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ON FRICTION COMPENSATION WITHOUT FRICTION MODEL

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Abstract: This paper presents a new methodology for friction compensation that is not based on any friction model. This is done by using finite-terms Fourier series to approximate the friction term. Updating laws for the coefficients of the series are easily derived from a Lyapunov approach to guarantee asymptotic convergence of the tracking error. Computer simulations are then proposed in which friction term is generated by existing models (unknown to the controller) to illustrate the efficiency of the proposed approach in compensating friction effects without friction model. Extension to the case of unknown inertia is also proposed. Robustness against velocity measurement errors is investigated by simulation showing better robustness than existing model-based schemes.

Keywords: Friction compensation; Fourier Series; free-model.

1. INTRODUCTION

Friction is a highly nonlinear inevitable phenomenon that causes deterioration in the performance of mechanical systems at low velocities. That is why intensive researches have been devoted to design friction compensation schemes. Almost all of these are model-based schemes trying to estimate some friction model parameters (Canudas-de Wit *et al.*, 1995; Bai, 1997; Canudas de Wit and Lischinsky, 1997; Olsson *et al.*, 1998; Armstrong-Helouvry *et al.*, 1994; Swevers *et al.*, 2000; Liaw and Huang, 1998)).
Since faithful model of such a nonlinear complex phenomenon need to be so, adaptive schemes become more and more complicated to built. The key point of this work is to remark that 1) faithful friction models can never be really found and 2) friction term appearing in the related problems is bounded and clearly meets the Dirichlet's conditions. It can therefore be well approximated by Fourier series expansion.
Based on this idea, a free-model compensation scheme is proposed that may in addition handle unknown system's inertia. performance and robustness (against inertia uncertainties and/or velocity measurement errors) of the controller so-obtained are tested by simulation and compared to the result of (Canudas de Wit and Lischinsky, 1997) that can fairly be considered as a very good representative of model-based friction compensation schemes.

2. PROBLEM STATEMENT

the dynamics of the process in which the friction is present is assumed to be represented by

$$m\ddot{x} = -F + u \tag{1}$$

where m is the mass, F the friction force, u is the actuator force and x is the position to be controlled. It goes without saying that (1) can be directly used in the case of angular control with m replaced by the moment of inertia J and u representing the torque.

The control objective is to track a given reference signal x_r in the following two cases :

- STANDARD PROBLEM: The mass m is perfectly known.
- PROBLEM WITH UNKNOWN MASS: Only a lower bound $\underline{m}$ on the mass is given, that is $m \geq \underline{m}$.

Solutions for the two above problems are presented in sections 4 and 5 and the resulting two controllers are denoted Controller#1 and Controller#2 respectively. These solutions resort to Fourier series expansions, that is why the following section recalls some related facts.

3. FUNCTION APPROXIMATION BY FOURIER SERIES

Any piecewise continuous real-valued function $f(t)$ satisfying the Dirichlet's conditions may be represented within any finite time-interval of length T by a Fourier series of the form (Hildebrand, 1976) :

$$f(t) = a_0 + \sum_{i=1}^{\infty}\Big(a_i \cos\omega_i t + b_i \sin\omega_i t\Big) \qquad (2)$$

where $\omega_i := 2i\pi/T$ ($i \in \mathbb{N}$). The values a_i, b_i are called the Fourier coefficients. Now, putting :

$$Z_F(t) := \Big[1, \cos\omega_1 t, \sin\omega_1 t, \dots, \cos\omega_{n_F} t, \sin\omega_{n_F} t\Big]^T \quad (3)$$

$$W_f := \Big[a_0, a_1, b_1, \dots, a_{n_F}, b_{n_F}\Big] \in \mathbb{R}^{2n_F+1} \qquad (4)$$

equation (2) can be rewritten in the following form $f(t) = W_f^T Z_F(t) + \epsilon(t)$ where $\epsilon(t)$ is the error due to the truncation satisfying (Rudin, 1976) :

$$|\epsilon| \leq \sum_{i>n_F}(|a_i| + |b_i|)$$

and since $|a_i|$ and $|b_i|$ are directly linked to the signal's energy at the corresponding frequency $2i\pi/T$, taking n_F sufficiently large ensures small approximation error ϵ since any physical signal is necessary practically band limited.

4. SOLUTION FOR THE STANDARD PROBLEM (CONTROLLER#1)

Let us denote the tracking error by $e := x - x_r$. Consider the stabilizing surface :

$$S = \dot{e} + \lambda_e e \quad ; \quad \lambda_e > 0$$

computing the time-derivative of S gives

$$\dot{S} = -\frac{F}{m} + \frac{1}{m}u - \ddot{x}_r + \lambda_e(v - \dot{x}_r) \qquad (5)$$

therefore, if one has a good estimation $\hat{F}_1$ of $F_1 := \frac{F}{m}$, a suitable stabilizing feedback may be given by :

$$u = m\Big[\hat{F}_1 + \ddot{x}_r - \lambda_e(v - \dot{x}_r) - \lambda_s S\Big] \qquad (6)$$

since (6) imposes the following dynamics on S :

$$\dot{S} = \hat{F}_1 - F_1 - \lambda_s S \qquad (7)$$

Now, suppose that n_F is chosen sufficiently large for the following to hold with good precision :

$$F_1(t) = W_{F_1}^T Z_F(t) \quad ; \quad W_{F_1} \in \mathbb{R}^{2n_F+1} \text{(fixed)} \quad (8)$$

using the following parameterization for $\hat{F}_1$:

$$\hat{F}_1 := W_{\hat{F}_1}^T(t) Z_F(t) \quad ; \quad W_{\hat{F}_1} \in \mathbb{R}^{2n_F+1} \qquad (9)$$

equation (7) becomes :

$$\dot{S} = \Big[W_{\hat{F}_1} - W_{F_1}\Big]^T Z_F - \lambda_s S =: \tilde{W}_{F_1}^T Z_F - \lambda_s S$$

where $\tilde{W}_{F_1} := W_{\hat{F}_1} - W_{F_1}$. Now, let us consider the following nonnegative function :

$$V := \frac{1}{2}S^2 + \frac{1}{2}\tilde{W}_{F_1}^T Q_F \tilde{W}_{F_1} \qquad (10)$$

and compute its time derivative under the control (6) in which (9) is injected :

$$\dot{V} = \tilde{W}_{F_1}^T\Big[S Z_F + Q_F \dot{\tilde{W}}_{F_1}\Big] - \lambda_s S^2 \qquad (11)$$

and using the fact that $\dot{\tilde{W}}_{F_1} = \dot{W}_{\hat{F}_1}$, (11) becomes :

$$\dot{V} = \tilde{W}_{F_1}^T\Big[S Z_F + Q_F \dot{W}_{\hat{F}_1}\Big] - \lambda_s S^2 \qquad (12)$$

This suggests the following updating law for $W_{\hat{F}_1}$:

$$\dot{W}_{\hat{F}_1} = -S Q_F^{-1} Z_F(t) \qquad (13)$$

Indeed, with this updating law, one has :

$$\dot{V} = -\lambda_s S^2 \qquad (14)$$

which implies by the invariance principle that $\lim_{t\to\infty} S = 0$ and hence $\lim_{t\to\infty} e = 0$.
To sum up, the solution of the standard friction compensation problem is given by the following dynamic output feedback :

$$u = m\Big[W_{\hat{F}_1}^T Z_F + \ddot{x}_r - \lambda_e(v - \dot{x}_r) - \lambda_s S\Big]$$
$$\dot{W}_{\hat{F}_1} = -S Q_F^{-1} Z_F(t)$$
$$S := (v - \dot{x}_r) + \lambda_e(x - x_r)$$

4.1 *Validation of the standard problem's solution (Controller#1)*

In this section, comparisons are done with the compensation scheme proposed in (Canudas de Wit and Lischinsky, 1997) that may be fairly considered as a good representative of model-based compensation schemes.

4.1.1. *Description of the simulations protocol*

Several kind of reference trajectories have been successfully tested (squared signal, sinusoidal, ...etc.) Because of the lack of space however, only the signal proposed in (Canudas de Wit and Lischinsky, 1997) is used with a lower amplitude in order to obtain very low velocities :

$$x_r(t) = 0.05\sin(2\pi t/5)\sin(2\pi t/100) \qquad (15)$$

The friction model's structure used in the simulation is the one used to design the compensation scheme in (Canudas de Wit and Lischinsky, 1997), namely :

$$\dot{z} = v - \frac{\sigma_0}{\alpha_0 + \alpha_1 e^{(-v/v_0)^2}} z|v| \qquad (16)$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 v \qquad (17)$$

In (Canudas de Wit and Lischinsky, 1997), two one-parameter based adaptive friction compensation schemes have been proposed to handle either variations in only the static parameters (σ_0,σ_1) or variations in all the friction parameters appearing in (16)-(17).

Since in the two proposed schemes, only one-parameter based adaptation is used, one may expect that when all the friction model's parameters (σ_0,σ_1, α_0, α_1, and α_2) indeed change, the adaptive compensation scheme of (Canudas de Wit and Lischinsky, 1997) performs less better that in the case where only σ_0 and σ_1 change while all the others are supposed to be perfectly known.

Based on the above discussion, the case where only static friction parameters σ_0 and σ_1 is chosen in the comparison in order to favour the model-based friction compensation scheme of (Canudas de Wit and Lischinsky, 1997). Note that This scheme is already favoured by the use of the friction model's structure it uses in its own design. Indeed, it has been shown in (Swevers *et al.*, 2000; Armstrong, 1995) that the friction model (16)-(17), while globally satisfactory, still presents some shortcomings preventing the accurate prediction of friction behaviour under some circumstances (over-dissipativity in presliding, fixed transition curve shape).

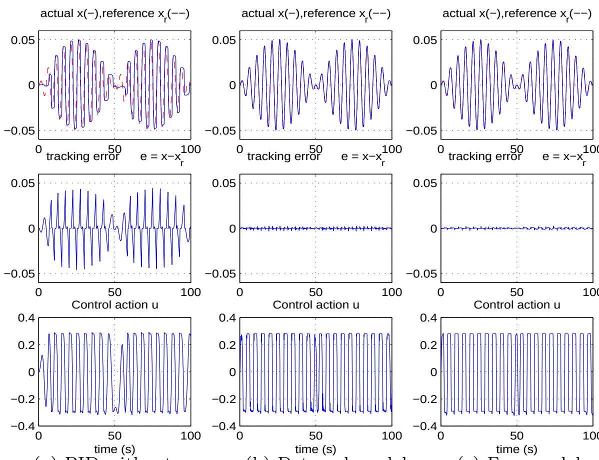
The nominal values for the friction model parameters have been taken equal to those used in (Canudas de Wit and Lischinsky, 1997), namely : $\alpha_0^{nom} = 0.285$, $\alpha_1^{nom} = 0.05$, $\alpha_2^{nom} = 0.01$, $v_0^{nom} = 0.01$, $\sigma_0^{nom} = 260$ and $\sigma_1^{nom} = 0.6$ while dependance w.r.t the velocity sign has been introduced according to the table I of the same paper.

The parameter values of the compensation scheme given in (Canudas de Wit and Lischinsky, 1997) have been used, namely $\omega_0 = 20, \xi = 0.999, n = 1.2, k = 1, \gamma = 10$. Note that ω_0, ξ and n are used to tune the PID related terms in the compensation action.

The friction model has been slightly detuned by changing only σ_0 and σ_1 by +30% and +5% respectively. All the other friction model's parameters are supposed to be exactly known by the model-based compensation controller while the free-model compensation scheme proposed in this paper ignores naturally everything about the friction model used to generate the friction term in the simulations. The free-model compensation scheme used the following parameters for all the related experiments $T = 25$, $n_F = 10$, $Q_F = 0.001$, $\lambda_e = \lambda_s = 50$

4.1.2. *Simulation Results and discussion*

Figure 1 shows the tracking performances for the PID without compensation (a), the detuned model-based controller of (Canudas de Wit and Lischinsky, 1997) (b) and the free-model compensator (Controller#1) proposed in the preceding section (c). Friction force estimations of both the model-based controller and the free-model compensation scheme (Controller#1) are shown on Figure 2. Finally, Figure 3 shows evolution of some of the Fourier coefficients vector $W_{\hat{F}} = mW_{\hat{F}_1}$.



(a) PID without compensation (b) Detuned model based controller (c) Free-model Controller#1

Fig. 1. Comparison of controllers performances

As expected, when low velocities arise ($v \approx 0$), tracking error increases in the absence of friction

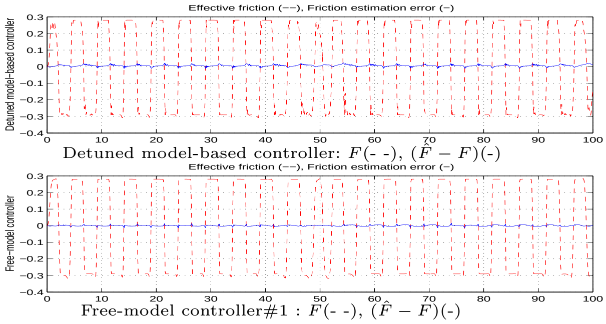


Fig. 2. Friction and friction estimation errors

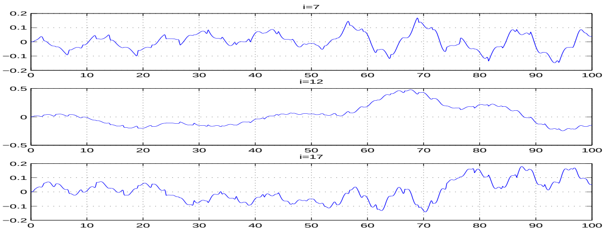


Fig. 3. Some of the components of the Fourier coefficient vector $W_{\hat{F}} = mW_{\hat{F}_1}$

compensation while it is maintained equally small by both nonlinear model-based and free-model controller.
Examination of Figure 2 shows that the free-model controller possesses an excellent friction estimation capacity since the friction estimation error remains in the neighborhood of 0.

5. SOLUTION FOR THE FRICTION COMPENSATION PROBLEM WITH UNKNOWN INERTIA (CONTROLLER#2)

In this section, the mass (or the moment of inertia for rotating systems) is supposed to be unknown. Only a lower bound $\underline{m}$ of m is given such that : $m \geq \underline{m}$. Using the notations of section 4, equation (5) is still valid :

$$\dot{S} = \frac{1}{m}\Big[-F + u - m\big(\ddot{x}_r - \lambda_e(v - \dot{x}_r)\big)\Big] \quad (18)$$

In order to simplify the expressions, the following notation is used for the measured quantity $E := \ddot{x}_r - \lambda_e(v - \dot{x}_r)$ so that (18) becomes

$$\dot{S} = \frac{1}{m}\Big[-F + u - mE\Big] \quad (19)$$

giving rise to the following control strategy

$$u = \hat{F} + \hat{m}E - \hat{m}\lambda_s S \quad (20)$$

where $\hat{m}$ and $\hat{F}$ are some instantaneous estimations of m and F respectively. Using (20) in (19) gives

$$\dot{S} = \frac{1}{m}\Big[\tilde{W}_F^T Z_F + \tilde{W}_m E - \hat{m}\lambda_s S\Big] \quad (21)$$

where, following the terminology of section 4, $\hat{F} = W_{\hat{F}}^T Z_F$, $\tilde{W}_F := W_{\hat{F}} - W_F$ and $\tilde{W}_m := \hat{m} - m$. Now consider the nonnegative function V given by :

$$V = \frac{1}{2}S^2 + \frac{1}{2m}\Big[\tilde{W}_F Q_F \tilde{W}_F + q_m \tilde{W}_m^2\Big] \quad (22)$$

where $Q_F \in \mathbb{R}^{(2n_F+1)\times(2n_F+1)}$ and $q_m \in \mathbb{R}$ are positive definite. After rather straightforward manipulations, it can be shown that computing the time derivative of V suggests the following solution of the friction compensation problem with unknown inertia :

$$\begin{aligned}
u &= W_{\hat{F}}^T Z_F + \hat{m}[\ddot{x}_r - \lambda_e(v - \dot{x}_r)] - \hat{m}\lambda_s S \\
\dot{W}_{\hat{F}} &= -\Big(Q_F^{-1} Z_F\Big)S \\
\dot{\hat{m}} &= \begin{cases} -SE/q_m & \text{if } \hat{m} > \underline{m} \\ 0 & \text{if } \hat{m} = \underline{m} \text{ and } SE < 0 \\ -SE/q_m & \text{if } \hat{m} = \underline{m} \text{ and } SE \geq 0 \end{cases} \\
E &= \ddot{x}_r - \lambda_e(v - \dot{x}_r) \\
S &:= (v - \dot{x}_r) + \lambda_e(x - x_r)
\end{aligned}$$

6. VALIDATION OF THE FREE-MODEL CONTROLLER UNDER UNKNOWN INERTIA

In this section, simulations are proposed to first illustrate the efficiency of the solution proposed in section 5 in handling uncertainties on the system's inertia and then to investigate the robustness of the controllers proposed in both sections 4 and 5 against velocity measurement errors. This is done while comparing the performances of these two controllers to that of the detuned model-based controller that has been used in section 4.

For easy references, the controllers proposed in sections 4 and 5 are denoted by Controller#1 and Controller#2 respectively.

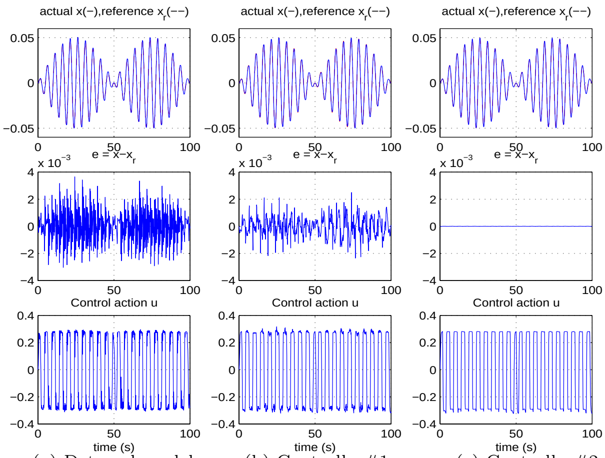
- ✓ Simulations with erroneous mass and perfect velocity measurements [Figures 4 and 5]

  In this simulation, the effective mass m used in the simulation is 4 times greater than the nominal mass m_{nom} used in all the controllers (as a constant value in the detuned model-based controller#1 and as an initial value of $\hat{m}$ in controller#2)

  $$m = 4 \times m_{nom} = 4 \times 0.0022$$

  Figure 4 shows the performances of the three controllers. The order of magnitude of the tracking error corresponding to the Controller#2 is ($\sim$ 10^{-6}) while that of the detuned model-based controller and Controller#1 is ($\sim 10^{-3}$).

Figure 5 shows the evolution of the esti-



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 4. Comparison of controllers performances under perfect velocity measurements with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$)

mated mass $\hat{m}$ used by Controller#2. Note that the result of section 5 do not guarantee the convergence of $\hat{m}$ to m but only the convergence of the tracking error.

This simulation suggests that the mass adaptation mechanism included in free-model Controller#2 enables a noticeable improvement of the tracking performances.

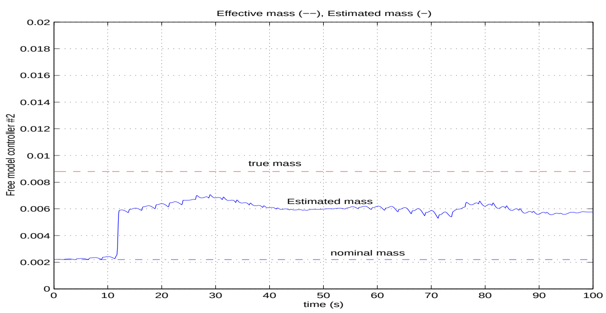


Fig. 5. Evolution of the estimated mass $\hat{m}$ used by Controller#2 using perfect velocity measurements and erroneous initial estimated mass $\hat{m}(0) = 0.25m$

✓ Simulations with erroneous mass and unperfect velocity measurements : Case of absolute error [Figures 6-7]

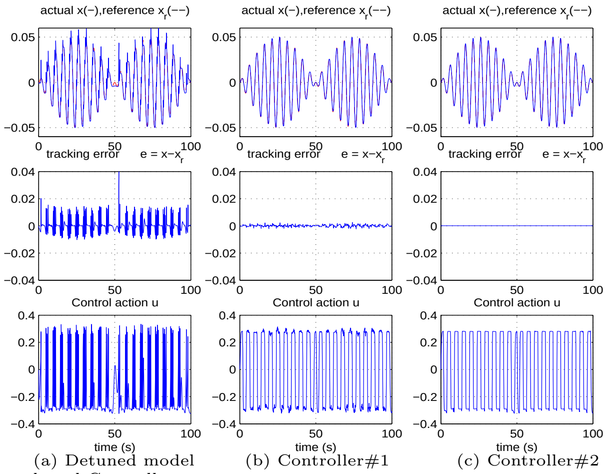
In this simulation, an absolute measurement error is introduced such that :

$$v_m = v - \varepsilon_v \quad ; \quad \varepsilon_v = 0.005 \qquad (23)$$

where $v = \dot{x}$ is the true velocity while v_m is the measured velocity used by the controllers.

This simulates a constant sensor offset. The error on the mass is maintained as in the preceding simulation. Figure 6 shows the performances of the three controllers. This simulation suggests that free-model controllers #1 and #2 are more robust to offset-like velocity measurements errors that the model-based controller.

Figure 7 shows the behaviour of the estimated mass $\hat{m}$ used by Controller#2 during this experiment.



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 6. Comparison of controllers performances with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$) and the constant offset error (23) on velocity measurements

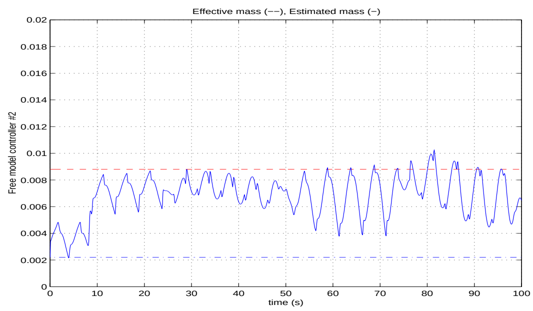


Fig. 7. Evolution of the estimated mass $\hat{m}$ used by Controller#2 with erroneous mass m_{nom} (true mass $m = 4 \times m_{nom}$) and the constant offset error (23) on the velocity measure

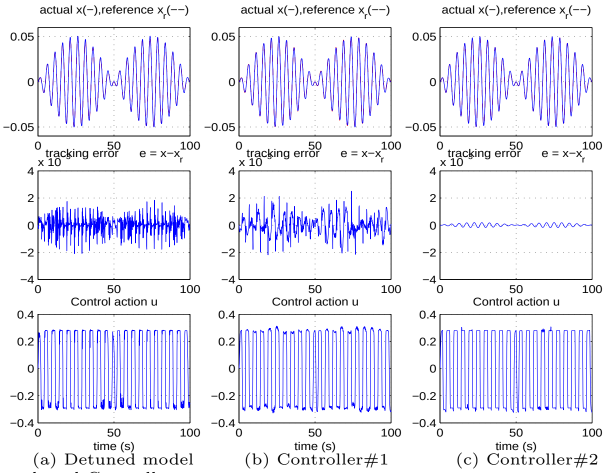
✓ Simulations with erroneous mass and unperfect velocity measurements : Case of relative error [Figure 8]

In this last experiment, a relative error is used on velocity measurements, namely

$$v_m = (1 + \varepsilon_v)v \quad ; \quad \varepsilon_v = 0.15 \qquad (24)$$

Results are presented on Figure 8.

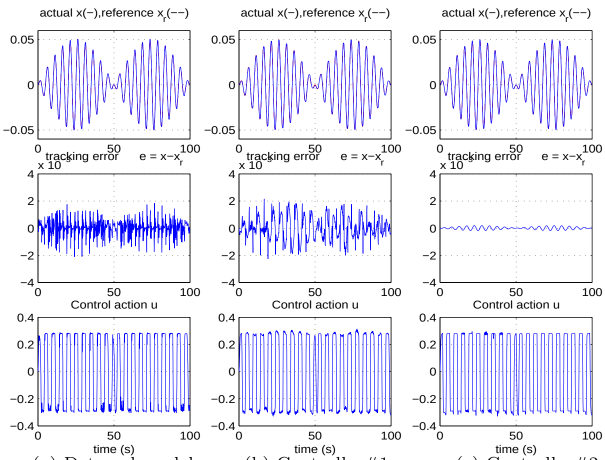
✓ Good performances are not due to the periodic nature of the reference signal. It may be thought that good performance are linked to the periodic nature of



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 8. Comparison of controllers performances with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$) and relative measurement error given by (24) on velocity measurements

the reference signal. This periodicity makes Fourier series approximation working. This objection may be rejected by underlying that in the above experiment $T = 25\ s$ is not the signal period. But since it still be a particular value (half a period) experiments have been remade with $T = 28\ s$ and the same level of performance has been observed. Figure 9 shows an example of the results so obtained.



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 9. The same experiment than that of Figure 8 with $T = 28$. Good performances are not due to the periodic nature of the reference signal.

7. CONCLUSION

In this paper, a free-model friction compensation scheme is proposed based on Fourier series expansion of the unknown friction term. Updating laws for the Fourier series coefficients are obtained by Lyapunov approach. This yields a dynamic output feedback. Tracking performances of the free-model controllers so-obtained have been compared to an existing and widely appreciated model-based nonlinear adaptive controller (Canudas de Wit and Lischinsky, 1997). Comparison shows that the free-model controllers performs at least equally well in the absence of mass uncertainty and using perfect velocity measures and much better when uncertainties on system's inertia and/or errors on velocity measurement are introduced.

To sum up, model-free compensation schemes presented in this paper seems to be very promising from both performance, robustness and implementation point of view since no friction model is needed nor preliminary identification experiments are necessary.

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