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ON DYNAMIC PHENOMENA IN A DC-DC BOOST CONVERTER SUBJECT TO VARIABLE STRUCTURE CONTROL

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Abstract: In this paper, the dynamics of a DC-DC boost power converter under variable structure control is studied applying bifurcation analysis techniques. The switching hyperplane coefficients are considered as bifurcation parameters. The dynamic phenomena detected in this work suggest new types of *sliding bifurcations.* This analysis is relevant in the context of sliding mode control with auxiliary adaptive strategies, since bifurcation analysis is a suitable tool to investigate global stability.

Keywords: Nonlinear Systems, Switching Systems, Boost Power Converter, Bifurcation Analysis, Variable Structure Control, Sliding Mode.

1. INTRODUCTION

Switching systems, also denominated piecewise smooth systems, appear quite frequently in different engineering areas. The dynamic models that describe them are generally composed by differential equations with discontinuous righthand sides (Filippov, 1988). The theory of *Variable Structure Systems (VSS)* provides useful tools to deal with difficulties associated to this class of dynamic models (Utkin, 1978).

Power electronic devices are a typical example of such systems. Their operation is based on switching between two or more different topologies. For that reason they can generally be described though piecewise smooth dynamic systems.

Synthesis of controllers for power electronic switching devices is traditionally done by means of the so called averaged state space models (Kassakian *et al.*, 1991). Continuous feedback control laws are implemented with the help of *Pulse Width Modulation (PWM).* Thus, the way to deal with the discontinuity of these systems is avoiding them through the use of high frequency PWM switching. This technique makes the averaging suitable. The main problem of that procedure is that, most of the time, these averaged models are nonlinear and the control scheme employed is based on linearization around the desired equilibrium point. As it is well known, this technique imposes limitations on the operation range of the system. On the other hand, nonlinear control laws can be designed directly from the averaged model (Escobar *et al.*, 1999)

In the last decade, a variety of complex behaviours associated with this PWM control scheme has been reported (Deane and Hamill, 1990), (Hamill *et al.*, 1992), (Tse, 1994), (Yuan *et al.*, 1998), (di Bernardo *et al.*, 1998), (Kousaka *et al.*, 2000). Bifurcations and Chaos were shown to be frequent in such systems. These features are possible even for second order systems because the presence of an external PWM sawtooth carrier wave makes these systems non-autonomous. Even featuring these somewhat strange phenomena, PWM based

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control is still largely used in power electronics. Besides, despite the fact that the power electronic devices are typical variable structure systems, their controllers are rarely designed employing the results of the VSS theory and sliding modes. One of the reasons for this is that sliding mode controllers applied to the boost converter make the latter susceptible to load resistance changes and source disturbances. This drawback can be overcome by the use of appropriate adaptive schemes as reported in Escobar *et al.* (1999).

The main purpose of this paper is to characterize the possible behavior modes of a DC-DC boost converter subject to a variable structure control. Bifurcation analysis is performed considering variation on the controller parameters which, in the present case, are the coefficients of the switching hyperplane. This analysis is relevant since any adaptive scheme is based on the dynamic update of these coefficients.

This paper is organized as follows. Some basic results on Variable Structure Systems are presented in Section 2. Section 3 describes the model of the boost converter. Section 4 deals with the equilibrium analysis of the system as well as the bifurcation analysis of such equilibria. Section 5 presents some final remarks concerning the results presented in this paper.

2. PREVIOUS RESULTS ON VSS

Consider the generic nonlinear dynamic variable structure system described by

$$\dot{x} = f(t,x) + B(t,x)u(t,x) \tag{1}$$

with

$$u(t,x) = \begin{cases} u^{+}(t,x), & \text{if } \sigma(x) > 0 \\ u^{-}(t,x), & \text{if } \sigma(x) < 0, \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is a discontinuous control function, $f : D_f \subset \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ and $B : D_B \subset \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ are continuous vector functions with continuous derivatives with respect to all their arguments and $\sigma : \mathbb{R}^n \to \mathbb{R}$ is a scalar function of the state vector.

The control function undergoes discontinuity when the state vector belongs to the set

$$S = \{x \in \mathbb{R}^n / \sigma(x) = 0\} \tag{3}$$

since S is a switching surface between the two different structures of the system. A sliding mode may occur on this surface if there is a region in the state space for which the phase velocity $\dot{x}$ points toward S. The subset of S where this happens is called *sliding domain*. In order to determine this domain it is necessary to find the region in the state space where the function σ satisfies

$$V(x) = \frac{1}{2}\sigma^2(x) > 0; \qquad \dot{V}(x) = \sigma \frac{\partial \sigma}{\partial x}\frac{dx}{dt} < 0. \tag{4}$$

(Itkis, 1976), (DeCarlo *et al.*, 1988). The conditions stated in (4) define the following set

$$\Psi = \left\{ x \in \mathbb{R}^n \Big/ \sigma \frac{\partial \sigma}{\partial x}\frac{dx}{dt} < 0 \right\}, \tag{5}$$

which is the domain where $V(x)$ is a Lyapunov function. The sliding domain will be $\Omega = S \cap \Psi$. If Ω is empty, sliding modes do not exist. Otherwise, the state trajectories that hit Ω start to move along the switching surface in a sliding mode. The state equations do not describe this dynamics in an explicit way. To compute the sliding mode equation one has to employ methods of VSS theory (Utkin, 1978), (Filippov, 1988). One possibility is to use the equivalent control given by

$$u_{eq} = -\left[J_\sigma(x)B(t,x)\right]^{-1} \cdot J_\sigma(x)f(t,x) \tag{6}$$

where $J_\sigma(x)$ is the jacobian matrix of the function σ with respect to the state vector x. In the case of the single input system (1) it becomes

$$J_\sigma(x) = \left[\ \tfrac{\partial \sigma}{\partial x_1} \quad \tfrac{\partial \sigma}{\partial x_2} \quad \cdots \quad \tfrac{\partial \sigma}{\partial x_n}\ \right]. \tag{7}$$

The equivalent control represents a fictitious continuous control function that can replace the discontinuous control u in (1) when the state vector moves along the discontinuity surface. Evaluating the state equation (1) for the equivalent control (6) the following dynamic equation is obtained

$$\dot{x}(t) = \left[I - B(t,x)\cdot\left[J_\sigma B(t,x)\right]^{-1}\cdot J_\sigma\right] f(t,x) \tag{8}$$

which describes the system dynamics on S in the presence of a sliding mode.

The sliding trajectory moves along the domain Ω until meeting its boundary. When this happens, the state vector leaves the sliding domain and follows a trajectory leading to: (i) some attractor that does not belong to Ω or (ii) a closed sliding orbit (Johansson *et al.*, 1997). Beside this possibility, there is another behaviour for the system trajectory in a sliding mode. It can move along the sliding domain converging on an equilibrium point in Ω. These two different behaviours suggest a distinction between *natural equilibria*, as the stationary points of all the different structure of the system will be referred to, and *sliding equilibria*, as the equilibria that lie on the sliding domain will be called. The natural equilibrium points admit further classification. A natural equilibrium $\overline{X}$ is said to be a *real equilibrium point* when it belongs to the region in the state space where the dynamics is determined by the same structure that has $\overline{X}$ as an equilibrium. On the other hand, if the point $\overline{X}$ is a fixed point of one particular structure of the system but lies on a region where the system dynamics is governed by any other structure, then the fixed point $\overline{X}$ is called a *virtual equilibrium point* (Costa *et al.*, 2000).

3. DC-DC BOOST CONVERTER

A schematic representation of the basic DC-DC boost converter with ideal switching is given in Fig. 1. All components are considered ideal and the boost is assumed to operate in *continuous conduction mode*.

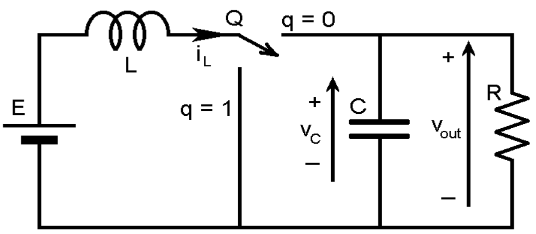

Fig. 1. Ideal DC-DC boost model.

In Fig. 1, R represents a load resistance and $E > 0$ is the external voltage source. The voltage v_{out} over R is the system output which should be driven to a regulated desired value $v_{out} = V_C > E$. The state variables are the indutor current i_L and the capacitor voltage v_C, which are both nonnegative.

An elementary analysis of the circuit in Fig. 1 leads to the mathematical dynamic model

$$\begin{cases} \dot{x}_1 = -\dfrac{1}{L}\big(1 - q(t,x)\big)x_2 + \dfrac{1}{L}E \\ \dot{x}_2 = \dfrac{1}{C}\big(1 - q(t,x)\big)x_1 - \dfrac{1}{RC}x_2 \end{cases} \tag{9}$$

where $x_1 = i_L$ and $x_2 = v_C$.

In real applications, changes in the parameters R and E represent load disturbances and voltage-source fluctuations, respectively.

Equation (9) can be rearranged as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{E - x_2}{L} \\ \frac{x_1}{C} - \frac{x_2}{RC} \end{bmatrix} + \begin{bmatrix} \frac{x_2}{L} \\ -\frac{x_1}{C} \end{bmatrix} \cdot q(t,x). \tag{10}$$

Note that (10) has the same form of (1) with q playing the role of u.

The possible equilibrium points of the considered system are obtained from Eq. (9) after eliminating the switching control function q. The set of equilibria obtained by this procedure is

$$\Gamma = \left\{ (x_1, x_2) \in \mathbb{R}^2 \Big/ x_1 = \frac{{x_2}^2}{RE} \right\}. \tag{11}$$

All possible equilibrium points of (9) must lie on the manifold Γ regardless of the function q. Note that the set Γ depends on the values of R and E.

Unlike PWM control, the variable structure control applied to the boost converter does not depend explicitly on time. Instead, it is a discontinuous function of the state vector of the form

$$q(x) = \begin{cases} 1, & \text{if } \sigma(x) > 0 \\ 0, & \text{if } \sigma(x) < 0 \end{cases} \tag{12}$$

and σ can be chosen as

$$\sigma(x) = s_0 + s_1 x_1 + s_2 x_2. \tag{13}$$

The function σ is responsible for the selection of the control structure in such a way that the set S as defined in (3) separates the state space into two regions. For each of them the switch Q is in a different position. When $\sigma < 0$, $q = 0$ and Eq. (9) has only one equilibrium point given by

$$\overline{X}_0 = \begin{bmatrix} \bar{x}_{0_1} \\ \bar{x}_{0_2} \end{bmatrix} = \begin{bmatrix} E/R \\ E \end{bmatrix} \tag{14}$$

with eigenvalues

$$\lambda_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}. \tag{15}$$

Note that $\overline{X}_0$ belongs to the set Γ.

For $\sigma > 0$, $q = 1$ and the system exhibits no equilibrium point. The trajectories are analytically obtained as

$$\begin{cases} x_1(t) = x_{10} + \frac{E}{L}t \\ x_2(t) = x_{20} e^{-\frac{t}{RC}} \end{cases} \tag{16}$$

where x_{10}, and x_{20} are the initial conditions. For $\sigma = 0$, the control function is undefined and sliding modes can exist or not. The switching surface S is defined in terms of the equation $\sigma = 0$, which can be rearranged as

$$x_2 = K + \alpha x_1 \tag{17}$$

with α and K defined as

$$\alpha = -\frac{s_1}{s_2}, \qquad K = -\frac{s_0}{s_2}. \tag{18}$$

The condition for the existence of sliding modes is that the set $\Omega = S \cap \Psi$ as defined in Section 2 is not empty. To compute Ω it is necessary to know the domain Ψ of the Lyapunov function $V(x) = 0.5\sigma^2$ for the surface S. This can be done evaluating (4) for the system formed by (9), (12) and (13) considering the two possible values of q. For $q = 0$ this procedure gives

$$\begin{cases} x_2 < \frac{s_1 RCE + s_2 RLx_1}{s_1 RC + s_2 L} & \text{for } s_2 > -s_1 \frac{RC}{L} \\ x_2 > \frac{s_1 RCE + s_2 RLx_1}{s_1 RC + s_2 L} & \text{for } s_2 < -s_1 \frac{RC}{L} \end{cases} \tag{19}$$

and for $q = 1$ it follows that

$$\begin{cases} x_2 > \frac{s_1}{s_2}\frac{RCE}{L} & \text{for } s_2 > 0 \\ \\ x_2 < \frac{s_1}{s_2}\frac{RCE}{L} & \text{for } s_2 < 0. \end{cases} \tag{20}$$

Thus, the region Ψ is defined as

$$\Psi = \left\{ x \in \mathbb{R}^2 / \, x \quad \text{satisfies (19) and (20)} \right\}. \tag{21}$$

If a segment of the surface S lies inside the region Ψ then this segment is the sliding mode domain Ω. Once the state vector hits Ω, a sliding motion takes place. Sliding equilibria will appear if Ω intercepts Γ. To determine these points, consider first the intersections of S and Γ. These points can be readily computed from Eqs. (11) and (17) as

$$\widehat{X}_1 = \begin{bmatrix} \hat{x}_{1_1} \\ \hat{x}_{1_2} \end{bmatrix} = \begin{bmatrix} -\frac{K}{\alpha} + \frac{RE - \sqrt{R^2E^2 - 4K\alpha RE}}{2\alpha^2} \\ \\ \frac{RE}{2\alpha} - \frac{\sqrt{R^2E^2 - 4K\alpha RE}}{2\alpha} \end{bmatrix} \tag{22}$$

$$\widehat{X}_2 = \begin{bmatrix} \hat{x}_{2_1} \\ \hat{x}_{2_2} \end{bmatrix} = \begin{bmatrix} -\frac{K}{\alpha} + \frac{RE + \sqrt{R^2E^2 - 4K\alpha RE}}{2\alpha^2} \\ \\ \frac{RE}{2\alpha} + \frac{\sqrt{R^2E^2 - 4K\alpha RE}}{2\alpha} \end{bmatrix} \tag{23}$$

The points $\widehat{X}_1$ and $\widehat{X}_2$ have physical meaning only if their components are real numbers. Otherwise, the intersections between S and Γ do not exist. Besides, in order to be sliding equilibrium points, $\widehat{X}_1$ and $\widehat{X}_2$ must belong to Ψ. When this happens they are renamed as $\overline{X}_1 = \widehat{X}_1$ and $\overline{X}_2 = \widehat{X}_2$ denoting sliding equilibrium points.

4. BIFURCATION ANALYSIS

This Section analyses the dynamical behaviour of the boost subject to variable structure control when parameters of the switching surface vary. The switching surfaces coefficients K and α are taken as bifurcation parameters. Changing their values, a bifurcation set in the plane (K, α) is obtained. For the sake of brevity, Fig. 2 only shows the case of $s_2 > 0$, despite the fact that this analysis can be applied to all cases. In this diagram each continuous line represents a bifurcation phenomenon. The different bifurcation curves can be determined by the analytic conditions described below.

The condition that makes $\widehat{X}_1$ and $\widehat{X}_2$ have real components (see Eqs. (22), (23)) is shown to be

$$\begin{cases} K > \frac{RE}{4\alpha} & \text{for } \alpha < 0, \\ \\ K < \frac{RE}{4\alpha} & \text{for } \alpha > 0. \end{cases} \tag{24}$$

The region defined by (24) is bounded by a hyperbola which is shown in Fig. 2. Note that only the positive branch of the hyperbola has physical meaning since, as the state variables are supposed to be nonnegative, S is required to lie at least partially in the first quadrant of the state space. This implies that K and α can not be both negative. Hence, only the second inequality in (24) is necessary.

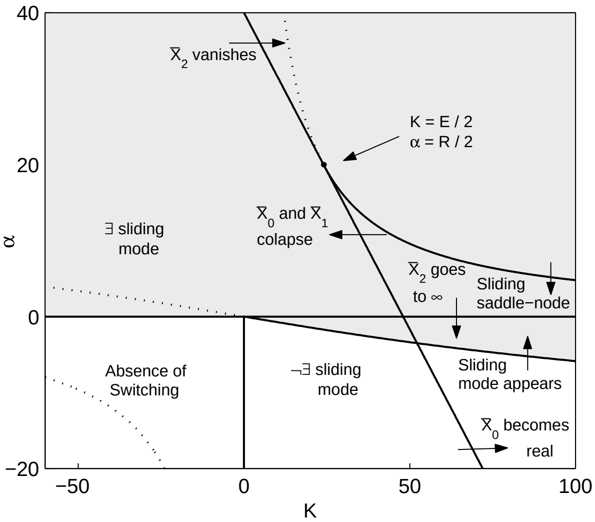

Fig. 2. Bifurcation set considering (K, α) as bifurcation parameters for $s_2 > 0$, $E = 48V$, $R = 40\Omega$, $C = 10\mu F$ and $L = 1.4mH$

The natural equilibrium point $\overline{X}_0$ changes its character according to the position of the switching line S. The condition for $\overline{X}_0$ to be a real equilibrium is

$$\alpha > R - \frac{R}{E}K. \tag{25}$$

In Fig. 2 this corresponds to the straight line tangent to the positive branch of the hyperbolic boundary.

It should be noticed that when α tends to zero, the point $\overline{X}_1$ remains finite and $\overline{X}_2$ goes to infinity since α is the slope of the switching line. When $\alpha < 0$ the point $\overline{X}_2$ has a negative component and hence, no physical meaning. This fact is illustrated in Fig. 2 by a continuous line in $\alpha = 0$.

To analyse the existence of sliding modes is equivalent to finding the condition under which the set $\Omega = S \cap \Psi$ is not empty. A merely geometric analysis based on Eqs. (17) and (21) permits to conclude that this condition is that the point (K, α) must belong to the set

$$\Phi = \left\{ (K, \alpha) \in \mathbb{R}^2 / \alpha > 0 \right\} \cup \left\{ (K, \alpha) \in \mathbb{R}^2 / K > -\alpha^3 \frac{C^2}{L^2} RE - \alpha \frac{C}{L} RE \right\}. \tag{26}$$

The region Φ is filled in grey as shown in Fig. 2.

The bifurcation set (shown in Fig. 2) captures all the behaviour modes that the analysed system can exhibit for $s_2 > 0$. In order to clarify the

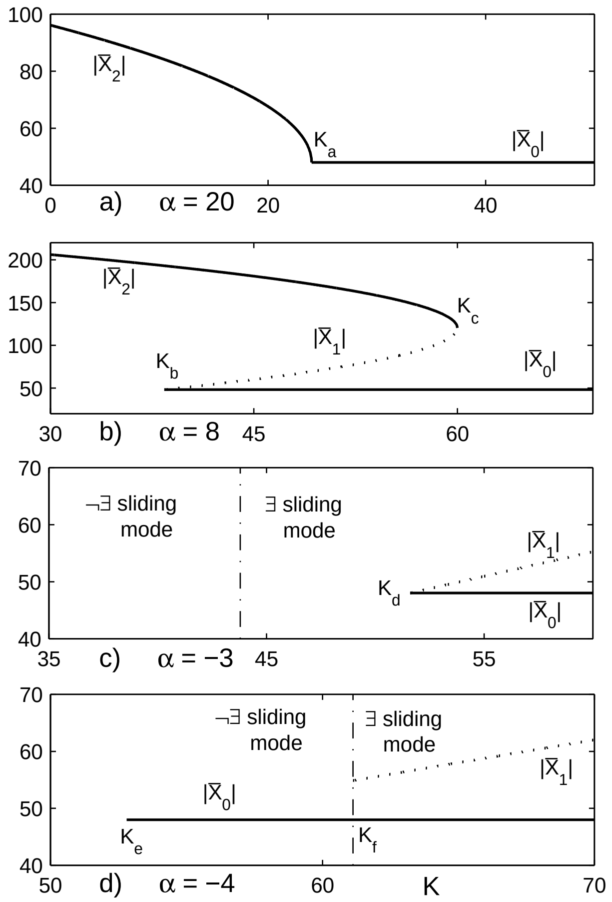

Fig. 3. Bifurcation diagrams considering K as bifurcation parameter for four different values of α. Continuous and dashed lines are stable and unstable equilibria respectively.

bifurcation phenomena described in Fig. 2, four different values of α were chosen to construct bifurcation diagrams. The variables represented in the y-axis of these diagrams are the 2-norm of the position vectors corresponding to the equilibrium points. For the system analysed in this work, only bifurcations of equilibria were found.

Fig. 3a shows the transformation of $\overline{X}_0$ in a real stable focus at the point K_a. At the same K_a, the stable sliding equilibrium $\overline{X}_2$ steps outside the sliding domain being no more an equilibrium point. These two simultaneous phenomena produce a continuous path in the bifurcation diagram that seems to correspond to the displacement of a single equilibrium point under a parameter variation. However, this apparently single continuous path is in fact the union of two different paths corresponding to two equilibria of different nature.

The second bifurcation diagram (see Fig. 3b) depicts two different phenomena. First, at K_b, two new equilibria appear, $\overline{X}_0$ becomes a real stable equilibrium point and $\widehat{X}_1$ turns into the sliding equilibrium $\overline{X}_1$. The latter one behaves like a saddle point since it divides the state space into two regions of attraction, one for $\overline{X}_0$ and

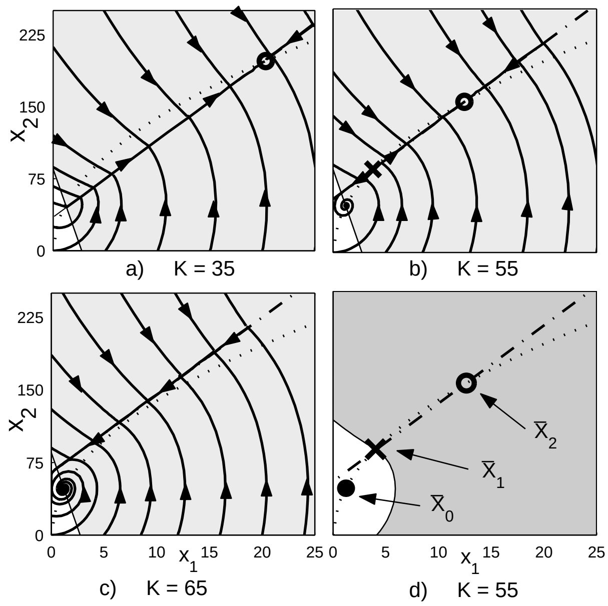

Fig. 4. Phase diagrams for different values of K. The dotted line corresponds to the set Γ and the dashed/dotted line is the switching surface S. The dark dot is the natural equilibrium point, the X is the unstable sliding equilibrium and the circle is the stable sliding equilibrium. The regions Ψ in a, b, and c are filed in light grey, and the dark grey region in d is the domain of attraction of $\overline{X}_2$

another for $\overline{X}_2$. As K increases, a *saddle-node-like* bifurcation takes place at K_c, where $\overline{X}_1$ and $\overline{X}_2$ collapse. Note that these two points lie in the sliding domain and therefore both are sliding equilibria.

Fig. 3c shows a bifurcation diagram for a negative α. As can be seen in Fig. 2, this case involves a transition between the nonexistence and the existence of sliding modes. This transition is indicated in the figure by a dashed/doted line. The phenomenon that occurs at K_d is similar to that at K_b.

Fig. 3d displays at the point K_e the sudden single birth of the equilibrium point $\overline{X}_0$, which correspond to the transition from a virtual to a real equilibrium. Increasing K a little further a sliding mode arises at K_f and brings the sliding unstable equilibrium point $\overline{X}_1$ with it.

All the phenomena described above are associated with topological changes of the state space flow and, as such, can be classified as bifurcations. To illustrate this idea the bifurcation diagram of Fig. 3b ($\alpha = 8$) was chosen in order to construct phase portraits for the three different possibilities of K. These portraits are shown in Fig. 4 together with a representation of the attraction basins for the case with $K = 55$. When $K = 35$, $\overline{X}_2$ is the only equilibrium point, which is globally asymptotically stable. For the sake of brevity this stability will not be demonstrated here. The state space flow for the region between the two bifurcations at K_b and K_c is topologically equivalent to that of Fig. 4*b*. The presence of the sliding saddle point $\overline{X}_1$ separates the attraction basins of $\overline{X}_0$ and $\overline{X}_2$ as can be seen in Fig. 4*d*. For values of K above the bifurcation point K_c the only equilibrium point is the natural focus at $\overline{X}_0$ that is globally asymptotically stable (as shown in Fig. 4*c*).

5. CONCLUSIONS

The analysis performed in this paper allowed for the detection of some bifurcation phenomena that, as far as the authors know, have not been reported yet. These phenomena can be classified into a wider family of sliding bifurcations, which includes the one reported by di Bernardo *et al.* (1999).

The variable structure control applied to the boost converter shows sliding regimes and some bifurcations of equilibrium points related to it.

As mentioned before, to reject load and source disturbances in the sliding mode controlled boost, some kind of adaptive scheme is necessary. This adaptation can change the values of the controller parameters and hence the position of the switching surface. The bifurcation analysis plays an important role in this scenario since, in order to guarantee global stability, the surface adaptation should not provoke bifurcations of the equilibrium points.

It is to be remarked that, despite the generality of the analytic results, the bifurcation set and diagrams graphically shown in this paper are partial. They only consider the case of $s_2 > 0$ and do not take into account the effects of variations in the physical components of the boost converter or consider the effects of load or source disturbances. However, the bifurcation set does not suffer topological changes when the parameters R and E vary. This fact can be checked by a simple examination of the equations that produce that set, namely (24), (25) and (26) where R and E appear as gain factors.

Further research will be conducted in order to obtain experimental results and to apply this method to other power converters.

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