z_{\varsigma_j}\in\partial_{S'}\}} + V_0(z_{\varsigma_{i+1}})] | \mathcal{F}_{\varsigma_1}\Big\}\Big\} \\ &= \inf_{\pi,u} E^{\pi,u}\Big\{\int_0^{\varsigma_{i+1}} e^{-\alpha s} f(z_s)\,ds \\ &+ \sum_{j=1}^{i+1} e^{-\alpha\tau_j} g(z_{\tau_j}) 1_{\{\tau_j < T_{i+1}\}} \\ &+ \sum_{j=1}^{i+1} e^{-\alpha\varsigma_j}[h(z_{\varsigma_j}), u(z_{\varsigma_j})) 1_{\{z_{\varsigma_j}\in\partial_{S'}\}} + V_0(z_{\varsigma_{i+1}})]\Big\}, \end{aligned}$$

showing the expression (23) for $i + 1$. □

Proposition 5. Assume that H_1 and H_2 hold. Then V is the biggest Borel measurable solution of the system:

$$\begin{cases} w = \mathcal{L}[w] \\ w \leq h^u \end{cases}$$

for $u \in U_\partial$ with $I : S \cup \partial S \to \mathbb{R}$ given by

$$\begin{aligned} I(z) := E_z^u\Big\{\int_0^\infty e^{-\alpha s} f(z_s)\,ds \\ + \sum_{i=1}^{\infty} e^{-\alpha T_i} h(z,u) 1_{\{z_{T_1}\in\partial_{S'}\}}\Big\} \end{aligned} \tag{25}$$

Moreover, for a function $V_0 \geq I$, the sequence V_i $i = 1, 2, \ldots$ defined by (22) is such that $V_i \downarrow V$ as $i \to \infty$.

Proof: From theorem 54.23 in (Davis, 1993), $V_i(z) = \mathcal{L}[V_{i-1}](z)$, $i = 1, 2, \ldots$ for all $z \in S$ is a monotone decreasing sequence that converges to a continuous function w such that $w = \mathcal{L}[w]$. In view of $\varsigma_i \to \infty$ when $i \to \infty$, we conclude from Lemma 4 that $V_i \to V$ and therefore $V = w$. □

3.3 *The Second Aproximation Method*

For $\phi \in C_b(S \cup \partial_S^*)$ let us consider the following operator:

$$\mathcal{P}[\phi](z) := \begin{cases} \mathcal{R}[\phi, \mathcal{N}[\phi]](z), & \text{for } z \in S' \\ (\mathcal{R}[\phi, \mathcal{N}[\phi]](\bar{z}) + g(z)) \wedge \mathcal{N}[\phi](z), & \text{for } z \in S''. \end{cases} \tag{26}$$

Theorem 3.1. Suppose that H_1–H_3 hold, then for any $W_0 \in C_b(S \cup \partial_S^*)$, each function $W_i, i = 1, 2, \ldots$ defined by:

$$W_i(z) := \mathcal{P}[W_{i-1}](z), \quad \forall z \in S, \tag{27}$$

is a viscosity solution of the variational inequalities:

$$\{\mathcal{B}[W_i](z) - \widehat{\lambda}(z)W_i(z) + \lambda(z)\mathcal{Q}_S[W_{i-1}](z) + f(z)\} \wedge \{\mathcal{N}[W_{i-1}](z) + g(z) - W_i(z)\} = 0 \text{ for } z \in S', \tag{28}$$

$$W_i(z) = [W_i(\bar{z}) + g(z)] \wedge \mathcal{N}[W_{i-1}](z), \quad \forall z \in S'', \tag{29}$$

with the boundary condition for $z \in \partial_{S'}^*$:

$$W_i(z) = \inf_{u \in U\partial} \{\mathcal{Q}_\partial^u[W_{i-1}](z) + h(z, u)\} \wedge \mathcal{N}[W_{i-1}](z) \tag{30}$$

In addition, $W_i \to V$ uniformly, as $i \to \infty$.

Proof: From Lemma 3 and the arguments presented in the proof of Theorems 3 and 4 in (Salles and do Val, 2001).

4. CONCLUSIONS

A production scheduling problem in a stochastic manufacturing system that produces different items has been formulated as an Impulse Control of Piecewise Deterministic Markov Processes. The production scheduling determines a sequence of setups in the middle of the production interval, that represent the decisions on starting or stopping the production of an item, and a sequence of setups in the boundary of the state pace, that represent the decision on switching the production from one item to another. Two sucessive aproximation methods that furnishe the solution of the scheduling problem have been obtained: the first requires a restriction in the initial function of the procedure, and the second presents an unifom convergence and is initialized by any continuous function. The last procedure explores the fact that the state space of the production process has general drift in one copy of the space state, and in the other copy the deterministic drift is null. At the moment, these methods are being implemented, and numerical results will be presented at the conference.

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