

FIRST ESTIMATES OF WIENER-HAMMERSTEIN SYSTEMS USING A RANDOM MULTISINE EXCITATION

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Abstract: Wiener-Hammerstein systems consist of a linear dynamic system followed by a static nonlinearity, followed by another linear dynamic system. These models are difficult to identify due to the presence of two dynamic systems. Usually, a nonlinear estimation procedure is used to estimate the parameters of the different parts. This nonlinear estimation procedure needs good starting values to converge quickly and/or reliably to a global minimum. This paper proposes a method to compute a first estimate based on one measurement record only. *Copyright © 2002 IFAC.*

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1. INTRODUCTION

Nonlinear models are used extensively in various domains. They allow to represent physical processes over a wider range of operating point than the linear models. This gives rise to a growing need for the identification of such models. For parametric identification, the model parameters are found as the minimizers of a cost function. The initialisation of these parameters is of vital importance, since it influences the number of optimization iterations of the numerical search procedure and a bad choice can also lead to local minima of the cost function. In this paper, we propose an initialisation method for Wiener-Hammerstein systems.

Wiener-Hammerstein systems model the nonlinearity as a series of fundamental building blocks : linear

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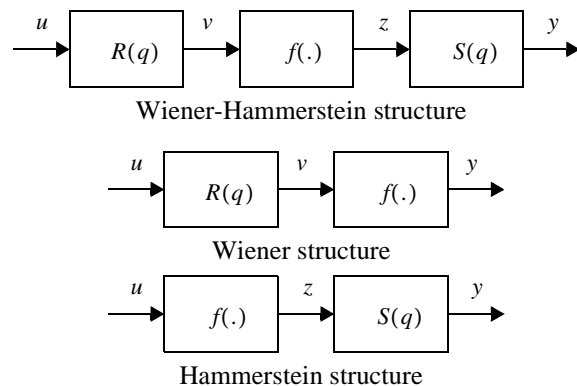


Fig 1. Related nonlinear model structures

dynamic systems and a static nonlinearity. It is plain to see that the static nonlinearity introduces the nonlinear behaviour, while the linear dynamic systems allow to model the memory that might be present in the system. Figure 1 shows the Wiener-Hammerstein structure and two special cases : the Wiener structure and the Hammerstein structure.

There have been some papers about the identification of the Wiener-Hammerstein structure (Boutayeb and Darouach, 1995; Chen and Fassois, 1992; Bershad, *et*

al., 2001). The methods (Boutayeb and Darouach, 1995; Chen and Fassois, 1992) explicitly state the importance of a proper initialisation of the estimated parameters.

While there is literature about initialisation of Wiener models (Hagenblad 1999; Crama and Schoukens 2001) or Hammerstein models (Crama and Schoukens 2001), no paper dedicated to the generation of a first estimate for Wiener-Hammerstein system could be found in the literature.

2. MULTISINES AND THEIR USE IN NONLINEAR SYSTEM IDENTIFICATION

This section summarizes the results from (Dobrowiecki and Schoukens, 2001) that are necessary to understand the principle of the proposed method.

2.1 Definition

The proposed periodic excitation signal, a random phase multisine, is defined in (1). The user can choose the amplitude distribution and basic frequency, but the phases are random. This is an advantage because the user can adapt the input signal spectrum to his needs : e.g. identification of only a frequency band or flat band-limited spectrum at the output.

$$u(t) = \sum_{k=-N}^N U_k \cdot e^{j\left(2\pi \frac{f_{max}}{N} kt + \varphi_k\right)} \quad (1)$$

In (1), U_k are the user-defined amplitudes, with $U_k = U_{-k}$ and $\varphi_k = -\varphi_{-k}$. φ_k are the phases taken from a random distribution such that $E[e^{j\varphi_k}] = 0$. U_k is scaled such that the excitation power remains the same as N increases.

2.2 Advantages

This choice of random multisine excitation signals has several advantages, but only the most important will be considered here. First of all, this signal is periodic, allowing us to average out the noise. This would not be possible with a noise excitation signal. A second point to consider is that the periodicity allows us to use a frequency domain representation of our data without leakage problems. The amplitudes of the exciting frequency lines are known whereas noise excitation suffers from dips in the power spectrum (for one particular realisation of the input vector), which degrade the signal to noise ratio of the frequency response function measurement seriously. The

proposed method could be extended to Gaussian inputs because similar properties exist for this class of excitation.

2.3 Specific properties for nonlinear systems

As explained in (Dobrowiecki and Schoukens, 2001), it is possible to obtain an estimation of the related dynamic system by dividing the output spectra with the input spectra. The result of this division is (2).

$$\begin{aligned} \frac{Y(l)}{U(l)} &= G^l(j\omega_l) + G_B(j\omega_l) + G_S(j\omega_l) \\ &= G_R(j\omega_l) + G_S(j\omega_l) \end{aligned} \quad (2)$$

The right-hand terms of this equality are respectively: the true linear part of the system ($G^l(l)$), the bias term due to the odd nonlinearities ($G_B(l)$) and a zero mean stochastic term ($G_S(l)$). The related linear dynamic system ($G_R(l)$) is the sum of the two first terms. In (Dobrowiecki and Schoukens, 2001), an expression for the related linear dynamic system for a nonlinear element surrounded by linear blocks is given.

$$G_R(j\omega_l) = R(j\omega_l)S(j\omega_l)C(U, R) \quad (3)$$

In (3), $R(l)$ and $S(l)$ are the frequency response functions of the linear systems evaluated at the l -th frequency line of the DFT (see Figure 1 for a definition of R and S), and $C(U, R)$ is a signal dependent constant.

G_R is an important source of information in the proposed method : G_R gives us a link between the Wiener and the Hammerstein part that isn't influenced by the static nonlinearity except for a scaling factor. This scaling factor can be ignored because of the way the proposed method deals with indetermination. If we can eliminate the influence of G_S in our measures of G , we have an estimation of G_R . There are two ways to exclude G_S from the G measurements (these two ways can even be combined) : repeating the experiment with different phase realisations of the random phase multisines or fitting a linear parametric model through the measurements. The first option makes sense because G_S has a zero mean. The second option works because the parametric model will not be able to follow the contributions of G_S (unless too much parameters are estimated) and the obtained model will converge to G_R (consistent estimator). The second possibility was retained because computation power is cheap and will continue to drop in price, whereas measurement time can be considered as expensive.

To conclude, it is possible to recover the product of the transfer function of the linear blocks of a Wiener-Hammerstein system. However, knowing the product

of two linear systems isn't sufficient to separate the two transfer functions : it is necessary to use the information introduced by the static nonlinearity.

3. OUTLINE OF THE METHOD

An iterative scheme was derived to separate the product of transfer functions into its two components : the input linear transfer function (R in Figure 1, called Wiener part in this paper) and the output linear transfer function (S in Figure 1, called Hammerstein part in this paper). All signal names (u , v , z and y) that will be used in the description of the method are defined in Figure 1.

3.1 Basic idea

The first step is to make the best estimation possible of G_R , because it is going to be used extensively afterwards. If the Wiener part was known, the Hammerstein part could be deduced immediately from the knowledge of G_R . Then, as in (Crama and Schoukens, 2001), the static nonlinearity could be estimated, yielding a complete model of the Wiener-Hammerstein system.

Since the Wiener part is not known in practice, an iterative solution is used, where the previous estimations of the Wiener part and of the Hammerstein part are used to generate a new estimation of the static nonlinearity. Because G_R alone hasn't enough information to separate the two linear parts, information from the nonlinear effects is brought in : the signal z is computed and used to form a new estimate of the Hammerstein part. Afterwards, a new estimate of the Wiener part is computed using (4). This completes the iteration.

$$R_k(j\omega_l) = \frac{G_R(j\omega_l)}{S_k(j\omega_l)} \quad (4)$$

3.2 Putting it all together

To clarify the outlined method, a complete iteration cycle will be given, using the measurement data presented in 4. MEASUREMENT RESULTS.

Step 1. Estimate the related linear dynamic system G_R . Figure 2 shows the influence of the stochastic nonlinear contributions and that the parametric estimation of G_R rejects the stochastic nonlinearities. It cannot be stressed enough that the noise-like behaviour of the measurements in Figure 2 is not due to a bad signal-to-noise ratio, as the values of the standard deviation in the same plot prove : the

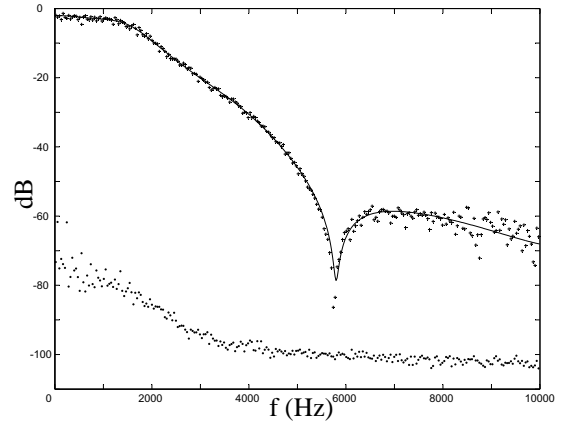


Fig 2. Fit of the Related Linear Dynamic System G_R (- :model; + :measurements; . :standard deviation)

standard deviation of the transfer function is well below the absolute value of the variations of the transfer function from the smooth model.

To keep this description short, some iterations will be neglected and the text will describe how from the estimations in iteration 25 the estimations in iteration 26 will be computed. Figure 3 shows the estimations of the linear parts of the Wiener-Hammerstein system compared to the true values.

Step 2. Using the knowledge of the Wiener part R_{k-1} and of the Hammerstein part S_{k-1} estimated at the previous iteration step (iteration 25), the static nonlinearity f_k is estimated. The nonlinearity is represented as a polynomial of degree 5. Other choices of basis functions will be discussed in paragraph 3.3. For each degree n , a signal y_n is computed (see (5)).

$$y_n^{(26)} = S^{(25)}[(R^{(25)}[u])^n] \quad (5)$$

The signals y_n are then compared with the measured output and the coefficients of the basis functions are then estimated such that the weighted sum of the

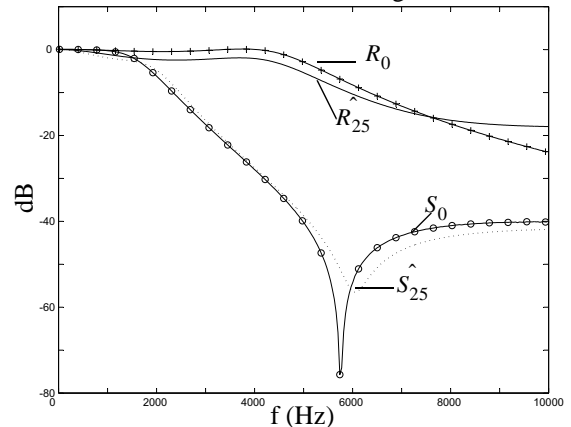


Fig 3. Comparison of R_{25} and S_{25} with the true R_0 and S_0 transfer function (- :Wiener part estimation; -- :Hammerstein part estimation; + :true Wiener part value; o :true Hammerstein part value)

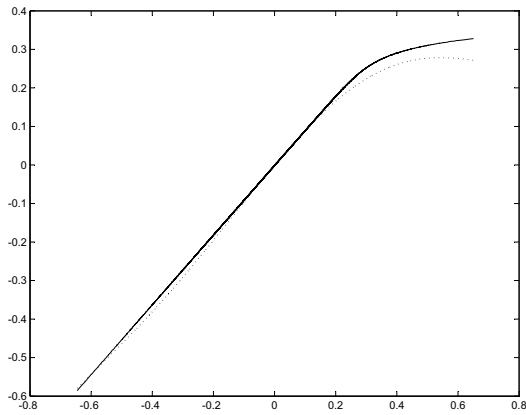


Fig 4. Comparison of the estimated and the measured static nonlinearity (- :true value; -- :estimation)

signals y_n approaches the measured output optimally in a least square sense.

Figure 4 shows the obtained model for the static nonlinearity (compared with a direct measurement of the static nonlinearity).

Step 3. Estimate the Hammerstein part S_k . Now the knowledge of our previous Wiener part R_{k-1} estimation (R_{25}) and our new estimation of the static nonlinearity can be used to form a new estimation of the Hammerstein part S_k . Figure 5 shows the data points used and the resulting model.

Step 4. Estimate the Wiener part R_k . The new estimate of the Wiener part S_k uses the information contained in the related linear dynamic system G_R and the new Hammerstein part estimation S_k . Figure 6 shows the data points used and the resulting model.

Once these steps are complete, the algorithm resumes at step 2.

At each iteration, the new estimates are then taken together to simulate the measured system and the prediction error power is remembered along with the estimated parameters. This data is used to choose the

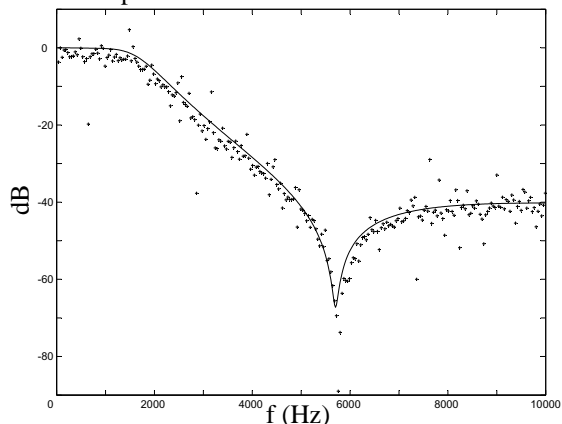


Fig 5. Estimation of the Hammerstein part S_{26} (+ :data points; - :fitted model)

model with the lowest prediction error power once the maximum number of iterations is reached.

3.3 Implementation details

Avoiding indetermination. A second point to consider is how to treat the indetermination present in our model : with only input and output measures of the Wiener-Hammerstein structure, it is impossible to know how the gain is distributed among the three blocks. The same goes for the delay : we can only measure the total delay introduced by both linear systems, but it is impossible to find out how the delay is split up.

To resolve the gain uncertainty, certain choices were made : for both linear transfer functions, the amplitude gain is fixed to 1 at a frequency chosen by the user. As a side-effect for the plots in this paper, the data has always been suitably scaled - where appropriate - to allow for an easier comparison.

For the delay, no assumption is made, but the delay is left free. The reason behind this is that the estimation of a parametric model limits the possible group delay realisations and that the algorithm will automatically converge to a delay distribution that allows it to fit the phase easily.

Representation of the static nonlinearity. The static nonlinearity may be represented by any set of basis functions. Besides x^n , a piecewise linear approximation of the static nonlinearity was considered. However, this idea was abandoned because the piecewise linear approximation resulted in slower convergence or even gave no result at all. Simulations showed that the problem was linked with the amount of nonlinear energy produced by the estimation. These nonlinear contributions are essential for the separation of the two linear parts because they give the necessary additional knowledge. However, the model of the static nonlinearity doesn't produce enough nonlinear contributions to cover the noise.

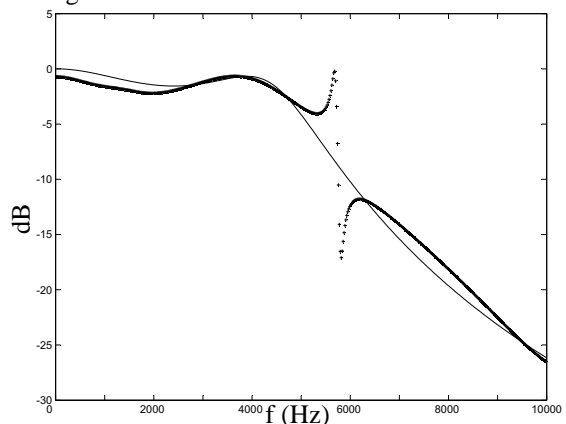


Fig 6. Estimation of the Wiener R_{26} (+ :data points; - :fitted model)

This is due to the choice of the breaking points of the piecewise linear approximation : the input at the modelled static nonlinearity has a probability density function concentrated around its mean value. This means that few points are sampled where the nonlinearity saturates. Because the break points are chosen for equal support from the data, the saturation of our example tended to be too weak, causing the nonlinear information to disappear beneath the noise. On the other hand, each polynomial basis function applied to the input introduces a substantial amount of nonlinearity, making the separation of the two linear parts of the Wiener-Hammerstein system much easier. So a polynomial representation is used even though it is less able to model a saturation efficiently.

Weighting. The scheme uses empiric rules to include the noise information in the estimation process. The Hammerstein part is estimated using the computed z signal. The weighting of the estimation of the Hammerstein part consists of the standard deviation of the transfer function that will be estimated : the output spectrum is divided by the computed spectrum of z for each data record. The weights computed this way contain the measurement noise and the stochastic nonlinear contributions. For the estimation of the Wiener part, the noise at the output was chosen as heuristic value for the weights.

Convergence conditions. It is very difficult to give exact information about the convergence of the proposed method. Empirically, the most important parameter seems to be the smoothness of the estimated transfer function. As was already said, this can be achieved in two ways : estimating a parametric model and/or repeating the experiment with many different input signals. The same method could be implemented with non parametric estimation of the transfer functions, but this would only converge for a high number of different input signals, due to the necessity of eliminating the stochastic contribution of G_S . On the other hand, if the right number of parameters is used, parametric modelling of the transfer function makes convergence possible with only a few or even only one realisation of the excitation signals.

Choice of the excitation signal. The excitation signal is chosen from the set of multisines. The excitation for the experiments in this contains only odd components. This is useful to see the even nonlinearities clearly, because the odd nonlinearities do not create components on the even lines. Another advantage is that the estimation of the related dynamic system G_R will not be disturbed by the even part of the static nonlinearity, because these contributions fall outside of the frequency grid used for its estimation.

This isn't the only possible choice of excitation signal, however. Of course, the usual restrictions from the

linear identification theory apply : it is impossible to identify a pole outside (or too close to the edge of) the excitation or measurement range. There is an additional condition for the identification process to be able to make the difference between a stochastic nonlinear contribution and ripple of the transfer function : there have to be enough frequency lines contained in the region where the ripple occurs.

Order selection. This is still an open question : if the model order for the identification of the linear parts is chosen too high, convergence problems might appear. Take for example Figure 6 that shows how the effect of a misplaced zero in the Hammerstein estimation is eliminated because the model order isn't high enough to incorporate the fallacious peak at ca. 5500Hz. On the other hand, the model order must be high enough. Without prior knowledge, there is no way out but to make a guess and try different combinations.

4. MEASUREMENT RESULTS

4.1 Measurement setup

The presented initialisation methods were applied to data measured on a physical system. This system was built as a test object, where all the possible signals were accessible ($u(t)$, $v(t)$, $z(t)$ and $y(t)$, see Figure 1), allowing to measure the transfer function of each block on its own and to compare the identified models with the reality.

The measurements were done with a VXI-system, measuring both the input and the output signals. The system was excited by an arbitrary function generator (HP E1445A) and measured with two samplers (HP E1437A).

As required by the described methods, odd random phase multisines were used to excite the systems. The excited band was from the lowest available frequency grid line to approximately 10kHz. The sampling

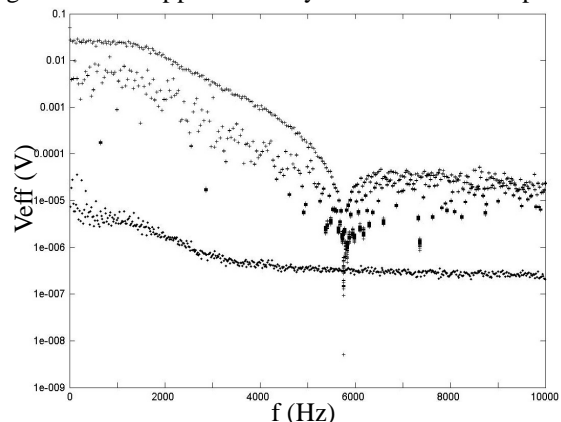


Fig 7. Measurement data (+ :data points; . :standard deviation)

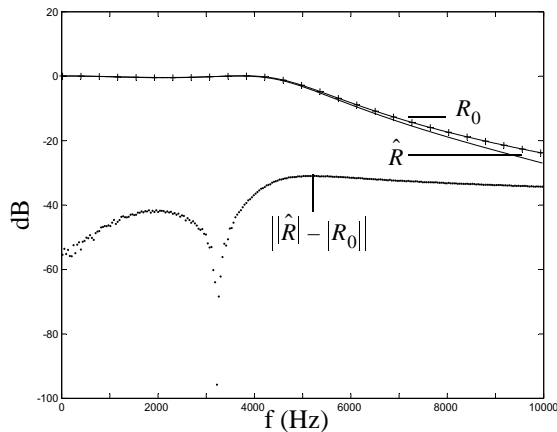


Fig 8. Identified Wiener part (- :identified model; + :true value; . :difference between the identified amplitude and the true amplitude)

frequency was 156.25kHz and each period was 8192 points long, resulting in a frequency resolution of ca. 19.1Hz. 32 periods were measured with a random phase multisine excitation, after the system was allowed 2s time to settle. Figure 7 shows the 32 measured periods together with the standard deviation.

4.2 Identified models

Figure 8, Figure 9 and Figure 10 show the identified models that best explain the measurements. In Figure 8 and Figure 10, only the quality of the amplitude estimation is assessed. The phase estimation is not evaluated due to the indetermination of the delay. The method has obviously succeeded in separating the main features of the linear systems : the zero of the Hammerstein system is correctly located, the bandwidths are correctly estimated. Of course, the fit isn't perfect yet, but the results are only intended to serve as initialization for a nonlinear estimation procedure that fits all parameters together.

5. CONCLUSION

The presented method looks promising when compared with the real transfer functions of the test

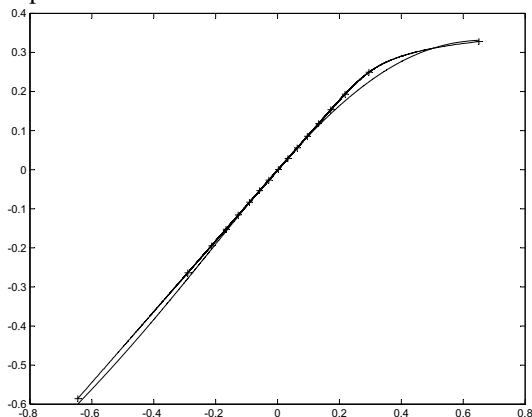


Fig 9. Identified static nonlinearity (- :identified model; + :true value)

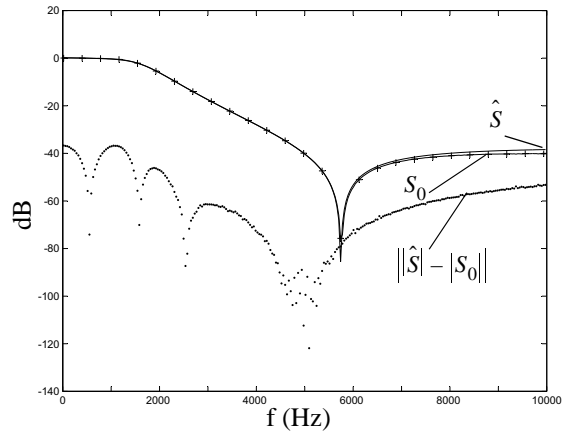


Fig 10. Identified Hammerstein part (- :identified model; + :true value; . :difference between the identified amplitude and the true amplitude)

system. The robustness to model errors has already been tested, but could perhaps be taken further. Until the complete nonlinear estimator has been implemented, the question remains whether the computed estimations give good convergence results.

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