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SOME RESULTS ON CONTROL SYSTEMS WITH MIXED PERTURBATIONS

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Abstract: The paper considers control systems with parametric as well as unstructured uncertainty. Parametric uncertainty is modelled by a transfer function whose numerator and denominator polynomials are independent uncertain polynomials of the form of $P(s,q) = l_0(q) + l_1(q)s + \ldots\ldots\ldots + l_n(q)s^n$ where the coefficients depend linearly on $q = [q_1, q_2, ..., q_q]^T$ and the uncertainty box is $Q = \{q : q_i \in [\underline{q_i}, \overline{q_i}], i = 1, 2, ..., q\}$. The unstructured uncertainty is modelled as H_∞ norm bounded perturbations and perturbations consisting of a family of nonlinear sector bounded feedback gains. Using the geometric structure of the value set of $P(s,q)$, some results are presented for determination of the robust small gain theorem, robust performance, strict positive realness and absolute stability problem of control systems with parametric as well as unstructured uncertainty.

Keywords: Nonlineer control systems; Robust stability; Robust performance; Popov criterion; Uncertain linear systems; Envelopes; Affine

1. INTRODUCTION

In most practical systems at least two types of uncertainties are present, namely unstructured (or nonparametric) uncertainty which represents unmodeled dynamics, nonlinearities etc., and structured (or parametric) uncertainty, representing a lack of precise knowledge of the actual parameters. The robust stability analysis of a control system in the presence of unstructured uncertainty is an important and well-developed subject in control theory. The well-known absolute stability problem (Vidyasagar, 1978) which was formulated in the 1950's is an important robustness problem regarding unstructured uncertainty as a fixed linear system is subjected to perturbations consisting of all possible nonlinear feedback gains lying in a sector. A similar problem was studied in the 1980's by modelling the perturbations as H_∞ norm bounded perturbations of a fixed linear system. And in recent years, a substantial amount of research concerning robustness analysis of control systems affected by real parametric uncertainty has been done (Barmish, 1994).

The aim of this paper is to study the determination of the robust small gain theorem, robust performance, strict positive realness and absolute stability problem of control systems with parametric as well as unstructured uncertainty. Thus, the system under investigation contains a mixed type uncertainty structure. These problems for systems with parametric uncertainty are studied in (Chapellat, *et al.*, 1990; Chapellat, *et al.*, 1991;

Dasgupta, 1987; Grujic and Petkoviski, 1987; Tesi and Vicino, 1991; Dahleh, *et al.*, 1993; Foo and Soh, 1994; Mori, *et al.*, 1995; Soh and Foo, 1992 and references therein). The majority of these results dealt with uncertain systems defined by an interval plant that is one whose numerator and denominator are interval polynomials whose coefficients are assumed to change independently. The major constraint on practical applications of these results is the assumption of interval polynomials and here results are derived assuming the numerator and the denominator polynomials of the transfer function of a given system are a polytopic polynomial family of the form

$$P(s,q) = l_0(q) + l_1(q)s + \cdots + l_n(q)s^n \qquad (1)$$

whose coefficients $l_i(q)$ depend linearly on $q = [q_1, q_2, ..., q_q]^T$ and the uncertainty box is $Q = \{q : q_i \in [\underline{q_i}, \overline{q_i}], i = 1, 2, ..., q\}$ where $\underline{q_i}$ and $\overline{q_i}$ are specified lower and upper bounds of the ith perturbation q_i, respectively. In other words, the system's transfer function is assumed to be

$$G(s,a,b) = \frac{N(s,b)}{D(s,a)} = \frac{k_0(b) + \cdots + k_m(b)s^m}{h_0(a) + \cdots + h_n(a)s^n} \quad (2)$$

where $a = [a_1, a_2, ..., a_a]^T$ and $b = [b_1, b_2, ..., b_b]^T$ and $A = \{a : a_i \in [\underline{a_i}, \overline{a_i}], i = 1, 2, ..., a\}$, $B = \{b : b_i \in [\underline{b_i}, \overline{b_i}], i = 1, 2, ..., b\}$. Recently, using the $2q$-convex parpolygonal value set of polynomials with affine linear uncertainty such as $P(s,q)$ of Eq.(1), some powerful procedures have been given in Tan and Atherton (2000) for computing the Bode, Nyquist and Nichols envelopes of the transfer function of Eq.(2). The approach given in Tan and Atherton (2000) eliminates some exposed edges of a polytope corresponding to an uncertain polynomial with affine linear coefficient perturbations. The eliminated edges are those which do not contribute to the boundary of the value sets. For each $s = j\omega$, the $2q$-convex parpolygon is defined as the outer edges of the image of the exposed edges($q2^{q-1}$ edges) of the Q-box. The idea of the $2q$-convex parpolygonal value set was first used in (Fu, 1989; Shaw and Jayasuriya, 1996) for robust stability analysis of uncertain polynomials. In this paper, using the boundary result for the transfer function of Eq.(2) derived in Tan and Atherton (2000), the classical small gain theorem, robust performance, strict positive realness and the absolute stability problem of a control system with an uncertain transfer function of the form of Eq.(2) are formulated. The results are extensions of those given in (Chapellat, *et al.*, 1990 and 1991) where the parametric uncertainty has been represented by an interval transfer function. Although the numerator and the denominator polynomials of the transfer function of Eq.(2) do not involve common uncertain parameters, it covers problems of greater generality than interval plant representations. In the case of common uncertain parameters between the numerator and denominator polynomials, the edge theorem can be used. However, the exponential growth of the exposed edges with respect to the number of uncertain parameters can lead to serious computational difficulties. Therefore, if the given uncertain transfer function has the structure of Eq.(2) the results of this paper can be more computationally efficient than existing results.

The outline of the paper is as follows: In Section 2 the construction procedures of the $2q$-convex parpolygon and Nyquist envelope are given and in Section 3, the robust version of the small gain theorem is derived. The robust performance of a feedback system with parametric uncertainty is discussed in Section 4. In Section 5, the strict positive realness conditions of the transfer function of Eq.(2) are investigated. The problem of robust absolute stability of systems with parametric uncertainty is discussed in Section 6. Section 7 includes concluding remarks.

2. CONSTRUCTION OF CONVEX PARPOLYGON AND NYQUIST ENVELOPE

In this section, the construction procedures of a $2q$-convex parpolygonal value set and the Nyquist envelope of a given transfer function of the form of Eq.(2) are discussed.

2.1 Construction of 2q-convex parpolygon

The corresponding polytope of a family of polynomials of Eq.(1) in the coefficient space has 2^q vertices and $q2^{q-1}$ exposed edges and it can be rewritten as

$$P(s,q) = f_0(s) + f_1(s)q_1 + \cdots + f_q(s)q_q, q \in Q \quad (3)$$

The 2^q vertex polynomials of the polytope of $P(s,q)$ can be written in the following pattern

$$\begin{aligned} c_1(s,q) &= f_0(s) + f_1(s)\underline{q_1} + \cdots + f_q(s)\underline{q_q} \\ c_2(s,q) &= f_0(s) + f_1(s)\overline{q_1} + \cdots + f_q(s)\underline{q_q} \\ &\cdots \\ &\cdots \\ c_{2^q}(s,q) &= f_0(s) + f_1(s)\overline{q_1} + \cdots + f_q(s)\overline{q_q} \end{aligned} \quad (4)$$

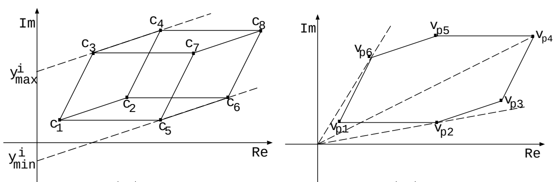
The value set of Eq.(1) can be obtained by mapping the $q2^{q-1}$ exposed edges in the complex plane or taking the convex hull of complex plane images of the vertices of the parameter box for each $s = j\omega$ and the outer edges of the value set define a $2q$-convex parpolygon. For example, let $e(c_i, c_j)$

denote the edge with end points c_i and c_j and for clarity of presentation consider Fig.1a which is the image of the exposed edges of a polytope with $q = 3$ parameters. It can be easily shown that the edges $e(c_1, c_2)$, $e(c_3, c_4)$, $e(c_5, c_6)$ and $e(c_7, c_8)$ are parallel to each other as shown in Fig.1a. Two of them which have the maximum and minimum intersections with the imaginary axis identify two edges of a $2q$-convex parpolygon as shown in Fig.1.

The following theorem is given in order to divide the frequency axis, $\omega\in[0, \infty)$, into a finite number of intervals where in each interval the edges of the $2q$-convex parpolygon remain images of the same edges of Q. The proof of the following theorem can be found in (Tan and Atherton, 2000; Shaw and Jayasuriya, 1996).
Theorem 1: The positive real roots of

$$Re[f_i]Im[f_j] - Re[f_j]Im[f_i] = 0$$
$$i, j = 1, 2, ..., q, i{\neq}j \tag{5}$$

divide the frequency axis into finite intervals where in each interval the $2q$ edges of the $q2^{q-1}$ exposed edges which constitute the outer boundary of the convex parpolygon remain unchanged. The frequencies where the outer edges of the convex parpolygon may change will be referred to as *transition frequencies.*



(a) (b)

Fig. 1. a) Image of exposed edges b) $2q$-convex parpolygon

2.2 Nyquist envelope

Consider the transfer function given in Eq.(2) and let v_{n1}, v_{n2}, v_{n3},...,v_{n2b} and v_{d1}, v_{d2}, v_{d3},...,v_{d2a} be the vertices of the $2b$ and $2a$-convex parpolygons of $N(s, b)$ and $D(s, a)$ at $s = j\omega^*$(see Fig.1.b), respectively. Then define the sets S_{N_V} and S_{N_E} which contain the vertices and the edges of the $2b$-convex parpolygon of $N(s, b)$ at $s = j\omega^*$ as

$$S_{N_V} = \{v_{n1}, v_{n2}, ..., v_{n2b}\} \tag{6}$$

$$S_{N_E} = \{e_{n1}, ..., e_{n2b}\} = \{(1 - \lambda)v_{n1} + \lambda v_{n2},$$
$$(1 - \lambda)v_{n2} + \lambda v_{n3}, ..., (1 - \lambda)v_{n2b} + \lambda v_{n1}\} \tag{7}$$

similarly define S_{D_V} and S_{D_E} for the denominator as

$$S_{D_V} = \{v_{d1}, v_{d2}, ..., v_{d2a}\} \tag{8}$$

$$S_{D_E} = \{e_{d1}, ..., e_{d2a}\} = \{(1 - \lambda)v_{d1} + \lambda v_{d2},$$
$$(1 - \lambda)v_{d2} + \lambda v_{d3}, ..., (1 - \lambda)v_{d2a} + \lambda v_{d1}\} \tag{9}$$

where $\lambda\in[0, 1]$. Using these vertex and edge sets, the extremal system which is a subset of $G(s, a, b)$ is given by

$$G_E(s) = \frac{v_{ni}(s)}{e_{dj}(s)} \cup \frac{e_{ni}(s)}{v_{dj}(s)} \tag{10}$$

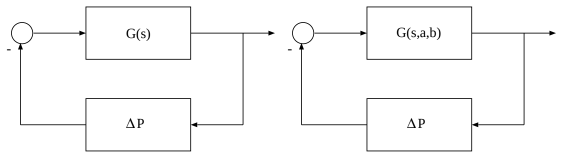
where $v_{ni}(s)\in S_{N_V}$, $e_{ni}(s)\in S_{N_E}$, $v_{dj}(s)\in S_{D_V}$, $e_{dj}(s)$ $\in S_{D_E}$, $i = 1, 2, ..., 2b$ and $j = 1, 2, ..., 2a$. Then, it is shown in Tan and Atherton (2000) that
Theorem 2: At $s = j\omega^*$,

$$\partial G(j\omega^*, a, b)\subset G_E(j\omega^*) \tag{11}$$

where ∂ denotes the boundary.

3. SMALL GAIN THEOREM

Generally, the classical small gain theorem studies the robust stability of the closed-loop system of Fig.2a where $G(s)$ is a stable linear time-invariant system which is perturbed via feedback by a stable transfer function ΔP with bounded H_∞ norm. It states that the configuration of Fig.2a remains stable for all feedback perturbations ΔP satisfying $\|\Delta P\|_\infty < \alpha$ if and only if $\|G\|_\infty \leq 1/\alpha$. In the following theorem this result is extended to the case where in addition to unstructured feedback perturbations, the $G(s)$ of Fig.2a is subject to parameter perturbations defined by Eq.(2).
Theorem 3(small gain theorem for configuration of



(a) (b)

Fig. 2. a) Closed loop system with H_∞ norm bounded perturbation b) Closed loop system with mixed perturbations

Fig.2b): Given an uncertain family $G(s, a, b)$ of stable proper plants, the closed loop system of Fig.2b remains stable for all stable perturbations ΔP such that $\|\Delta P\|_\infty < \alpha$ if and only if

$$\alpha \leq \frac{1}{max_{G\in\frac{S_{N_V}}{S_{D_E}}} \|G\|_\infty} \tag{12}$$

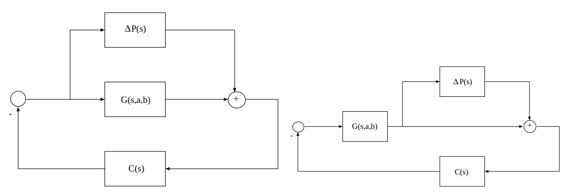
where the vertex set for the numerator (S_{N_V}) and the edge set for the denominator (S_{D_E}) are given by Eq.(6) and Eq.(9).
Proof: $\|G(s)\|_\infty < 1/\alpha$ for all $G(s)\in G(s, a, b)$ if and only if when the numerator of $G(s)$ is a vertex polynomial (S_{N_V}) and the denominator is an edge polynomial (S_{D_E}). Since the numerator and the denominator polynomials of $G(s, a, b)$ are

affine linear polynomials, it is known that for an arbitrary element $G(s)=\frac{N(s)}{D(s)}\in G(s,a,b)$ and for any fixed ω, the following holds:

$$|\frac{N(j\omega)}{D(j\omega)}|\le|\frac{v_{ni}(j\omega)}{D(j\omega)}|$$

In the above inequality, $v_{ni}(s)$ denotes a vertex polynomial which is a member of S_{N_V}. Therefore, $\|G(s)\|_\infty<1/\alpha$ for all $G(s)\in G(s,a,b)$ if and only if $\|\frac{v_{ni}(j\omega)}{D(j\omega)}\|_\infty<1/\alpha$ for all vertex polynomials $v_{ni}(s)\in S_{N_V}$ and for all $D(s)\in D(s,a)$. Now, for a fixed i, $\|\frac{v_{ni}(j\omega)}{D(j\omega)}\|_\infty<1/\alpha$ is equivalent to the Hurwitz stability of $D(s)+\alpha e^{j\theta}v_{ni}(s)$ where $\theta\in[0,2\pi)$. Since the value set of $D(s,a)$ at each frequency is contained in the images of the edge set S_{D_E} and $\alpha e^{j\theta}v_{ni}(s)$ is a fixed complex number for a fixed θ and i, the value set of $D(s,a)+\alpha e^{j\theta}v_{ni}(s)$ is still a $2a$-convex parpolygon. Therefore, the stability of $S_{D_E}+\alpha e^{j\theta}v_{ni}(s)$ implies the stability of $D(s,a)+\alpha e^{j\theta}v_{ni}(s)$. This completes the proof. □

Now, consider the control system block diagrams given in Figs.3a and 3b where $C(s)$ is a stabilizing



(a) (b)

Fig. 3. a) Closed loop system with additive perturbations b) Closed loop system with multiplicative perturbations

controller for the entire family. In order to determine the amount of unstructured perturbations that can be tolerated by the additively perturbed uncertain system shown in Fig.3a, one needs to find the maximum of the H_∞ norm of the closed loop transfer function $C(s)(1+C(s)G(s))^{-1}$ over all elements $G(s)\in G(s,a,b)$. In the case of the multiplicative perturbations shown in Fig.3b, it is necessary to find the maximum of the H_∞ norm of the closed loop transfer function $C(s)G(s)(1+C(s)G(s))^{-1}$ for all elements $G(s)\in G(s,a,b)$. The following theorem is given for computing the level of unstructured perturbations that can be tolerated in both the additive and multiplicative cases shown in Figs.3a and 3b, respectively.

Theorem 4(small gain theorem for Figs. 3a and 3b): Let $G(s,a,b)$ of Figs.3a and 3b be a proper family of plants and $C(s)$ be a stabilizing controller then the closed loop systems in Figs.3a and 3b remains stable for all stable perturbations ΔP satisfying $\|\Delta P\|_\infty<\alpha$ if and only if,

$$\alpha\le\frac{1}{sup_{G(s)\in G_E(s)}\|C(s)(1+C(s)G(s))^{-1}\|_\infty} \quad (13)$$

and for Fig.3a

$$\alpha\le\frac{1}{sup_{G(s)\in G_E(s)}\|C(s)G(s)(1+C(s)G(s))^{-1}\|_\infty} \quad (14)$$

for Fig.3b where $G_E(s)$ is defined by Eq.(10).

Proof: Let the polynomial, $P(s,q)$, of Eq.(1) be multiplied with a fixed polynomial $\delta(s)=\delta_0+\delta_1 s+....+\delta_n s^n$. At $s=j\omega^*$, the value set of $P(j\omega^*,q)$ is contained in a $2q$-convex parpolygon. For $s=j\omega^*$, $\delta(s)$ can be written as

$$\delta(j\omega^*)=M(\omega^*)e^{j\theta(\omega^*)} \quad (15)$$

where $M(\omega^*)=(Re[\delta(j\omega^*)]^2+Im[\delta(j\omega^*)]^2)^{1/2}$ and $\theta(\omega^*)=tan^{-1}(Im[\delta(j\omega^*)]/Re[\delta(j\omega^*)])$. Thus, geometrically, the affect of multiplying $P(j\omega^*,q)$ by $\delta(j\omega^*)=M(\omega^*)e^{j\theta(\omega^*)}$ is to rotate and scale the value set of $P(s,q)$ at $s=j\omega^*$, but not to distort its shape. Therefore, the value set of $\delta(j\omega^*)P(j\omega^*,q)$ is still a $2q$-convex parpolygon. Thus, $\partial(C(j\omega)(1+C(j\omega)G(j\omega,a,b))^{-1})\subset C(j\omega)(1+G_E(j\omega)C(j\omega))^{-1}$. □

Example

Consider an uncertain transfer function of the form of Eq.(2) with $N(s,b)=b_2s+b_1$ and $D(s,a)=(a_2+a_3)s^3+(a_1+a_2+a_3)s^2+(a_1+a_2)s+a_1+a_2+a_3$ where $B=\{b=[b_1\ b_2]^T: b_1\in[1,2], b_2\in[0.2,0.8]\}$ and $A=\{a=[a_1\ a_2\ a_3]^T: a_1\in[4,5], a_2\in[0.5,1.2], a_3\in[0.5,0.8]\}$. Since $N(s,b)$ is an interval polynomial, the sets S_{N_V} and S_{N_E} $\forall\omega\in(0,\infty)$ are: $S_{N_V}=\{0.2s+1,\ 0.2s+2,\ 0.8s+2,\ 0.8s+1\}$ and $S_{N_E}=\{0.2s+1+\lambda,\ (0.2+0.6\lambda)s+2,\ 0.8s+2-\lambda,\ (0.8-0.6\lambda)s+1\}$ where $\lambda\in[0,1]$. Using theorem 1 it is easily shown that there is no *transition frequency* for $D(s,a)$. Thus, one single value of frequency within $\omega\in(0,\infty)$ is sufficient to identify the edges of the $2a$-convex parpolygons of $D(s,a)$. The edges, $e(c_1,c_2)$, $e(c_2,c_4)$, $e(c_5,c_7)$, $e(c_4,c_8)$, $e(c_1,c_5)$ and $e(c_7,c_8)$, are found to constitute the boundary of the $2a$-convex parpolygons of $D(s,a)$. Thus for all $\omega\in(0,\infty)$, from Eqs.(6-9) the vertex, S_{D_V}, and the edge, S_{D_E}, sets of $D(s,a)$ can be obtained. Using these vertex and edge sets the maximum H_∞ norm of the family was computed as 0.6782. Thus, from theorem 3, the entire family of systems remains stable under any unstructured feedback perturbations of H_∞ norm less than

$$\alpha=\frac{1}{0.6782}=1.47$$

The Nyquist envelope of the extremal system, $G_E(s)$, of $G(s,a,b)$ is shown in Fig. 4 which also confirms that the maximum H_∞ norm of the family is about 0.67.

4. ROBUST PERFORMANCE

The error, output and control signal transfer functions from the input for a unity feedback control system with a fixed controller $C(s)$ and an

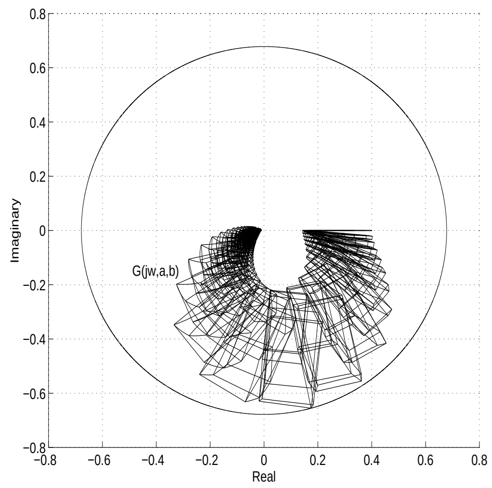


Fig. 4. Nyquist envelope of $G(s,a,b)$ and H_∞ stability margin

uncertain transfer function $G(s,a,b)$ of the form of Eq.(2) are: $T^e(s) = (1 + C(s)G(s,a,b))^{-1}$, $T^y(s) = C(s)G(s,a,b)(1 + C(s)G(s,a,b))^{-1}$ and $T^u(s) = C(s)(1 + C(s)G(s,a,b))^{-1}$. In the H_∞ approach to robust control problems, system performance is measured by the size of the H_∞ norm of error, output and other transfer functions. The worst case performance of these transfer functions can be determined by the following theorem

Theorem 5(robust performance): Let $G(s,a,b)$ be an uncertain transfer function of the form of Eq.(2) then the maximum value of the H_∞ norms of $T^e(s)$, $T^y(s)$ and $T^u(s)$ are: $sup_{G(s)\in G(s,a,b)}\|(1+C(s)G(s))^{-1}\|_\infty = sup_{G(s)\in G_E(s)}\|(1+G(s)C(s))^{-1}\|_\infty$, $sup_{G(s)\in G(s,a,b)}\|C(s)G(s)(1+C(s)G(s))^{-1}\|_\infty = sup_{G(s)\in G_E(s)}\|C(s)G(s)(1 + G(s)C(s))^{-1}\|_\infty$ and $sup_{G(s)\in G(s,a,b)}\|C(s)(1 + C(s)G(s))^{-1}\|_\infty = sup_{G(s)\in G_E(s)}\|C(s)(1+G(s)C(s))^{-1}\|_\infty$ where $G_E(s)$ is defined by Eq.(10).

Proof: The proof of this theorem is similar to the proof of theorem 4. Therefore, it is omitted.

5. SPR(STRICT POSITIVE REALNESS) CONDITIONS

Strictly positive real transfer functions are of importance in control theory. Such transfer functions are stable and their Nyquist diagrams are in the first and fourth quadrants of the complex plane. A definition of a strictly positive real transfer function is given as

Definition 1: A proper transfer function, $G(s) = N(s)/D(s)$, is called strictly positive real if

i) $D(s)$ is Hurwitz and

ii) $Re[G(j\omega)] > 0, \forall\omega\geq 0$

In other words, a transfer function is strictly positive real if it is stable and its Nyquist plot is completely contained in the right half complex plane. With this definition, we can proceed to investigate the SPR of the transfer function of the form of Eq.(2) which is given by the following theorem:

Theorem 6: Let the vertex and edge sets of $N(s,b)$ and $D(s,a)$ be given by Eqs.(6-9). Then, $G(s,a,b)$ is a strictly positive real transfer function family if

a) S_{D_E} has at least one stable member and the $2a$-convex parpolygons of $D(s,a)$ do not include the origin for all frequencies.

b) For all frequencies

$$|arg_{v_{ni}\in S_{N_V}}[v_{ni}] - arg_{v_{dj}\in S_{D_V}}[v_{dj}]| < \frac{\pi}{2} \quad (16)$$

where $i = 1, 2, ..., 2b$ and $j = 1, 2, ..., 2a$.

Proof: a) From definition 1, $D(s,a)$ must be stable. The stability of $D(s,a)$ can be checked by using the *zero exclusion principle* and *value set concept*. The value set of $D(s,a)$ at $s = j\omega^*$ is contained in a $2a$-convex parpolygon. Thus, from the *zero exclusion principle*, for stability of $D(s,a)$, the edge set, S_{D_E}, of $D(s,a)$ must be stable. This implies that there must be at least one stable member of S_{D_E} and the $2a$-convex parpolygonal value set of $D(s,a)$ will not include the origin.

b) The value set of the numerator $N(s,b)$ and the denominator $D(s,a)$ for a fixed $s = j\omega^*$ are $2b$ and $2a$-convex parpolygons. The phase condition of $G(s,a,b)$ to be strictly positive real is

$$|arg[G(j\omega,a,b)]| < \frac{\pi}{2} \quad (17)$$

This means that for each frequency both the $2b$ and $2a$-convex parpolygonal value sets must be included in a $\pi/2$-sector. This is guaranteed if the transfer functions obtained by the vertex sets (S_{N_V} and S_{D_V}) achieve this phase condition. □

6. ABSOLUTE STABILITY PROBLEM

The robust versions of the classical absolute stability criteria for the block diagram of Fig. 5a with parametric uncertainty defined by Eq.(2) were derived in Tan and Atherton (1999). In this section, the results given in Tan and Atherton (1999) are further developed for the system of Fig. 5b. From the proof of the theorem 4, it can be shown for the system of Fig. 5b that $\partial C(s)G(s,a,b)(1 + C(s)G(s,a,b))^{-1} \subset C(s)G_E(s)\ (1 + C(s)G_E(s))^{-1}$ where $G_E(s)$ is given by Eq.(10). Thus, using this boundary result, the robust versions of absolute stability criteria of Lur'e, Popov and the Circle criterion for the uncertain system of Fig. 5b are given in the following theorems:

Theorem 7:(Lur'e Criterion) If the characteristic polynomial, $1 + C(s)G(s,a,b) = 0$, of the inner loop of Fig. 5b is stable and the nonlinearity ϕ belongs to the sector $[0, k_l]$ then the condition for absolute stability is

$$\frac{1}{k_l} + Re[C(j\omega)G_E(j\omega)(1+ C(j\omega)G_E(j\omega))^{-1}] > 0, \forall\omega\geq 0 \qquad (18)$$

where $G_E(s)$ is given by Eq.(10).

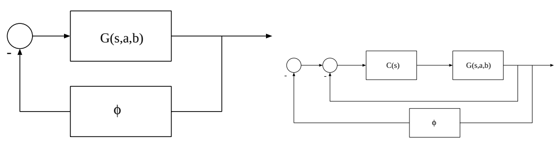
Theorem 8:(*Popov Criterion*) If the characteristic polynomial, $1 + C(s)G(s, a, b) = 0$, of the inner loop of Fig. 5b is stable and ϕ is a time-invariant nonlinearity which belongs to the sector $[0, k_p]$ then the condition for absolute stability is that there exists a real number θ such that $\forall\omega\geq 0$

$$\frac{1}{k_p} + Re[(1+\theta j\omega)C(j\omega)G_E(j\omega)(1+ C(j\omega)G_E(j\omega))^{-1}] > 0 \qquad (19)$$

where $G_E(s)$ is defined by Eq.(10).

For the circle criterion define a circle C which is centered on the negative real axis at the point $(-(k_1+k_2)/2k_1k_2, 0)$ and cutting the negative real axis at $-1/k_1$ and $-1/k_2$ where $k_1 > 0$, $k_2 > 0$ and $k_1 < k_2$. Then,

Theorem 9:(*Circle Criterion*) If the characteristic polynomial, $1 + C(s)G(s, a, b) = 0$, of the inner loop of Fig. 5b is stable and ϕ is a time-invariant nonlinearity which belongs to the sector $[k_1, k_2]$ then the condition for absolute stability is that the Nyquist plots of $C(j\omega)G_E(j\omega)(1 + C(j\omega)G_E(j\omega))^{-1}$ stays out of the circle C.



(a) (b)

Fig. 5. a) A plant with nonlinear feedback perturbations b) A closed loop system with nonlinear feedback perturbations

7. CONCLUSION

Some results on the robust small gain theorem, robust performance, strict positive realness conditions and absolute stability problem of control systems with mixed perturbations have been given. A novel feature of the results given in this paper is the use of the $2q$-convex parpolygonal value set of a polynomial with affine linear uncertainty and the transition frequency concept to reduce the number of computations. The $2q$-convex parpolygonal value set allows one to eliminate some exposed edges of a polytope corresponding to a polynomial with affine linear uncertainty which are not useful for construction of the Nyquist envelope of transfer functions with affine linear perturbations. The results given in this paper are particularly advantageous over the existing ones especially in cases where the number of transition frequencies is low and that of uncertain parameters is high.

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