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ADAPTIVE GAIN-SCHEDULED H_∞ CONTROL OF LINEAR PARAMETER-VARYING SYSTEMS

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Abstract : This paper concerns with a new class of adaptive gain-scheduled H_∞ control of linear parameter-varying (LPV) systems. The plants in this manuscript are assumed to be polytopic LPV systems, but the time-varying parameters in those plants are not available for measurement, and thus, the conventional gain-scheduled control strategy cannot be applied. In the proposed adaptive schemes, the estimates of those unknown parameters are obtained successively, and the current estimates are fed to the controllers to stabilize the plants and to attain H_∞ control performance adaptively. Stability analysis of the adaptive control systems is carried out by utilizing Lyapunov approaches based on linear matrix inequalities in the bounded real lemma.

Keywords: adaptive control; gain-scheduled control; linear parameter-varying systems; H_∞ control; linear matrix inequality

1. INTRODUCTION

Recently, there has been much progress in the field of gain-scheduled control of linear parameter-varying (LPV) systems with guaranteed control performances (Packard, 1994; Becker and Packard, 1994; Apkarian, *et al.*, 1995; Apkarian and Gahinet, 1995; Gahinet, *et al.*,1996; Watanabe, *et al.*, 1996). Those results are based on linear matrix inequalities (LMI) techniques in control engineering and computation tools solving LMI. The general descriptions of LPV systems, and specified forms such as LFT parameter dependence and polytopic system structures have been discussed in those studies, and especially, the gain-scheduled control schemes for polytopic LPV systems (Apkarian, *et al.*, 1995) has been one of the several standard techniques with useful computation tools (Gahinet, *et al.*, 1995). However, in those approaches, the time-varying process parameters are assumed to be known a priori. Those parameters are fed to the gain-scheduled controllers as scheduled variables to attain stability and certain control performances. If those scheduled parameters are unknown, or inaccurate, then even the stability of the resulting control systems is not assured.

This paper concerns with a new class of adaptive gain-scheduled H_∞ control of LPV systems. The plants in this manuscript are assumed to be polytopic LPV systems, but the time-varying parameters in those plants are not available for measurement, and thus, the conventional gain-scheduled control strategy cannot be applied. In the proposed adaptive schemes, the estimates of those unknown parameters are obtained successively, and the current estimates are fed to the controllers as scheduled variables to stabilize the plants and to attain H_∞ control performance adaptively. Stability analysis of the adaptive control systems is carried out by utilizing Lyapunov approaches based on LMI in the bounded real lemma.

2. GAIN-SCHEDULED H_∞ CONTROL OF LPV SYSTEMS

The gain-scheduled H_∞ control schemes for polytopic LPV systems (Apkarian, *et al.*, 1995) are reviewed, where time-varying parameters in those systems are assumed to be available for measurement.

2.1 Gain-Scheduled Control via State Feedback.

Consider the following LPV system

$$\frac{d}{dt}x = A_p(\alpha)x + B_p u + B_1(\alpha)\omega, \qquad (1)$$
$$z = C_1(\alpha)x + D_{21}u + D_{22}(\alpha)\omega, \qquad (2)$$

where $A_p(\alpha)$, $B_1(\alpha)$, $C_1(\alpha)$, $D_{22}(\alpha)$ depend affinely on the time-varying parameter α and satisfy

$$\begin{bmatrix} A_p(\alpha) & B_1(\alpha) \\ C_1(\alpha) & D_{22}(\alpha) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{pi} & B_{1i} \\ C_{1i} & D_{22i} \end{bmatrix}, \qquad (3)$$

with time-invariant matrices A_{pi}, B_{1i}, C_{1i}, D_{22i}. The parameter α ranges over a fixed polytope such that

$$\alpha \equiv [\alpha_1, \alpha_2, \cdots, \alpha_r]^T, \qquad (4)$$
$$\sum_{i=1}^{r} \alpha_i = 1, \quad \alpha_i \geq 0. \qquad (5)$$

The control objective is to stabilize that polytopic LPV system and to make $\mathcal{L}_2$ gain from disturbances ω to generalized outputs z less than $\gamma(> 0)$ for all possible α. For that purpose, the following gain-scheduled state feedback control is chosen.

$$u = F(\alpha)x, \qquad (6)$$
$$F(\alpha) = \sum_{i=1}^{r} \alpha_i F_i, \qquad (7)$$

where F_i are time-invariant matrices. Then the feedback system is described by

$$\frac{d}{dt}x = A_{cl}(\alpha)x + B_1(\alpha)\omega, \qquad (8)$$
$$z = C_{cl}(\alpha)x + D_{22}(\alpha)\omega, \qquad (9)$$

where

$$\begin{bmatrix} A_{cl}(\alpha) & B_1(\alpha) \\ C_{cl}(\alpha) & D_{22}(\alpha) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{cli} & B_{1i} \\ C_{cli} & D_{22i} \end{bmatrix}$$
$$= \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{pi} + B_p F_i & B_{1i} \\ C_{1i} + D_{21} F_i & D_{22i} \end{bmatrix}. \qquad (10)$$

The controlled system is stabilized and the $\mathcal{L}_2$ gain from ω to z is made less than $\gamma(> 0)$, if there exists a positive definite matrix P satisfying the following LMI for all possible α (5) (Bounded Real Lemma).

$$\begin{bmatrix} A_{cl}(\alpha)^T P + P A_{cl}(\alpha) & P B_1(\alpha) & C_{cl}(\alpha)^T \\ B_1(\alpha)^T P & -\gamma I & D_{22}(\alpha)^T \\ C_{cl}(\alpha) & D_{22}(\alpha) & -\gamma I \end{bmatrix}$$
$$= \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{cli}^T P + P A_{cli} & P B_{1i} & C_{cli}^T \\ B_{1i}^T P & -\gamma I & D_{22i}^T \\ C_{cli} & D_{22i} & -\gamma I \end{bmatrix}$$
$$< 0. \qquad (11)$$

The condition (11) is equivalent to the existence of the positive definite P satisfying the next systems of LMIs (12).

$$\begin{bmatrix} A_{cli}^T P + P A_{cli} & P B_{1i} & C_{cli}^T \\ B_{1i}^T P & -\gamma I & D_{22i}^T \\ C_{cli} & D_{22i} & -\gamma I \end{bmatrix} < 0, \qquad (12)$$
$$(1 \leq i \leq r).$$

2.2 Gain-Scheduled Control via Dynamic Compensator.

Consider the following polytopic LPV system

$$\frac{d}{dt}x = A_p(\alpha)x + B_p u + B_1(\alpha)\omega, \qquad (13)$$
$$y = C_p x + D_{12}\omega, \qquad (14)$$
$$z = C_1(\alpha)x + D_{21}u + D_{22}(\alpha)\omega, \qquad (15)$$

where $A_p(\alpha)$, $B_1(\alpha)$, $C_1(\alpha)$, $D_{22}(\alpha)$ are defined by

$$\begin{bmatrix} A_p(\alpha) & B_1(\alpha) \\ C_1(\alpha) & D_{22}(\alpha) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{pi} & B_{1i} \\ C_{1i} & D_{22i} \end{bmatrix}, \qquad (16)$$

with time-invariant matrices A_{pi}, B_{1i}, C_{1i}, D_{22i}, and the time-varying parameter α satisfies the same condition (5).

The control objective is to stabilize the process, and make $\mathcal{L}_2$ gain from disturbance ω to generalized output z less than $\gamma(> 0)$, for all possible α. For that purpose, the following gain-scheduled dynamic compensator is introduced.

$$\frac{d}{dt}x_K = A_k(\alpha)x_K + B_K(\alpha)y, \qquad (17)$$
$$u = C_k(\alpha)x + D_K(\alpha)y, \qquad (18)$$
$$\begin{bmatrix} A_K(\alpha) & B_K(\alpha) \\ C_K(\alpha) & D_K(\alpha) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{Ki} & B_{Ki} \\ C_{Ki} & D_{Ki} \end{bmatrix}, \qquad (19)$$

where A_{Ki}, B_{Ki}, C_{Ki}, D_{Ki} are time-invariant matrices. Then, the feedback system is written by

$$\frac{d}{dt}x_{cl} = A_{cl}(\alpha)x_{cl} + B_{cl}(\alpha)\omega, \qquad (20)$$
$$z = C_{cl}(\alpha)x_{cl} + D_{cl}(\alpha)\omega, \qquad (21)$$
$$x_{cl} = \begin{bmatrix} x \\ x_K \end{bmatrix}, \qquad (22)$$

where

$$\begin{bmatrix} A_{cl}(\alpha) & B_{cl}(\alpha) \\ C_{cl}(\alpha) & D_{cl}(\alpha) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{cli} & B_{cli} \\ C_{cli} & D_{cli} \end{bmatrix}, \qquad (23)$$
$$A_{cli} = \begin{bmatrix} A_{pi} + B_p D_{Ki} C_p & B_p C_{Ki} \\ B_{Ki} C_p & A_{Ki} \end{bmatrix}, \qquad (24)$$
$$B_{cli} = \begin{bmatrix} B_p D_{Ki} D_{12} + B_{1i} \\ B_{Ki} D_{12} \end{bmatrix}, \qquad (25)$$
$$C_{cli} = \begin{bmatrix} C_{1i} + D_{21} D_{Ki} C_p & D_{21} C_{Ki} \end{bmatrix}, \qquad (26)$$
$$D_{cli} = D_{21} D_{Ki} D_{12} + D_{22i}. \qquad (27)$$

The controlled process is stabilized, and $\mathcal{L}_2$ gain from ω to z is made less than $\gamma(>0)$, if there exists a positive definite P satisfying the following LMI for all possible α (5) (Bounded Real Lemma).

$$\begin{bmatrix} A_{cl}(\alpha)^T P + PA_{cl}(\alpha) & PB_{cl}(\alpha) & C_{cl}(\alpha)^T \\ B_{cl}(\alpha)^T P & -\gamma I & D_{cl}(\alpha)^T \\ C_{cl}(\alpha) & D_{cl}(\alpha) & -\gamma I \end{bmatrix}$$
$$= \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{cli}^T P + PA_{cli} & PB_{cli} & C_{cli}^T \\ B_{cli}^T P & -\gamma I & D_{cli}^T \\ C_{cli} & D_{cli} & -\gamma I \end{bmatrix} < 0. \tag{28}$$

The condition (28) is equivalent to the existence of the positive definite P satisfying the next systems of LMIs (29).

$$\begin{bmatrix} A_{cli}^T P + PA_{cli} & PB_{cli} & C_{cli}^T \\ B_{cli}^T P & -\gamma I & D_{cli}^T \\ C_{cli} & D_{cli} & -\gamma I \end{bmatrix} < 0, \quad (1 \leq i \leq r). \tag{29}$$

3. ADAPTIVE GAIN-SCHEDULED H_∞ CONTROL OF LPV SYSTEMS

The adaptive gain-scheduled H_∞ controllers for polytopic LPV systems are constructed for the case where the parameter α is not available for measurement. First, it is assumed that the parameter α is time-invariant, and that $\omega \in \mathcal{L}^\infty \cap \mathcal{L}^2$, $\dot{\omega} \in \mathcal{L}^\infty$, and the basic structure of the proposed adaptive control systems is shown.

3.1 Adaptive Gain-Scheduled Control via State Feedback.

Consider the polytopic LPV system (1), (2), where the system matrices A_{pi}, B_{1i}, C_{1i}, D_{22i} $(1 \leq r)$, B_p, D_{21} are known, but the parameter α is not available for measurement. The current estimate of α is defined by $\hat{\alpha}$, and that estimate is fed to the gain-scheduled state feedback controller as follows:

$$u = F(\hat{\alpha})x = \sum_{i=1}^{r} \hat{\alpha}_i F_i x. \tag{30}$$

Then, the controlled system is described by

$$\dot{x} = \sum_{i=1}^{r} \alpha_i A_{cli} x + \sum_{i=1}^{r} \alpha_i B_{1i}\omega + B_p \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i) F_i x, \tag{31}$$

$$z = \sum_{i=1}^{r} \alpha_i C_{cli} x + \sum_{i=1}^{r} \alpha_i D_{22i}\omega + D_{21} \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i) F_i x. \tag{32}$$

It is assumed that the LMI (11), (12) is solvable for the LPV system (1), (2), and there exists a positive definite matrix P. The LMI (11) is divided into the form

$$\sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{cli}^T P + PA_{cli} & PB_{1i} & C_{cli}^T \\ B_{1i}^T P & -\gamma I & D_{22i}^T \\ C_{cli} & D_{22i} & -\gamma I \end{bmatrix}$$
$$= \sum_{i=1}^{r} \alpha_i \left\{ \begin{bmatrix} PA_{cli} & PB_{1i} & 0 \\ 0 & -\frac{\gamma}{2} I & 0 \\ C_{cli} & D_{22i} & -\frac{\gamma}{2} I \end{bmatrix} + \begin{bmatrix} PA_{cli} & PB_{1i} & 0 \\ 0 & -\frac{\gamma}{2} I & 0 \\ C_{cli} & D_{22i} & -\frac{\gamma}{2} I \end{bmatrix}^T \right\} < 0. \tag{33}$$

Then, the following relation holds for any vector x, ω, d with proper dimensions and δ_1, δ_2, $\delta_3 > 0$.

$$\begin{aligned} 0 &\geq -\delta_1 \|x\|^2 - \delta_2 \|\omega\|^2 - \delta_3 \|d\|^2 \\ &\geq \sum_{i=1}^{r} \alpha_i \begin{bmatrix} x^T \, \omega^T \, d^T \end{bmatrix} \begin{bmatrix} PA_{cli} & PB_{1i} & 0 \\ 0 & -\frac{\gamma}{2} I & 0 \\ C_{cli} & D_{22i} & -\frac{\gamma}{2} I \end{bmatrix} \begin{bmatrix} x \\ \omega \\ d \end{bmatrix} \\ &= x^T P \left(\sum_{i=1}^{r} \alpha_i A_{cli} x + \sum_{i=1}^{r} \alpha_i B_{1i} \omega \right) \\ &\quad + d^T \left(\sum_{i=1}^{r} \alpha_i C_{cli} x + \sum_{i=1}^{r} \alpha_i D_{22i} \omega - \frac{\gamma}{2} d \right) \\ &\quad - \frac{\gamma}{2} \|\omega\|^2. \end{aligned} \tag{34}$$

By considering (31), (32), and by setting d as

$$d \equiv \frac{z}{\gamma}, \tag{35}$$

the inequality (34) is rewritten into

$$\begin{aligned} 0 &\geq -\delta_1 \|x\|^2 - \delta_2 \|\omega\|^2 - \delta_3 \frac{\|z\|^2}{\gamma^2} \\ &\geq x^T P \left\{ \dot{x} - B_p \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i) F_i x \right\} \\ &\quad + \left(\frac{z}{\gamma} \right)^T \left\{ z - D_{21} \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i) F_i x - \frac{1}{2} z \right\} \\ &\quad - \frac{\gamma}{2} \|\omega\|^2 \\ &= \frac{1}{2} \frac{d}{dt} (x^T P x) + \frac{1}{2\gamma} \|z\|^2 - \frac{\gamma}{2} \|\omega\|^2 \\ &\quad - \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i) \left\{ x^T P B_p F_i x + \left(\frac{z}{\gamma} \right)^T D_{21} F_i x \right\}. \end{aligned} \tag{36}$$

Here, define the positive function W

$$W = \frac{1}{2} x^T P x + \frac{1}{2} \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i)^2 / g_i, \quad (g_i > 0), \tag{37}$$

and take the time derivative of it along the trajectories of x and $\hat{\alpha}_i$.

$$\begin{aligned} \dot{W} &= \frac{1}{2} \frac{d}{dt} (x^T P x) + \sum_{i=1}^{r} (\hat{\alpha}_i - \alpha_i) \dot{\hat{\alpha}}_i / g_i \\ &\leq -\frac{1}{2\gamma} \|z\|^2 + \frac{\gamma}{2} \|\omega\|^2 \end{aligned}$$

$$+\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)\left\{x^TPB_pF_ix+\left(\frac{z}{\gamma}\right)^TD_{21}F_ix\right\}$$
$$+\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)\dot{\hat{\alpha}}_i/g_i. \quad (38)$$

From that, the adaptive laws of $\hat{\alpha}_i$ are determined as follows ($1\le i\le r$):

$$\dot{\hat{\alpha}}_i=-g_i\left\{x^TPB_pF_ix+\left(\frac{z}{\gamma}\right)^TD_{21}F_ix\right\}. \quad (39)$$

Then, $\dot{W}$ is evaluated such that

$$\dot{W}\le-\frac{1}{2\gamma}||z||^2+\frac{\gamma}{2}||\omega||^2, \quad (40)$$

and the next relation is obtained.

$$\int_0^t||z(\tau)||^2d\tau$$
$$+\gamma\sum_{i=1}^{r}\{\hat{\alpha}_i(t)-\alpha_i\}^2/g_i+\gamma x(t)^TPx(t)$$
$$\le\gamma^2\int_0^t||\omega(\tau)||^2d\tau+\gamma\sum_{i=1}^{r}\{\hat{\alpha}_i(0)-\alpha_i\}^2/g_i$$
$$+\gamma x(0)^TPx(0). \quad (41)$$

Hence, it is shown that x, z, $\hat{\alpha}_i\in\mathcal{L}^\infty$; x, $z\to0$ for $\omega\in\mathcal{L}^\infty\cap\mathcal{L}^2$, $\dot{\omega}\in\mathcal{L}^\infty$, and that $\mathcal{L}_2$ gain from ω to z is prescribed by γ, where initial errors of tuning parameters $\sum_{i=1}^{r}\{\hat{\alpha}_i(0)-\alpha_i\}^2$ are also included (*adaptive H_∞ control performance*).

Theorem 1 : *It is assumed that the LMI (11), (12) is solvable for the LPV system (1), (2). Then, the adaptive gain-scheduled control schemes (30), (39) stabilize the process (x, z, $\hat{\alpha}_i\in\mathcal{L}^\infty$; x, $z\to0$), and attain the adaptive H_∞ control performance (41), for time-invariant α and for $\omega\in\mathcal{L}^2\cap\mathcal{L}^\infty$, $\dot{\omega}\in\mathcal{L}^\infty$.*

3.2 Adaptive Gain-Scheduled Control via Dynamic Compensator.

Consider the polytopic LPV system (13), (14), (15), where the system matrices A_{pi}, B_{1i}, C_{1i}, D_{22i} ($1\le r$), B_p, C_p, D_{12}, D_{21} are known, but the parameter α is unknown. The current estimate $\hat{\alpha}$ is defined similarly, and $\hat{\alpha}$ is fed to the gain-scheduled dynamic compensators as follows:

$$\frac{d}{dt}x_K=A_k(\hat{\alpha})x_K+B_K(\hat{\alpha})y, \quad (42)$$
$$u=C_k(\hat{\alpha})x+D_K(\hat{\alpha})y, \quad (43)$$
$$\begin{bmatrix}A_K(\hat{\alpha}) & B_K(\hat{\alpha})\\ C_K(\hat{\alpha}) & D_K(\hat{\alpha})\end{bmatrix}=\sum_{i=1}^{r}\hat{\alpha}_i\begin{bmatrix}A_{Ki} & B_{Ki}\\ C_{Ki} & D_{Ki}\end{bmatrix}. \quad (44)$$

Then, overall controlled process is written by

$$\frac{d}{dt}x_{cl} = A_{cl}(\alpha)x_{cl}+B_{cl}(\alpha)\omega$$
$$+\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)\begin{bmatrix}B_p(C_{Ki}x_K+D_{Ki}y)\\ A_{Ki}x_K+B_{Ki}y\end{bmatrix}, \quad (45)$$
$$z = C_{cl}(\alpha)x_{cl}+D_{cl}(\alpha)\omega$$
$$+\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)D_{21}(C_{Ki}x_K+D_{Ki}y). \quad (46)$$

It is assumed that the LMI (28), (29) is solvable for the LPV system (13), (14), (15) and there exists a positive definite matrix P. The LMI (28) is divided into the form

$$\sum_{i=1}^{r}\alpha_i\begin{bmatrix}A_{cli}^TP+PA_{cli} & PB_{cli} & C_{cli}^T\\ B_{cli}^TP & -\gamma I & D_{cli}^T\\ C_{cli} & D_{cli} & -\gamma I\end{bmatrix}$$
$$=\sum_{i=1}^{r}\alpha_i\left\{\begin{bmatrix}PA_{cli} & PB_{cli} & 0\\ 0 & -\frac{\gamma}{2}I & 0\\ C_{cli} & D_{cli} & -\frac{\gamma}{2}I\end{bmatrix}\right.$$
$$\left.+\begin{bmatrix}PA_{cli} & PB_{cli} & 0\\ 0 & -\frac{\gamma}{2}I & 0\\ C_{cli} & D_{cli} & -\frac{\gamma}{2}I\end{bmatrix}^T\right\}<0. \quad (47)$$

Then, the following relation holds for any vector x_{cl}, ω, d with proper dimensions and δ_1, δ_2, $\delta_3>0$

$$0\ge-\delta_1||x_{cl}||^2-\delta_2||\omega||^2-\delta_3||d||^2$$
$$\ge\sum_{i=1}^{r}\alpha_i\left[x_{cl}^T\,\omega^T\,d^T\right]\begin{bmatrix}PA_{cli} & PB_{cli} & 0\\ 0 & -\frac{\gamma}{2}I & 0\\ C_{cli} & D_{cli} & -\frac{\gamma}{2}I\end{bmatrix}\begin{bmatrix}x_{cl}\\ \omega\\ d\end{bmatrix}$$
$$=x_{cl}^TP\left(\sum_{i=1}^{r}\alpha_iA_{cli}x_{cl}+\sum_{i=1}^{r}\alpha_iB_{cli}\omega\right)$$
$$+d^T\left(\sum_{i=1}^{r}\alpha_iC_{cli}x_{cl}+\sum_{i=1}^{r}\alpha_iD_{cli}\omega-\frac{\gamma}{2}d\right)$$
$$-\frac{\gamma}{2}||\omega||^2. \quad (48)$$

By considering (45), (46), and by setting d as

$$d\equiv\frac{z}{\gamma}, \quad (49)$$

the inequality (48) is reduced into

$$0\ge-\delta_1||x_{cl}||^2-\delta_2||\omega||^2-\delta_3\frac{||z||^2}{\gamma^2}$$
$$\ge x_{cl}^TP\left\{\dot{x}_{cl}-\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)\begin{bmatrix}B_p(C_{Ki}x_K+D_{Ki}y)\\ A_{Ki}x_K+B_{Ki}y\end{bmatrix}\right\}$$
$$+\left(\frac{z}{\gamma}\right)^T\left\{z-\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)D_{21}(C_{Ki}x_K+D_{Ki}y)-\frac{1}{2}z\right\}$$
$$-\frac{\gamma}{2}||\omega||^2$$
$$=\frac{1}{2}\frac{d}{dt}(x_{cl}^TPx_{cl})+\frac{1}{2\gamma}||z||^2-\frac{\gamma}{2}||\omega||^2$$
$$-\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)\left\{x_{cl}^TP\begin{bmatrix}B_p(C_{Ki}x_K+D_{Ki}y)\\ A_{Ki}x_K+B_{Ki}y\end{bmatrix}\right.$$
$$\left.+\left(\frac{z}{\gamma}\right)^TD_{21}(C_{Ki}x_K+D_{Ki}y)\right\}. \quad (50)$$

Here, define W by

$$W = \frac{1}{2}x_{cl}^TPx_{cl}+\frac{1}{2}\sum_{i=1}^{r}(\hat{\alpha}_i-\alpha_i)^2/g_i, \quad (51)$$

$(g_i > 0)$,

and take the time derivative of it along the trajectories of x_{cl} and $\hat{\alpha}_i$.

$$\begin{aligned}\dot{W} &= \frac{1}{2}\frac{d}{dt}(x_{cl}^T P x_{cl}) + \sum_{i=1}^{r}(\hat{\alpha}_i - \alpha_i)\dot{\hat{\alpha}}_i/g_i \\ &\le -\frac{1}{2\gamma}\|z\|^2 + \frac{\gamma}{2}\|\omega\|^2 \\ &\quad + \sum_{i=1}^{r}(\hat{\alpha}_i - \alpha_i)\left\{x_{cl}^T P \begin{bmatrix} B_p(C_{Ki}x_K + D_{Ki}y) \\ A_{Ki}x_K + B_{Ki}y \end{bmatrix}\right. \\ &\quad \left. + \left(\frac{z}{\gamma}\right)^T D_{21}(C_{Ki}x_K + D_{Ki}y)\right\} \\ &\quad + \sum_{i=1}^{r}(\hat{\alpha}_i - \alpha_i)\dot{\hat{\alpha}}_i/g_i. \end{aligned} \tag{52}$$

From that, the adaptive law of $\hat{\alpha}_i$ are chosen such that

$$\begin{aligned}\dot{\hat{\alpha}}_i &= -g_i\left\{x_{cl}^T P \begin{bmatrix} B_p(C_{Ki}x_K + D_{Ki}y) \\ A_{Ki}x_K + B_{Ki}y \end{bmatrix}\right. \\ &\quad \left. + \left(\frac{z}{\gamma}\right)^T D_{21}(C_{Ki}x_K + D_{Ki}y)\right\}. \end{aligned} \tag{53}$$

Then, $\dot{W}$ is evaluated as follows:

$$\dot{W} \le -\frac{1}{2\gamma}\|z\|^2 + \frac{\gamma}{2}\|\omega\|^2, \tag{54}$$

and finally, the next relation is obtained.

$$\begin{aligned}&\int_0^t \|z(\tau)\|^2 d\tau + \gamma\sum_{i=1}^{r}\{\hat{\alpha}_i(t) - \alpha_i\}^2/g_i \\ &\quad + \gamma x_{cl}(t)^T P x_{cl}(t) \\ &\le \gamma^2\int_0^t \|\omega(\tau)\|^2 d\tau + \gamma\sum_{i=1}^{r}\{\hat{\alpha}_i(0) - \alpha_i\}^2/g_i \\ &\quad + \gamma x_{cl}(0)^T P x_{cl}(0). \end{aligned} \tag{55}$$

Hence, it follows that x_{cl}, z, $\hat{\alpha}_i \in \mathcal{L}^\infty$, x_{cl}, $z \to 0$ for $\omega \in \mathcal{L}^\infty \cap \mathcal{L}^2$, $\dot{\omega} \in \mathcal{L}^\infty$, and that $\mathcal{L}_2$ gain from ω to z is prescribed by γ, where initial errors of tuning parameters $\sum_{i=1}^{r}\{\hat{\alpha}_i(0) - \alpha_i\}^2$ are also included (*adaptive H_∞ control performance*).

Theorem 2 : *It is assumed that the LMI (28), (29) is solvable for the LPV system (13), (14), (15). Then, the adaptive gain-scheduled control schemes (42), (43), (53) stabilize the process (x_{cl}, z, $\hat{\alpha}_i \in \mathcal{L}^\infty$; x_{cl}, $z \to 0$), and attain adaptive H_∞ control performance (55), for time-invariant α and for $\omega \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, $\dot{\omega} \in \mathcal{L}^\infty$.*

4. ROBUST ADAPTIVE GAIN-SCHEDULED H_∞ CONTROL OF LPV SYSTEMS

In the present section, it is assumed that the parameters α is unknown and time-varying (α, $\dot{\alpha} \in \mathcal{L}^\infty$), and that $\omega \in \mathcal{L}^\infty$. The robust adaptive schemes (Ioannou and Sun, 1996) are introduced.

4.1 Robust Adaptive Gain-Scheduled Control via State Feedback.

The same gain-scheduled state feedback control (30) is chosen. Define W by (37), and take the time derivative of it along the trajectories of x, $\hat{\alpha}_i$ and α_i. Contrary to the previous case, $\dot{\alpha} \ne 0$ should be taken into consideration.

$$\begin{aligned}\dot{W} &= \frac{1}{2}\frac{d}{dt}(x^T P x) + \sum_{i=1}^{r}(\hat{\alpha}_i - \alpha_i)\{\dot{\hat{\alpha}}_i - \dot{\alpha}_i\}/g_i \\ &\le -\frac{1}{2\gamma}\|z\|^2 + \frac{\gamma}{2}\|\omega\|^2 \\ &\quad + \sum_{i=1}^{r}(\hat{\alpha}_i - \alpha_i)\left\{x^T P B_p F_i x + \left(\frac{z}{\gamma}\right)^T D_{21}F_i x\right\} \\ &\quad + \sum_{i=1}^{r}(\hat{\alpha}_i - \alpha_i)\{\dot{\hat{\alpha}}_i - \dot{\alpha}_i\}/g_i \\ &\quad -\delta_1\|x\|^2 - \delta_2\|\omega\|^2 - \delta_3\|d\|^2. \end{aligned} \tag{56}$$

The projection-type adaptive laws (Ioannou and Sun, 1996) are chosen for the tuning of $\hat{\alpha} = [\hat{\alpha}_1, \cdots, \hat{\alpha}_r]^T$.

$$\begin{aligned}&\text{if } \xi(\hat{\alpha}) < 0, \\ &\quad \text{or } \xi(\hat{\alpha}) = 0 \ \& \ \nabla\xi(\hat{\alpha})^T G v \ge 0, \\ &\qquad \dot{\hat{\alpha}}(t) = -Gv, \\ &\text{if } \xi(\hat{\alpha}) = 0 \ \& \ \nabla\xi(\hat{\alpha})^T G v < 0, \\ &\qquad \dot{\hat{\alpha}}(t) = -Gv + G\frac{\nabla\xi(\hat{\alpha})\nabla\xi(\hat{\alpha})^T}{\nabla\xi(\hat{\alpha})^T G \nabla\xi(\hat{\alpha})}Gv, \\ &\qquad (\hat{\alpha}(0) \in \mathcal{S}), \end{aligned} \tag{57}$$

where

$$v = [v_1\, v_2\, \cdots\, v_r]^T,$$

$$v_i = x^T P B_p F_i x + \left(\frac{z}{\gamma}\right)^T D_{21}F_i x, \quad (1 \le i \le r), \tag{58}$$

$$G = \text{diag}\,(g_1,\, g_2,\, \cdots,\, g_r). \tag{59}$$

$\xi(\alpha)$ is a differentiable function of α satisfying

$$\xi(\alpha) \le 0, \quad \alpha \in \mathcal{S}\,(\in \mathbf{R}^r), \tag{60}$$

where $\mathcal{S}(\in \mathbf{R}^r)$ is a bounded region and the constraints of α (5) is included in $\mathcal{S}$. From the properties of projection-type adaptive laws, it follows that $\hat{\alpha} \in \mathcal{S}$ ($\alpha_i \in \mathcal{L}^\infty$) and that

$$(\hat{\alpha} - \alpha)^T G^{-1}\dot{\hat{\alpha}} \le -(\hat{\alpha} - \alpha)^T v. \tag{61}$$

Then, $\dot{W}$ is evaluated by

$$\begin{aligned}\dot{W} &\le -\frac{1}{2\gamma}\|z\|^2 + \frac{\gamma}{2}\|\omega\|^2 \\ &\quad -\delta_1\|x\|^2 - \delta_2\|\omega\|^2 - \delta_3\|d\|^2 \\ &\quad + \sum_{i=1}^{r}(\alpha_i - \hat{\alpha}_i)\dot{\alpha}_i/g_i \end{aligned} \tag{62}$$

$$\leq -\frac{1}{2\gamma}||z||^2 + \frac{\gamma}{2}||\omega||^2 + \sum_{i=1}^{r}(\alpha_i - \hat{\alpha}_i)\dot{\alpha}_i/g_i. \quad (63)$$

Since α_i, $\dot{\alpha}_i \in \mathcal{L}^\infty$ and $\hat{\alpha} \in \mathcal{L}^\infty$, the next relation is obtained from (62).

$$\dot{W} \leq -\delta W + D, \quad (0 < \delta,\, D < \infty). \quad (64)$$

Hence, it is shown that $W \in \mathcal{L}^\infty$, and that x, $z \in \mathcal{L}^\infty$. Also, the next inequality is derived from (63).

$$\int_0^t ||z(\tau)||^2 d\tau + \gamma \sum_{i=1}^{r}\{\hat{\alpha}_i(t) - \alpha_i\}^2/g_i + \gamma x(t)^T P x(t) \leq \gamma^2 \int_0^t ||\omega(\tau)||^2 d\tau + \gamma \sum_{i=1}^{r}\{\hat{\alpha}_i(0) - \alpha_i\}^2/g_i + \gamma x(0)^T P x(0) + 2\gamma \sum_{i=1}^{r}\int_0^t (\alpha_i - \hat{\alpha}_i)\dot{\alpha}_i/g_i d\tau. \quad (65)$$

Then, the $\mathcal{L}_2$ gain from ω to z is prescribed by γ, where initial error of tuning parameters $\sum_{i=1}^{r}\{\hat{\alpha}_i(0)-\alpha_i\}^2$ and time-varying elements of $\sum_{i=1}^{r}(\alpha_i-\hat{\alpha}_i)\dot{\alpha}_i/g_i$ are also included (*adaptive H_∞ control performance*).

Theorem 3 : *It is assumed that the LMI (11), (12) is solvable for the LPV system (1), (2). Then, the adaptive gain-scheduled control schemes (30), (57) stabilize the process (x, z, $\hat{\alpha} \in \mathcal{L}^\infty$), and attain adaptive H_∞ control performance (65) for time varying α (α, $\dot{\alpha} \in \mathcal{L}^\infty$) and for $\omega \in \mathcal{L}^\infty$.*

4.2 Robust Adaptive Gain-Scheduled Control via Dynamic Compensator.

The same gain-scheduled dynamic compensators (42), (43) are adopted. The projection-type adaptive laws (Ioannou and Sun, 1996) are chosen similarly to (57), but v (58) is defined by

$$v_i = x_{cl}^T P \begin{bmatrix} B_p(C_{Ki}x_K + D_{Ki}y) \\ A_{Ki}x_K + B_{Ki}y \end{bmatrix} + \left(\frac{z}{\gamma}\right)^T D_{21}(C_{Ki}x_K + D_{Ki}y), \quad (66)$$
$$(1 \leq i \leq r).$$

Then, in the same way as *4.1*, it is shown that x_{cl}, $z \in \mathcal{L}^\infty$, and that the next inequality holds.

$$\int_0^t ||z(\tau)||^2 d\tau + \gamma \sum_{i=1}^{r}\{\hat{\alpha}_i(t) - \alpha_i\}^2/g_i + \gamma x_{cl}(t)^T P x_{cl}(t) \leq \gamma^2 \int_0^t ||\omega(\tau)||^2 d\tau + \gamma \sum_{i=1}^{r}\{\hat{\alpha}_i(0) - \alpha_i\}^2/g_i + \gamma x_{cl}(0)^T P x_{cl}(0) + 2\gamma \sum_{i=1}^{r}\int_0^t (\alpha_i - \hat{\alpha}_i)\dot{\alpha}_i/g_i d\tau. \quad (67)$$

Theorem 4 : *It is assumed that the LMI (28), (29) is solvable for the LPV system (13), (14), (15). The adaptive gain-scheduled control schemes (42), (43), (57), (58), (66) stabilize the process (x_{cl}, z, $\hat{\alpha} \in \mathcal{L}^\infty$), and attain adaptive H_∞ control performance (67) for time varying α (α, $\dot{\alpha} \in \mathcal{L}^\infty$) and for $\omega \in \mathcal{L}^\infty$.*

Remark : The projection-type adaptive laws are also applied to **Theorem 1** and **Theorem 2**.

5. CONCLUDING REMARKS

A class of adaptive gain-scheduled H_∞ control scheme of polytopic LPV systems is presented in this manuscript. The current estimates of parameters are fed to the gain-scheduled controllers, and the stability and certain H_∞ control performance are attained for time-invariant and time-varying parameters. One drawback is that z and x are needed even for the construction via dynamic compensators, which is to be solved in the future study. Also, the proposed design methods are derived from the parameter-independent Lyapunov matrices P (Apkarian, *et al.*, 1995), and thus potentially conservative approaches. The introduction of parameter-dependent Lyapunov matrices $P(\alpha)$ (Gahinet, *et al.*, 1996; Watanabe, *et al.*, 1996) into the adaptive control schemes is also left in the future.

REFERENCES

Apkarian, P., P. Gahinet, and G. Beckers (1995). Self-scheduled H_∞ control of linear parameter-varying systems : a design example. *Automatica*, **31**, 1251-1261.

Apkarian, P. and P. Gahinet (1995). A convex characterization of gain-scheduled H_∞ controllers. *IEEE Trans. Autom. Control*, **AC-40**, 853-864.

Becker, G. and A. Packard. (1994). Robust performance of linear parameterically-varying systems using parametrically-dependent linear feedback. *Sys. Contr. Lett.*, **23**, 436-442.

Gahinet, P., A. Nemirovski, A. J. Laub and M. Chilali. (1995). *LMI Control Toolbox User's Guide*,Math. Work Inc.

Gahinet, P., P. Apkarian and M. Chilali. (1996). Affine parameter-dependent Lyapunov functions and real parametric uncertainty. *IEEE Trans. Autom. Control*, **AC-41**, 436-442.

Ioannou, P.A. and J. Sun (1996). *Robust Adaptive Control.* Prentice-Hall, New Jersey.

Packard, A. (1994). Gain scheduling via linear fractional transformations. *Sys. Contr. Lett.*, **22**, 436-442.

Watanabe, R., K. Uchida and M. Fujita (1996). A new LMI approach to analysis of linear systems with scheduling parameter based on finite number of LMI conditions. *Proc. 35th Conf. on Decision and Control*, 1663-1665.