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INFINITE-IMPULSE AND FINITE-IMPULSE RESPONSE FILTERS FOR CONTINUOUS-TIME PARAMETER ESTIMATION

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Abstract: This paper examines two classes of algorithms that estimate a continuous time ARX type of models from discrete data: one is based on infinite impulse response (IIR) filters while the other is based on finite impulse response (FIR) filters. The IIR filters use continuous time state variable filters, and discretisation is performed on the filtered derivatives. In contrast, the FIR filters are in a discrete form with carefully chosen coefficients to approximate the derivatives of the continuous time variables. The strength and weakness of each approach are discussed and demonstrated by a set of simulation examples.

Keywords: continuous time systems, least squares estimation, parameter estimation.

1. INTRODUCTION

Continuous time system identification has been studied in the literature over a three decade period, see for instance: Young (1981), Gawthrop (1982),Gawthrop (1984*b*),Gawthrop (1984*a*), Unbehauen and Rao (1987), Gawthrop (1987), Gawthrop *et al.* (1989), Unbehauen and Rao (1990), Sinha and Rao (1991)Söderström *et al.* (1997), Gawthrop and Wang (2000) and Wang and Gawthrop (2001). In comparison with the discrete time counterpart, continuous time system identification raises several technical issues. The key point is related to implementation: at first sight, the least squares problem for direct parameter estimation involves differentiation of both input and output signals. There are a number of ways of avoiding the physically-unrealisable differentiation:

(1) discard the continuous-time approach and use a discrete-time formulation instead – this is a common approach but the apparent advantages are arguably illusory
(2) use the δ operator approach (Gawthrop, 1980; Middleton and Goodwin, 1990)
(3) reformulate the system equations into a realisable form whilst retaining the same parameterisation and a linear-in-the-parameters form – the state-variable filter approach (Young, 1981; Unbehauen and Rao, 1987) is one such reformulation
(4) approximate the derivatives using FIR (finite-impulse response) digital filters (Söderström *et al.*, 1997).

This paper compares and contrasts the latter two approaches.

The class of continuous-time systems considered is of the form:

$$y_0 = \frac{b(s)}{a(s)}u_0 + \frac{d(s)}{a(s)} \tag{1}$$

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where u_0 and y_0 are the system input and output respectively, s is the Laplace operator, $\frac{b(s)}{a(s)}$ is the system transfer function and $d(s)$ represents the effect of system initial conditions. In practical situations, the system input and output are subject to measurement noise. This is represented here by assuming the *measured* system input (u) and output (y) are given by:

$$u = u_0 + v_u \tag{2}$$

$$y = y_0 + v_y \tag{3}$$

where v_y and v_u are white noise processes with variance σ_y and σ_u respectively.

Combining equations (1)–(3) gives:

$$y = \frac{b(s)}{a(s)}u + \frac{d(s)}{a(s)} + \frac{a(s)v_y - b(s)v_u}{a(s)} \tag{4}$$

Using standard spectral factorisation results Equation (4) can be rewritten as:

$$y = \frac{b(s)}{a(s)}u + \frac{d(s)}{a(s)} + \frac{e(s)}{a(s)}v \tag{5}$$

where v is unit variance white noise and:

$$e(s)e(-s) = \sigma_y^2 a(s)a(-s) + \sigma_u^2 b(s)b(-s) \tag{6}$$

Note that if the system is stable and $\sigma_u = 0$ then $e(s) = \sigma_y a(s)$.

The model structure (5) is to be used in this study. Due to lack of space, the theoretical implications of (1) – (6) are not explored further in this paper.

2. PARAMETER ESTIMATION

This section describes the parameter estimation algorithms used in this study. The filter specific parts are described in Sections 2.1 and 2.2 and the parts of the algorithm that are common to the IIR and FIR approaches are then described.

2.1 *FIR filter approach*

The FIR approach to the estimation of ARX models is given by Söderström *et al.* (1997). The purpose of the FIR filters is to generate approximate *derivatives* of the signal in special forms to overcome bias.

Consider the sampled signal w_i where $1 \leq i \leq N$ and define the $N - m + 1 \times m$ matrix W

$$W = \begin{pmatrix} w & w_{m-1} & \dots & w_1 \\ w_{m+1} & w_m & \dots & w_2 \\ \dots & \dots\dots & & \\ w_N & w_{N-1} & \dots & w_{N-m+1} \end{pmatrix} \tag{7}$$

together with the $n \times m$ matrix F of filter coefficients. Then the $N - m + 1 \times n$ matrix X_w is given by

$$X_w = WF^T \tag{8}$$

By appropriate choice of F, the ijth row of X_w is an approximation to the $n+1-j$th derivative of w at sample instant i. In other words, the first column of X_w contains an approximation to $\frac{d^n w}{dw^n}$ and the last column contains an approximation to w.

The key result of Söderström *et al.* (1997) is that by careful choice of FIR coefficients, the bias in the resulting estimates can be reduced. A number of choices of coefficients are examined by Söderström *et al.* (1997) one choice that is recommended when $n = 2$ is the version "ZF1" of Table I of (Söderström *et al.*, 1997). This choice corresponds to the matrix:

$$F = \begin{pmatrix} -2 & 7 & -8 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2047 & 0.8860 & -1.3860 & 0.2953 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2.2 *IIR filter approach*

Suppose that an all-pole filter having denominator $C(s) = s^{n+1} + c_n s^n + c_{n-1}s^{n-1} + \ldots + c_0$ is selected for the identification procedure. By passing both input and output measurements $u(t)$ and $y(t)$ through this filter, we obtain filtered input and output signals. This operation when applied to the model of Equation 5 yields

$$\frac{a(s)}{c(s)}y = \frac{b(s)}{c(s)}u + \frac{d(s)}{c(s)} + \frac{e(s)}{c(s)}v \tag{9}$$

Here the filter operation is equivalent to the prefiltering operation in discrete time system save that in the continuous time case, the filter structure is restricted to all pole form.

To formulate a least squares problem for parameter estimation, the next step is to generate the derivatives of the filtered input and output responses. This step is simplified when a state variable filter implementation procedure is used (Gawthrop, 1987; Gawthrop, 1984*a*; Gawthrop, 1984*b*). Let $\bar{y}_n(t)$, $\bar{y}_{n-1}(t)$, $\ldots$, $\bar{y}_0(t)$ denote, respectively, $\frac{p^n}{c(p)}y(t)$, $\frac{p^{n-1}}{c(p)}y(t)$, $\ldots$, $\frac{1}{c(p)}y(t)$; let $\bar{u}_n(t), \bar{u}_{n-1}(t), \ldots, \bar{u}_0(t)$ denote, respectively, $\frac{p^n}{c(p)}u(t)$, $\frac{p^{n-1}}{c(p)}u(t)$, $\ldots$, $\frac{1}{c(p)}u(t)$. To obtain the derivatives of filtered output responses, we define a state variable vector

$$X^y(t) = [\bar{y}_n(t) \;\; \bar{y}_{n-1}(t) \;\; \ldots \bar{y}_0(t)]^T$$

Then, by choosing the state space model in a control canonical form, we have

$$\begin{bmatrix} \frac{d\bar{y}_n(t)}{dt} \\ \frac{d\bar{y}_{n-1}(t)}{dt} \\ \dots \\ \frac{d\bar{y}_0(t)}{dt} \end{bmatrix} = \begin{bmatrix} -c_n & -c_{n-1} & \dots & -c_0 \\ 1 & 0 & \dots & 0 \\ \dots & & & \\ 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{y}_n(t) \\ \bar{y}_{n-1}(t) \\ \dots \\ \bar{y}_0(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix} y(t)$$

$$= G \begin{bmatrix} \bar{y}_n(t) \\ \bar{y}_{n-1}(t) \\ \dots \\ \bar{y}_0(t) \end{bmatrix} + Ky(t) \tag{10}$$

The solution of the state space equation (10), assuming zero initial conditions, gives the derivatives of the filtered output responses. Similarly, define

$$X^u(t) = [\bar{u}_n(t)\ \ \bar{u}_{n-1}(t)\ldots\bar{u}_0(t)]^T$$

and

$$X^z(t) = [z_n(t)\ \ z_{n-1}(t)\ldots z_0(t)]^T$$

$X^z(t)$ will be used to capture the initial conditions of the state variables (when necessary). These state variables satisfy the following differential equations, respectively:

$$\begin{aligned}\dot{X}^u(t) &= GX^u(t) + Ku(t)\\ X^u(0) &= 0_n\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{X}^z(t) &= GX^z(t)\\ X^z(0) &= I_n\end{aligned} \tag{12}$$

where 0_n is a zero column vector of length n and I_n is a column vector of length n with the first element unity and the rest zero.

From Equation 9, it can be seen that the best choice for $c(s)$ is $e(s)$. However, $e(s)$ is dependent on the (unknown) system and is thus not known a-priori. One possibility is to adjust $c(s)$ in an iterative fashion – for simplicity this is not done here.

2.3 *The common algorithm*

In each case, the measured output y is filtered to give the *output* data vector

$$X_y = \left(y_{n_y}\ y_{n_y-1}\ \ldots\ y_0\right)^T \tag{13}$$

Where (as discussed in Sections 2.1 and 2.2) y_j is related to the jth derivative of the output measurement $s^j y$.

According to context, the measured input u may also be filtered to give the *input* data vector:

$$X_u = \left(u_{n_u}\ u_{n_u-1}\ \ldots\ u_0\right)^T \tag{14}$$

In the SVF case, the transient signal X_t is also generated

$$X_t = \left(z_{n_z-1}\ z_{n_z-1}\ \ldots\ u_0\right)^T \tag{15}$$

In this study, the ordinary least-squared method is used to extract parameters from the filtered data. As briefly discussed in Section 3.2, more sophisticated approaches would yield better results.

3. APPARENT STRENGTHS AND WEAKNESSES OF EACH APPROACH

The apparent strengths and weaknesses of the two approaches are informally discussed in the following sections. Section 4 provides a simulation study which verifies the expected behaviour suggested in this section.

3.1 *The FIR Approach*

As discussed by Söderström *et al.* (1997), the FIR approach assumes that the measured signals y and u are smooth enough to be differentiated the appropriate number of times. Thus, in principle, good performance is not expected if (in Equations 2 and 3) $\sigma_y > 0$ or $\sigma_u > 0$.

Although the FIR approach of Söderström *et al.* (1997) is very much designed for pure AR processes, it would be reasonable to suppose that it would perform well on the deterministic process defined by Equation 5 when both σ_y and σ_u are zero.

3.2 *The IIR Approach*

As discussed in Section 2.2, the effective noise is white only if $c(s) = e(s)$. If this is *not* true, then it would be expected that the parameter estimates would be biased. This bias depends on the signal to noise ratio at the system input and output.

In fact, this bias can be overcome using IV methods (Young and Jakeman, 1979; Jakeman and Young, 1979; Young and Jakeman, 1980); but this is not pursued further here.

4. SIMULATION STUDY

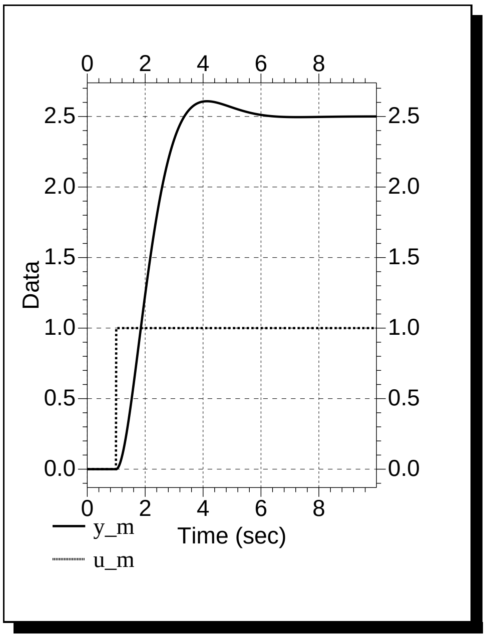

Fig. 1. *Step data:* $\sigma_y = \sigma_u = 0$

A simulation study was carried out to evaluate the performance of the two approaches in a number of cases. In each case:

- $a(s) = s^2 + 2s + 2$ (as also used by (Söderström *et al.*, 1997)).
- $b(s) = 5$

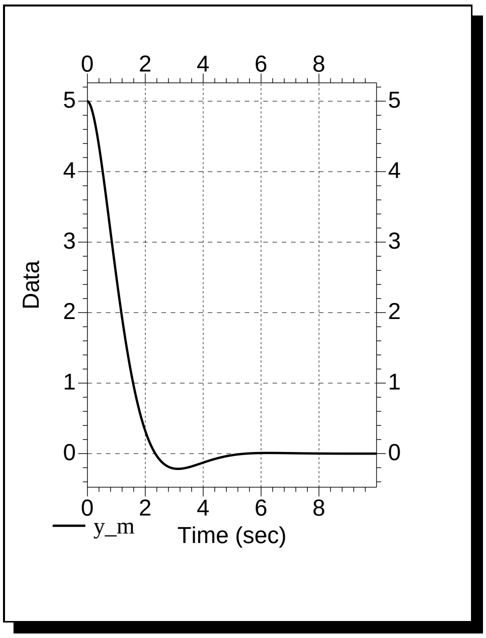

Fig. 2. *Transient data:* $\sigma_y = \sigma_u = 0$

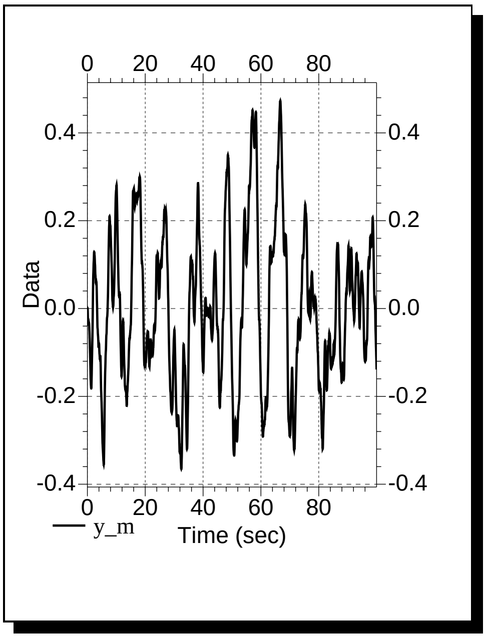

Fig. 3. *AR data:* $\sigma_y = \sigma_u = 0$

Data	Filter	a_1	a_2	b_1	b_2
step	IIR	2.00	2.00	0.00	5.00
step	FIR	1.92	1.96	0.00	4.91
free	IIR	2.00	2.00	–	–
free	FIR	1.96	2.00	–	–
AR	IIR	-0.01	0.41	–	–
AR	FIR	1.88	1.94	–	–

Table 1. $\sigma_y = \sigma_u = 0$

- Where random sequences were involved, the parameter estimates were (following (Söderström *et al.*, 1997)) averaged over 50 realisations.

Three types of data were used:

Step data where the input u was given by

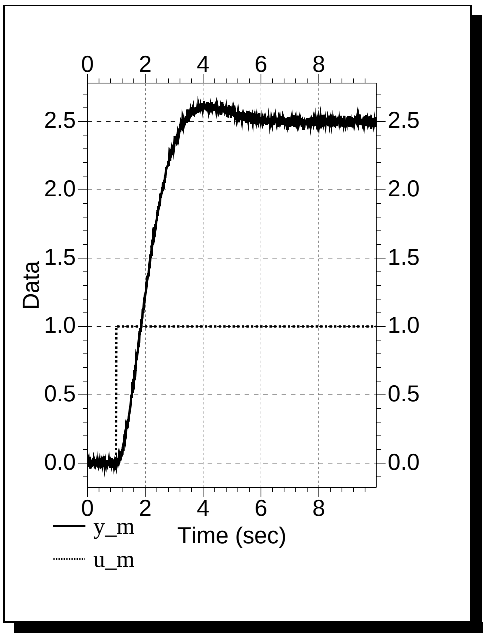

Fig. 4. *Step data:* $\sigma_y = 0.02, \sigma_u = 0$

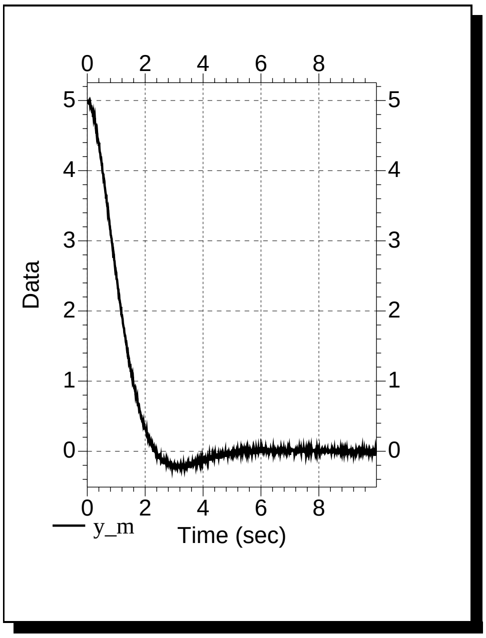

Fig. 5. *Transient data:* $\sigma_y = 0.02, \sigma_u = 0$

Data	Filter	a_1	a_2	b_1	b_2
step	IIR	2.00	2.00	0.00	5.00
step	FIR	-17.44	71.44	-2.57	154.47
free	IIR	2.00	2.00	–	–
free	FIR	-19.26	5.03	–	–
AR	IIR	-0.01	0.37	–	–
AR	FIR	-24.17	1094.96	–	–

Table 2. $\sigma_y = 0.02, \sigma_u = 0$

Data	Filter	a_1	a_2	b_1	b_2
step	IIR	1.99	2.00	0.00	4.99
step	FIR	1.37	1.44	0.00	3.56

Table 3. $\sigma_y = 0, \sigma_u = 0.1$

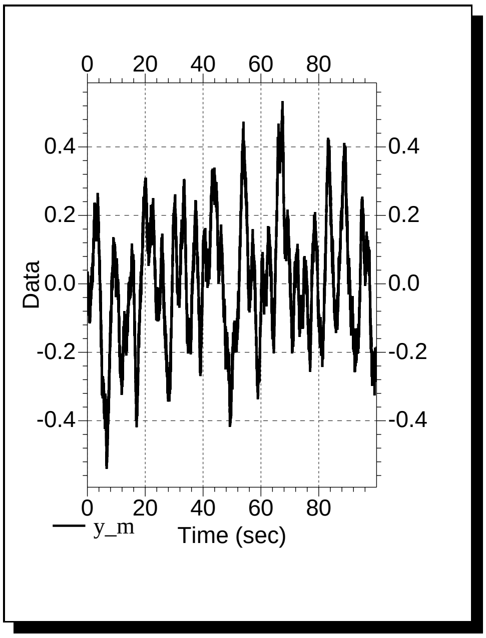

Fig. 6. *AR data:* $\sigma_y = 0.02, \sigma_u = 0$

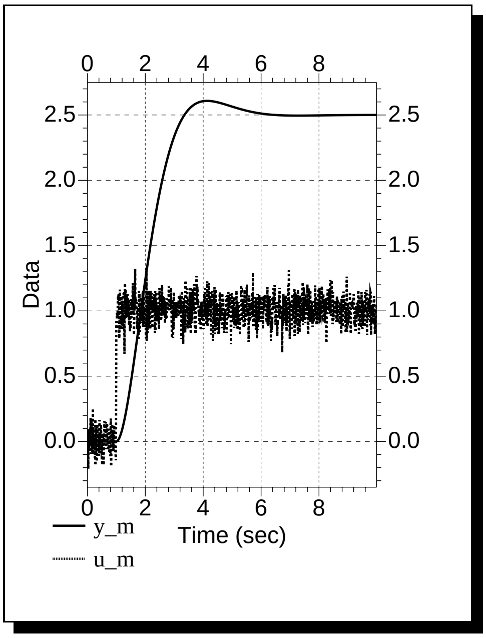

Fig. 7. *Step data:* $\sigma_y = 0, \sigma_u = 0.1$

$$u = \begin{cases} 0 & \text{if } t < 1 \\ 1 & \text{if } t \geq 1 \end{cases} \tag{16}$$

and the system initial condition was zero. The sample interval was $h = 0.01$ and the total time 10 – a total of 1000 data points.

Transient data where the input $u = 0$ and the system was started from rest at $y = 5$. The sample interval was $h = 0.01$ and the total time 10 – a total of 1000 data points.

AR data where the input u was unit white noise (and not measured) and the system initial condition was zero. The sample interval was $h = 0.01$ and the total time 100 – a total of 10000 data points.

Three noise conditions were used

$\sigma_y = \sigma_u = 0$ no measurement noise – see Table 1 and Figures 1–3.

$\sigma_y = 0.02$, $\sigma_u = 0$ output measurement noise – see Table 2 and Figures 4–6.

$\sigma_y = 0$, $\sigma_u = 0.1$ input measurement noise (only relevant for the step data) – see Table 3 and Figure 7.

The following conclusions may be drawn from this study:

(1) The performance of the IIR approach is poor on AR data but good on the rest of the data – with or without measurement noise.
(2) The performance of the FIR approach is (as indicated by (Söderström *et al.*, 1997)) excellent on the AR data, and good on the rest of the data, *when* $\sigma_y = 0$. However the performance is very poor *when* $\sigma_u > 0$.

In fact, further studies (not shown) show that the IIR approach gracefully degrades as σ_y and σ_u increase.

5. EXTRUDER DATA

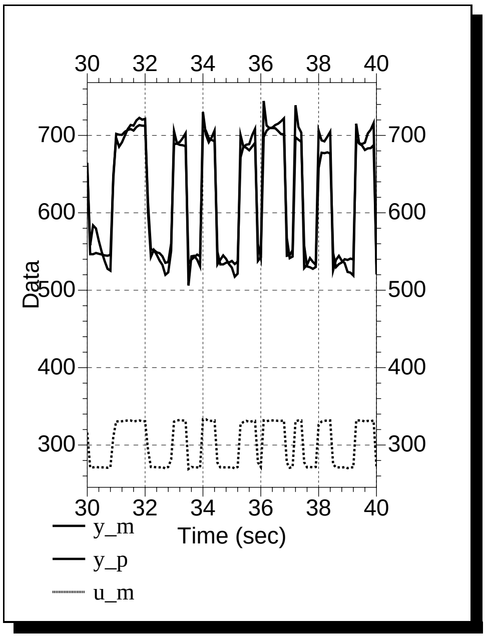

Fig. 8. *Extruder data*

Data	Filter	a_1	a_2	b_1	b_2
Extruder	IIR	11.56	33.29	8.10	31.46

Table 4. Extruder data

The food extruder under study belongs to Food Science Australia. An extrusion cooker simultaneously transports, mixes, shapes, stretches and shears material under elevated pressure and temperature. More details are given by Wang and Gawthrop (2001). For this particular case study, the manipulated variable is screw speed and the output variable is specific mechanical energy (SME). The process sampling interval was chosen as 1 sec. and samples of input and output variables were collected in the experiment.

Presumably because the input and output data was noisy, it proved impossible to get sensible results using the FIR approach. The IIR approach was used to estimate the parameters of the following polynomials

$$\begin{aligned} a(s) &= s^2 + a_1 s + a_2 \\ b(s) &= b(s^2 + b_1 s + b_2) \\ d(s) &= d_1 s + d_2 \end{aligned} \tag{17}$$

The values of d_1 and d_2 are of no direct interest, but are needed to correctly identify $a(s)$ and $b(s)$.

Figure 8 shows the following data:

(1) the upper graph is the measured system output (SME) together with the value of SME predicted from the estimated model and the measured system input (Screw Speed).
(2) the lower graph is the measured input (Screw Speed).

Table 4 gives the estimated parameter values.

Because of the close fit to the data, this model is regarded as accurate.

6. CONCLUSION

Given the backgrounds of the two approaches, it is hardly surprising that the FIR method works well on AR data and the IIR method on deterministic data with added noise. More interesting are the two cases where the methods do not work

(1) the FIR approach in the presence of measurement noise and
(2) the IIR approach when using AR data

We believe that the way forward involves a melding of the two approaches by either:

(1) adding further filtering to the FIR approach to remove high-frequency measurement noise or
(2) extending the synthesis approach of (Söderström *et al.*, 1997) to reduce bias in the IIR case.

It should also be emphasised that the IIR method could also be improved by making use of an unbiased approach such as instrumental variables (Young and Jakeman, 1979; Jakeman and Young, 1979; Young and Jakeman, 1980) to replace the least-squares method. This, together with a theoretical interpretation of our results, will be the subject of future work.

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