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DEVELOPMENT METHOD FOR A TAKAGI-SUGENO PI-FUZZY CONTROLLER

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Abstract: The paper proposes a new development method for a Takagi-Sugeno PI-fuzzy controller meant for a class of plants applicable to the fields of electrical drives or servo systems. The developed fuzzy control systems are quasi-optimal in terms of some quadratic performance indices defined in dynamic regimes with respect to the step modifications of the reference input and of four types of disturbance inputs. The method is validated by a case study together with digital simulation results that can correspond to the speed control of a servo system. *Copyright © 2002 IFAC*

Keywords: fuzzy systems, PI controllers, quadratic performance indices, minimization, servo systems, digital simulation.

1. INTRODUCTION

It is well considered that nowadays more than 90 % of control loops use conventional PI / PID controllers due to the very good control system performance they can offer (Åström and Hägglund, 1995). Furthermore, fuzzy controllers as nonlinear elements without dynamics can ensure control system performance enhancement (Driankov, *et al.*, 1993).

The introduction of dynamics in the structure of fuzzy controllers leads to PD-, PI- or PID-fuzzy controllers. In some well-stated conditions it is generally acknowledged the approximate equivalence between linear and fuzzy controllers (Tang and Mulholland, 1987).

This is one of the reasons why there are widely accepted development methods for fuzzy controllers by employing the merge between the knowledge on conventional linear PI controllers and the experience of experts in controlling the plant.

The considered control system structure is the conventional one, presented in Fig.1, with the nomenclature:

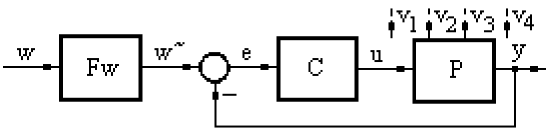

Fig. 1. Control system structure.

C – controller, P – controlled plant, Fw – reference filter, w – reference input, $w^{\sim}$ – filtered reference input, e – control error, u – control signal, y – controlled output, v_1, v_2, v_3, v_4 – disturbance inputs (generally denoted by v).

For the class of minimum phase plants characterized by transfer functions (1):

$$H_P(s) = k_P/[s(1+sT_\Sigma)] \,, \tag{1}$$

with k_P – gain and T_Σ – small time constant or sum of all parasitic time constants, the use of a linear PI controller having the transfer function (2):

$$H_C(s)=k_C[1+ 1/(sT_i)]=k_c(1+ sT_i)/s \,,\ k_C = k_cT_i \,, \tag{2}$$

with k_C – controller gain and T_i – integration time constant, can ensure relatively good control system performance (Åström and Hägglund, 1995).

The class of systems with the simplified structure from Fig.1 and transfer function (1) is specific to the fields of electrical drives or servo systems. For these systems, Preitl and Precup (1996) proposed a development method based on the generalization of the optimization relations specific to the Symmetrical Optimum (SO) method (Åström and Hägglund, 1995), called the Extended Symmetrical Optimum (ESO) method. The main results related to the ESO method are briefly presented as follows.

The parameters of the controller are tuned on the basis of the relations (3) that can guarantee the desired control system performance by means of a design parameter β:

$$k_c = 1/(\beta\, \beta^{1/2} k_P T_\Sigma^2)\ ,\ \ T_i = \beta T_\Sigma\ . \tag{3}$$

The tuning relations (3) ensure a maximum phase margin (φ_r) when there are controlled plants with constant k_P and, at the same time, a minimum guaranteed phase margin for plants with time-varying k_P.

By the choice of the parameter β in the domain $1<\beta\leq 20$, the control system performance indices can be accordingly modified and a compromise between these indices can be reached. The control system performance indices (CSPIs) defined as in Fig.2, with respect to the unit step modification of reference input, are $\{\sigma_1$ – overshoot, t_s – settling time, t_1 – first settling time$\}$. The control system performance diagrams $\{\sigma_1, \hat{t}_1 = t_1/T_\Sigma, \hat{t}_s = t_s/T_\Sigma, \varphi_r[^\circ]\}$ as function of β are presented in Fig.3.

The closed-loop transfer functions with respect to the filtered reference input, $H_w(s)$, will be expressed as:

$$H_w(s)=(\beta T_\Sigma s+1)/[\beta\, \beta^{1/2} T_\Sigma^3 s^3+\beta\, \beta^{1/2} T_\Sigma^2 s^2+\beta T_\Sigma s+1]\ . \tag{4}$$

The accordingly tuned reference filter (Fw) can further improve control system performance. One of the two filters proposed by Preitl and Precup (1996, 1999) has the following transfer function:

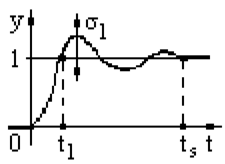

Fig. 2. CSPIs defined in unit step modification of w.

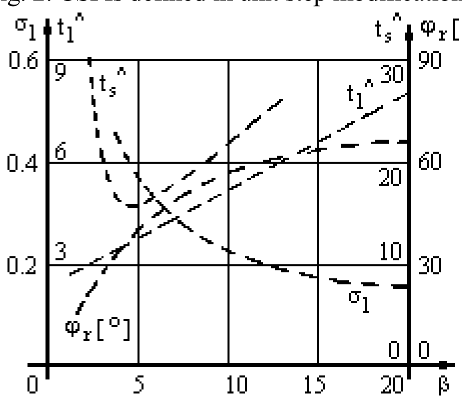

Fig. 3. Control system performance indices versus β.

$$H_{Fw}(s) = 1/(\beta T_\Sigma s + 1)\ , \tag{5}$$

and it compensated the zero from (4).

By taking into account the fact that all coefficients of $H_w(s)$ depend on the parameter β and that the PI controller development is reduced to the choice of this single parameter, β, it results that, with respect to this parameter, the optimization based on the minimization of integral quadratic criteria will have a unique solution. The following quadratic performance indices (integral quadratic optimization criteria) are widely used in the controller parameter tuning:

- the Integral of Square Error (ISE) – I_{2e}:

$$I_{2e} = \int_0^\infty e^2(t)dt\ , \tag{6}$$

- the generalized ISE – I_{2g}:

$$I_{2g} = \int_0^\infty [e^2(t) + \tau^2 \dot{e}^2(t)]dt = I_{2e} + \tau^2 \int_0^\infty \dot{e}^2(t)dt\ , \tag{7}$$

- the cross-optimization index – I_{2c}:

$$I_{2c} = \int_0^\infty [e^2(t) + \rho^2 u^2(t)]dt = I_{2e} + \rho^2 \int_0^\infty u^2(t)dt\ , \tag{8}$$

where τ and ρ are weighting coefficients.

By considering the dynamic regimes with respect to the step modification of the reference input w, it is shown in (Preitl and Precup, 2000) that the use of the integral quadratic optimization criteria $\{I_{2e}, I_{2g}, I_{2c}\}$ is no more necessary because there can be derived direct connections between the weighting coefficients τ and ρ and the design parameter β.

The PI controller development with respect to w is also performed in (Voda and Landau, 1995; Loron, 1997), where the behavior with respect to the disturbance input is only reported. Therefore, the minimization of the three mentioned quadratic performance indices (QPIs) will be done in this paper by deriving connections of type $\tau(\beta)$ and $\rho(\beta)$.

The development of fuzzy controllers for the linear plant (1) must not be viewed as a goal in itself, but as a first step in the development of more complex control structures. These structures can involve from one application to another either variable parameters of controlled plant or the plant placed at the lower hierarchical level of large-scale systems. There was presented in (Precup and Preitl, 1999) a development method for Mamdani-type PI-fuzzy controllers meant for controlling the plant (1).

The paper proposes a development method for a Takagi-Sygeno PI-fuzzy controller (TS-PI-FC). The considered TS-PI-FC is a type III fuzzy system according to (Koczy, 1996; Sugeno, 1999), and it combines the optimal PI controllers with respect to the reference input w and with respect to four possible types of disturbance inputs $\{v_1, v_2, v_3, v_4\}$.

Within the paper there will be addressed the following aspects. The connections between the ESO method and the optimization with respect to the minimization of I_{2e}, I_{2g} and I_{2c} will be discussed in the following Section. Then, Section 3 is dedicated to the presentation of the TS-PI-FC structure and of the proposed development method. In Section 4 there is applied the method to a case study that can correspond to the speed control of a servo system, and there are presented some digital simulation results. The final part of the paper highlights the conclusions.

2. CONNECTIONS BETWEEN ESO METHOD AND MINIMIZATION OF QUADRATIC PERFORMANCE INDICES

For the sake of expressing the three QPIs I_{2e}, I_{2g} and I_{2c} as function of β, there are computed the Laplace transforms of the time functions of the square factors appearing in the integrals from (6) … (8) resulting in the general expressions (9):

$$m(s) = (b_2s^2 + b_1s + b_0)/(a_3s^3 + a_2s^2 + a_1s + a_0)\,, \quad (9)$$

where m stands for e, $\dot{e}$ or u. Then, the use of Parseval relations leads to:

$$\int_0^\infty m^2(t)dt = [b_2^2a_0a_1 + (b_1^2-2b_0b_2)a_0a_3 + b_0^2a_2a_3] \,/\, / [2a_0a_1(a_1a_2-a_0a_3)]\,. \quad (10)$$

The dynamic regimes taken into consideration are: the unit step modification of the reference input w, called "w regime" (wr), and four types of unit step modifications of the disturbance inputs v_1, …, v_4, called "v_1 regime" (v_1r), …, "v_4 regime" (v_4r) (generally speaking, vr). Depending on the place of feeding the disturbance inputs to the plant and on the P structure, the types of disturbance inputs are defined in terms of Fig.4. It can be seen that v_4r is identical to wr from the point of view of the optimization.

The optimization results regarding the minimization of the QPIs I_{2e}, I_{2g} and I_{2c} with respect to β are gathered in Table 1.

There can be derived direct connections between the values of weighting coefficients τ and ρ, and the values of β for which the indices I_{2g} and I_{2c} reach their minimum. These connections are presented in Table 2 for the dynamic regimes {wr, v_1r, v_2r, v_3r, v_4r}.

Two aspects result from the analysis of Table 1 and Table 2.

Firstly, the optimization procedures based on minimizing the three considered QPIs can be reduced to the proper choice of the design parameter β. Therefore, the use of the indices I_{2e}, I_{2g} and I_{2c} becomes no more necessary.

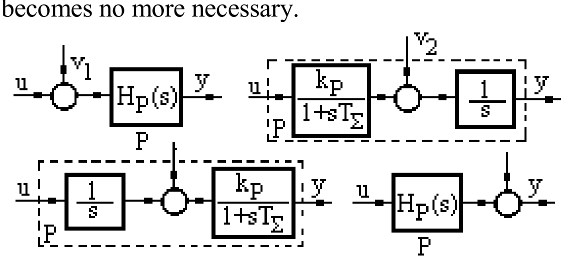

Fig. 4. Definition of disturbance input types.

Table 1 Optimization results of minimizing $I_{2e}(\beta)$, $I_{2g}(\beta)$ and $I_{2c}(\beta)$

Regime	Expression of I_{2+}	Value of β for I_{2emin}, I_{2gmin}, I_{2cmin}
I_{2e}		
wr or v_4r	$\beta T_\Sigma/[2(\beta^{1/2}-1)]$	4 (Kessler's case)
v_1r	$\beta^3k_P^2T_\Sigma^3/[2(\beta-1)]$	1.5
v_2r	$\beta\beta^{1/2}(\beta-\beta^{1/2}+1)T_\Sigma^3/[2(\beta^{1/2}-1)]$	$\beta=x^2$, $x\in(1, 2)$ – solution of $4x^3 - 8x^2 + 6x - 3 = 0$
v_3r	$\beta\beta^{1/2}k_P^2T_\Sigma/[2(\beta-1)]$	3
I_{2g}		
wr or v_4r	$[\beta T_\Sigma^2+\tau^2]/[2T_\Sigma(\beta^{1/2}-1)]$	$\{1+[1+(\tau/T_\Sigma)^2]^{1/2}\}^2$
v_1r	$\beta[\beta^2 +(\tau/T_\Sigma)^2\beta^{1/2}]k_P^2T_\Sigma^3/[2(\beta-1)]$	$\beta=x^2$, $x\in(1.5^{1/2}, 3^{1/2})$ – solution of $4x^5 - 6x^3 + (\tau/T_\Sigma)^2(x^2-3) = 0$
v_2r	$\beta[\beta^{1/2}(\beta-\beta^{1/2}+1)+(\tau/T_\Sigma)^2]T_\Sigma^3/[2(\beta^{1/2}-1)]$	$\beta=x^2$, $x\in(1, 2)$ – solution of $4x^4 - 8x^3 + 6x^2 - 3x + (\tau/T_\Sigma)^2(x-2) = 0$
v_3r	$\beta[\beta^{1/2}+(\tau/T_\Sigma)^2]k_P^2T_\Sigma/[2(\beta-1)]$	$\beta=x^2$, $x>3$ – solution of $x^3- 3x-2(\tau/T_\Sigma)^2 = 0$
I_{2c}		
	$[\beta^{5/2}T_\Sigma^2+(\rho/k_P)^2\beta\beta^{1/2}+\beta-\beta^{1/2}+1)]/[2T_\Sigma\beta\beta^{1/2}(\beta^{1/2}-1)]$	$\beta=x^2$, $x>2$ – solution of $T_\Sigma^2(x^6-2x^5)-(\rho/ k_P)^2(x^4+2x^3-4x^2+ 6x-3)= 0$
v_1r	impossible application of Parseval relations	
v_2r	impossible application of Parseval relations	
v_3r	$[\beta\beta^{1/2}T_\Sigma^2+(\rho/k_P)^2(\beta^{1/2}+1)]/[2T_\Sigma(\beta-1)]$	$\beta=x^2$, $x>3$–solution of $T_\Sigma^2(x^4-3x^2)-(\rho/k_P)^2(x+1)^2=0$

Secondly, as it has been expected, there can be observed quite different behaviors of the linear control systems with respect to the reference input w and the disturbance input $v \in \{v_1, v_2, v_3, v_4\}$. This fact will require the use of two values of the parameter β: β^w for the optimization in the case of the dynamic regime caused by the step modification of w (wr), and β^v for the optimization in the case of the dynamic regimes caused by the step modifications of several types of disturbance inputs (vr). The result will be, after applying the tuning relation (3) specific to the ESO method, in two sets of controller tuning parameters, of two linear PI controllers, $\{k_c^w, T_i^w\}$ for the PI-C-w (corresponding to wr) and $\{k_c^v, T_i^v\}$ for the PI-C-v (corresponding to vr), computed as:

$$k_c^w = 1/[\beta^w(\beta^w)^{1/2}k_PT_\Sigma^2]\,,\ T_i^w = \beta^wT_\Sigma\,,$$
$$k_c^v = 1/[\beta^v(\beta^v)^{1/2}k_PT_\Sigma^2]\,,\ T_i^v=\beta^vT_\Sigma\,. \quad (11)$$

The existence of two PI controllers, PI-C-w and PI-C-v, leads to the idea of a TS-PI-FC that should observe the dynamic regime (wr or vr) and should blend these linear controllers resulting in quasi-optimal behaviors with respect to both w and v.

Table 2 Expressions of $\tau(\beta)$ and $\rho(\beta)$

Regime	Expression of τ	Expression of ρ
I_{2g}		
wr or v_4r	$(\beta-2\beta^{1/2})^{1/2}T_\Sigma$ for $\beta > 4$	it is not the case
v_1r	$[2\beta 2\beta^{1/2}(3-2\beta)/(\beta-3)]^{1/2} T_\Sigma$ for $\beta\in(1.5, 3)$	it is not the case
v_2r	$[(4\beta^2-8\beta\beta^{1/2}+6\beta-3\beta^{1/2})/(2-\beta^{1/2})]^{1/2}T_\Sigma$ for $\beta\in(1, 4)$	it is not the case
v_3r	$[(\beta^{1/2}(\beta-3)/2\,]^{1/2}T_\Sigma$ for $\beta>3$	it is not the case
I_{2c}		
wr or v_4r	it is not the case	$\beta[\beta^{1/2}(\beta^{1/2}-2)/(\beta^2+2\beta\beta^{1/2}-4\beta+6\beta^{1/2}-3)]^{1/2}k_PT_\Sigma$ for $\beta > 4$
v_1r	impossible application of Parseval relations	
v_2r	impossible application of Parseval relations	
v_3r	it is not the case	$(\beta^2-3\beta)^{1/2}/(\beta^{1/2}+1)k_PT_\Sigma$ for $\beta>3$

3. OUTLINE OF DEVELOPMENT METHOD FOR TAKAGI-SUGENO FUZZY CONTROLLER

For the development of the TS-PI-FC it is necessary to discretize the continuous linear PI controllers with the parameters from (11). The use of Tustin's discretization method results in two quasi-continuous digital PI controllers in their incremental versions:

$$\Delta u_k = \Delta u_k^w = K_P^w\Delta e_k + K_I^w e_k\,,$$
$$\Delta u_k = \Delta u_k^v = K_P^v\Delta e_k + K_I^v e_k\,, \qquad (12)$$

where k – current sampling interval, $\Delta e_k=e_k-e_{k-1}$ – increment of control error, and $\Delta u_k=u_k-u_{k-1}$ – increment of control signal. The parameters of the two incremental digital PI controllers, $\{K_P^w, K_I^w\}$ and $\{K_P^v, K_I^v\}$, are computed in terms of (13):

$$K_P^w = k_c^w(T_i^w-h/2)\,,\ K_I^w = k_c^w h\,,$$
$$K_P^v = k_c^v(T_i^v-h/2)\,,\ K_I^v = k_c^v h\,, \qquad (13)$$

where h represents the sampling period.

The fuzzy control system structure is the conventional one from Fig.1, where the block C is replaced by the Takagi-Sugeno PI-type fuzzy controller (TS-PI-FC).

The structure of the proposed TS-PI-FC is presented in Fig.5, and it consists of: the strictly speaking PI-fuzzy controller (PI-FC), the fuzzy block FB1 for computing the current regime r_k, the fuzzy block FB2 for computing the current status s_k, and the linear blocks with dynamics.

All three fuzzy blocks {PI-FC, FB1, FB2} are Takagi-Sugeno fuzzy systems (Takagi and Sugeno, 1985), use the max and min operators in the inference engine and employ the weighted average method for defuzzification (Babuska and Verbruggen, 1996).

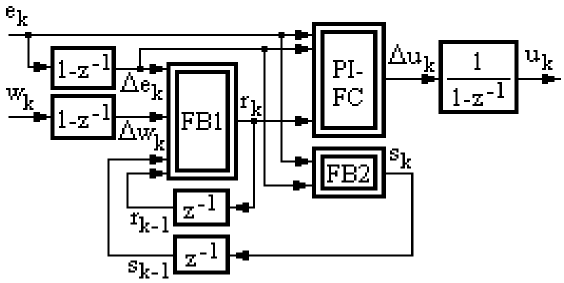

Fig. 5. Structure of TS-PI-FC.

The fuzzification is done by using the membership functions presented in Fig.6 ($\Delta w_k=w_k-w_{k-1}$ – increment of reference input). Fig.6 points out the strictly speaking positive parameters of the TS-PI-FC to be determined by the development method: $\{B_e, B_{\Delta e}, B_{\Delta w}, B_s, B_w, B_v\}$.

The fuzzy block FB1 has the role of observing the dynamic regime by computing the variable r_k. The linguistic terms "WR" and "VR" correspond to the dynamic regimes caused by the modification of w (wr) and v (vr), respectively. The inference engine of FB1 is assisted by the rule base presented as decision table in Table 3.

The fuzzy block FB2, which operates in parallel with PI-FC, computes the variable s_k characterizing the current status of the fuzzy control system. The linguistic term "ZE" corresponds to an accepted steady-state regime with almost zero e_k and Δe_k, and the linguistic term "P" corresponds to the situations when either e_k is non-zero or e_k is zero but it has the tendency to modify. The rule base of FB2 is illustrated by the decision table presented in Table 4.

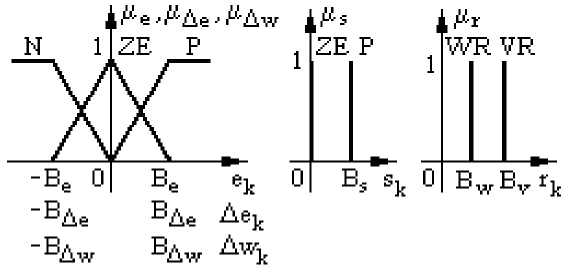

Fig. 6. Accepted input membership functions.

Table 3 Decision table of FB1

			Δw_k								
			N			Z			P		
			Δe_k			Δe_k			Δe_k		
			N	ZE	P	N	ZE	P	N	ZE	P
r_{k-1}	WR	s_{k-1} P	B_w	B_w	B_w	B_w	B_w	B_w	B_w	B_w	B_w
		ZE	B_w	B_w	B_w	B_v	B_w	B_v	B_w	B_w	B_w
	VR	s_{k-1} P	B_w	B_w	B_w	B_v	B_v	B_v	B_w	B_w	B_w
		ZE	B_w	B_w	B_w	B_v	B_w	B_v	B_w	B_w	B_w

Since FB1 and FB2 produce singleton consequents, these fuzzy blocks can be considered as type I fuzzy systems according to (Koczy, 1996; Sugeno, 1999). This is the reason why r_k and s_k are in fact not defuzzified as it results from the TS-PI-FC structure.

Table 4 Decision table of FB2

		e_k		
		N	ZE	P
	P	B_s	B_s	B_s
Δe_k	ZE	B_s	0	B_s
	N	B_s	B_s	B_s

The result this approach will be in the avoidance of supplementary fuzzification because r_k and s_k are fed to FB1 input at the next sampling time.

The inference engine of the strictly speaking PI-fuzzy controller (PI-FC) employs the rule base gathered in the decision table from Table 5.

Such a decision table ensures a quasi-PI behavior of the PI-FC. An additional parameter α, $\alpha \in (0, 1]$, was introduced for the sake of performance enhancement by alleviating the overshoot (and downshoot in the case of non-minimum-phase systems (Precup and Preitl, 1998)) in situations when e_k and Δe_k have the same sign. The cost of doing this is in a more complex rule base of the PI-FC block; otherwise the rule base of Table 5 can be reduced to only two rules.

Concerning the computation of controller parameters, the simplest to choose are B_w and B_v, which have to be different in order to create a clear difference between the two regimes, wr and vr. There are suggested the following values: $B_w = 1$, $B_v = 2$.

Then, the values of $B_{\Delta w}$ and B_s must be sufficiently small to clearly point out the constant values of w_k, and of e_k and Δe_k, respectively. If there is accepted a unit step modification of w and a 2 % settling time, the recommended values for these two parameters are: $B_{\Delta w} = 0.02$, $B_s = 0.02$.

For the computation of B_e and $B_{\Delta e}$ the modal equivalences principle is applied resulting in the relation (14) used by Precup and Preitl (1998) for a Mamdani PI-fuzzy controller:

$$B_{\Delta e}=2hB_e/(2T_i^m-h),\ T_i^m=(T_i^w+T_i^v)/2=(\beta^w+\beta^v)T_\Sigma/2, \quad (14)$$

where the parameter B_e is chosen in accordance with the experience of an expert in control systems. The relation (14) will ensure the approximate equivalence, mentioned in Section 1, between the TS-PI-FC and the linear PI controllers.

By taking into consideration all presented aspeects, the proposed development method for the TS-PI-FC consists of the following steps to be proceeded:
- express the simplified mathematical model of P in the form (1);
- choose the values of β^w and β^v as a compromise between desired control system performance resulted from Fig.3 (for σ_1, t_1, t_s and φ_r) and Table 2 (for QPIs);
- obtain the parameters $\{k_c^w, T_i^w\}$ and $\{k_c^v, T_i^v\}$ of PI-C-w and PI-C-v, respectively, from (11);
- choose a sufficiently small sampling period, h, accepted by quasi-continuous digital control and take into account the presence of a zero-order hold;

Table 5 Decision table of PI-FC

	r_k					
	WR			VR		
Δe_k /	e_k			e_k		
	N	ZE	P	N	ZE	P
P	Δu_k^w	Δu_k^w	$\alpha\ \Delta u_k^w$	Δu_k^v	Δu_k^v	$\alpha\ \Delta u_k^v$
ZE	Δu_k^w	Δu_k^w	Δu_k^w	Δu_k^v	Δu_k^v	Δu_k^v
N	$\alpha\ \Delta u_k^w$	Δu_k^w	Δu_k^w	$\alpha\ \Delta u_k^v$	Δu_k^v	Δu_k^v

- discretize the two continuous PI controllers and compute the parameters of the two quasi-continuous digital PI controllers, $\{K_P^w, K_I^w\}$ and $\{K_P^v, K_I^v\}$, by means of (13);
- choose the values of the parameters α and B_e of the TS-PI-FC and apply (14) resulting in the value of $B_{\Delta e}$;
- choose the values of the rest of TS-PI-FC parameters, $\{B_w, B_v, B_{\Delta w}, B_s\}$ by using the recommended calues.

Among the advantages of the presented method for development, implementation and use, it can be highlighted that the method can be seen also as a low cost solution. This solution ensures also a bumpless transfer from one linear PI controller to another and a quasi-optimal behavior of the control system with respect to both reference and disturbance inputs.

4. APPLICATION

A simple application of the proposed development method is exemplified for a case study that can correspond to the speed control of a separately excited armature controlled DC motor. The P is characterized in its linearized simplified form by the transfer function (1), with $k_P = 1$ and $T_\Sigma = 1$ sec. The development steps presented in Section 3 enable the development of the TS-PI-FC as follows.

There are chosen the two values of the design parameter, $\beta^w = 9$ and $\beta^v = 16$ (the case $\beta=4$ corresponds to Kessler's case (Åström and Hägglund, 1995)), for a $v = v_3$ disturbance input. These values correspond, in terms of Table 2, to $\tau = 1.7321$ sec and $\rho = 2.8844$ for minimizing I_{2g} in wr and I_{2c} in vr, respectively. Then, the tuning parameters of the continuous PI controllers are obtained from (11): $k_c^w= 0.037$ and $T_i^w= 9$ sec for PI-C-w, and $k_c^v= 0.0156$ and $T_i^v= 16$ sec for PI-C-v. Then, for the TS-PI-FC there is chosen the value of the sampling period as $h = 0.02$ sec, and the discretization of PI-C-w and PI-C-v by using (13) results in: $K_P^w = 0.333$, $K_I^w = 0.00074$, $K_P^v = 0.2498$, $K_I^v = 0.00031$. By choosing $\alpha= 0.9$ and $B_e= 0.5$ as in (Precup and Preitl, 1998) for Mamdani PI-fuzzy controllers, there is applied (14): $B_{\Delta e} = 0.0008$. Finally, the rest of TS-PI-FC parameters take the recommended values.

This developed TS-PI-FC is compared with two conventional PI controllers, PI-C-w and PI-C-v. The behavior of the three resulted control systems is analyzed by taking into account the following simulation scenario: a unit step modification of w followed by a –0.5 step modification of v_4 (after 7.5

sec). The three control system structures comprise also a reference filter Fw (Fig.1) computed for the medium value of β, β = 12.5 (see the relation (5)).

With respect to this scenario, part of the digital simulation results are presented in Fig.7 … Fig.9. It can be observed that the presented digital simulation results validate the development method.

5. CONCLUSIONS

The paper proposes a new development method for a Takagi-Sugeno PI-fuzzy controller meant for a class of second-order plants with integral character. The method is based on applying the modal equivalences principle by starting from two basic linear PI controllers obtained by minimizing three quadratic performance indices.

There are also derived in this paper new connections between the design parameter β specific to the ESO method and the weighting coefficients specific to the QPIs in case of dynamic regimes with respect to step modifications of four types of disturbance inputs (v).

The use of a Takagi-Sugeno-type fuzzy controller is successful in this case due to the linear dependence of each rule on the inputs that makes the TS-PI-FC to play the role of a bumpless interpolator between the two linear PI controllers separately designed with respect to w and v. Hence, the fuzzy control systems containing the proposed TS-PI-FC can be considered as quasi-optimal with respect to w and v.

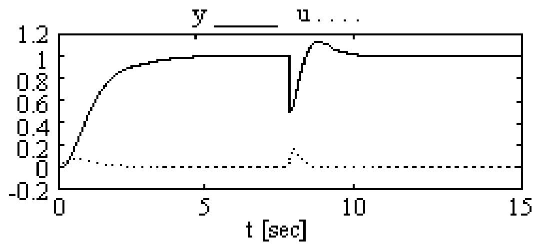

Fig. 7. Variables y and u versus time for PI-C-w.

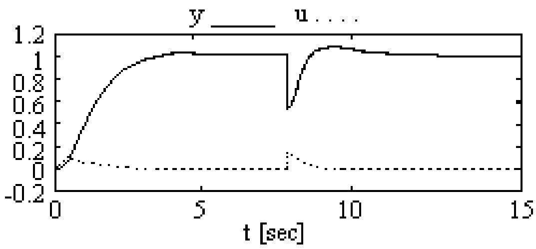

Fig. 8. Variables y and u versus time for PI-C-v.

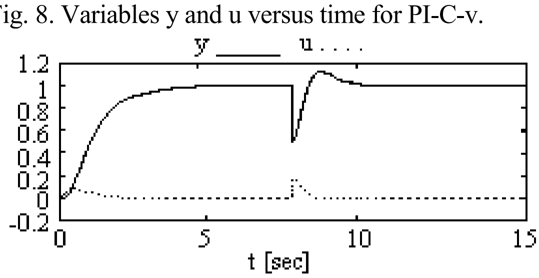

Fig. 9. Variables y and u versus time for TS-PI-FC.

Another version of bumpless interpolation is to re-compute the initial conditions (past values) of variables from the digital PI controllers.

The simplicity of TS-PI-FC structure and the flexibility and transparency of the development method makes it a low cost solution that can be extended with supplementary features to more complex applications. Nevertheless, the rigorous analysis in all applications must be performed.

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