15th Triennial World Congress, Barcelona, Spain

ANALYSIS OF FLATNESS USING BOND GRAPHS AND BICAUSALITY

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Abstract: Considering one of the most popular examples of flat system, namely the two dimensional crane, this paper shows how bicausality can be used in order to analyse its bond graph model as far as flatness is concerned. More generally, it will be seen how applying this concept both allows verifying the flatness property of a system and deriving the open-loop control laws that result from it in a systematic fashion, provided that flat outputs have been identified. *Copyright © 2002 IFAC*

Keywords: flatness, bond graph, bicausality, inverse dynamics.

1. INTRODUCTION

The issue of flatness from a bond graph point of view has already been addressed by Gil, *et al*. (1997). The method described in that contribution, which aims at determining the flatness property directly on a bond graph model, is based upon the analysis of causal paths between the inputs as well as the state variables on the one hand, and the candidate flat outputs on the other hand. It is restricted, however, to SISO systems. Moreover, it does only work in the case where flat outputs are state variables. The purpose of the present paper is to propose a more general approach. When searching for a structural way to verify flatness from a bond graph, one could first think of the notion of relative degrees, since flatness has to do with input-output invertibility. Thus a necessary condition for a system to be flat is that the relative degree associated with each of its input-flat output pairs be equal to its order. Now there exist systematic methods to determine relative degrees from bond graphs given a preferred integral causality assignment (Wu and Youcef-Toumi, 1993). Unfortunately, the previous condition is not sufficient since flatness also implies some kind of state-output invertibility, as will be seen in the next section. As a consequence, it seems that the most relevant answer to the question of flatness is provided by the notion of inverse dynamics (Gawthrop, 1998). Now this notion is strongly related to the concept of bicausality, introduced by Gawthrop (1995). It is why the latter concept will be investigated as a tool to analyse the flatness of systems modelled by bond graphs. The paper is organized as follows. In section 2, some background is given about flatness, then about bond graph modelling and finally about bicausality. Section 3 illustrates the proposed analysis approach through the famous example of a 2-D crane, which is first studied in a basic context before being refined in section 4. Lastly a general methodology is deduced in section 5.

2. BACKGROUND

2.1 Differential flatness

The concept of differential flatness was introduced by Fliess et al. as a new nonlinear extension of Kalman’s controllability. Flat systems, indeed, are equivalent to linear controllable ones via a special type of feedback called endogenous (Fliess, *et al.*, 1995). As a consequence, such systems are controllable whether they are linear or not. It is very important a result since many systems met in various engineering fields actually are flat (Rudolph, 1999).

Let now briefly recall the mathematical definition of this notion. Consider a nonlinear multivariable system characterized by the following generalized state representation:

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state vector and $u \in \mathbb{R}^m$ the input one.

This system is called (differentially) flat if there exists a new vector $y = h(x, u, \dot{u}, \ldots, u^{(\alpha)})$, where $y \in \mathbb{R}^m$ and $\alpha \in \mathbb{N}$, such that there exist two functions A and B, as well as an integer β, verifying the double property

$$\begin{cases} x = A\left(y, \dot{y}, \ldots, y^{(\beta)}\right) \\ u = B\left(y, \dot{y}, \ldots, y^{(\beta+1)}\right) \end{cases} \qquad (2)$$

Vector y is called a flat (or linearizing) output. It has the same dimension as input vector u, and its components are differentially independent real-analytic functions of x, u and a finite number of its time derivatives. Besides, it generally has a well defined physical meaning, although there is no uniqueness in its choice. The major property about it is that any variable of the system can be expressed as a differential function of its components, and thus calculated without integration of the differential equation governing the system.

Several important implications of flatness exist as far as control is concerned. The most obvious one is motion planning. Indeed, any desired trajectory of the linearizing output can be obtained in a straightforward manner via an open-loop control, since x and u trajectories are exactly and explicitly deduced from y ones. By another way, closed-loop strategies can be easily applied in order to control a flat system, since the latter can be transformed into a linear controllable one in Brunovsky canonical form, namely

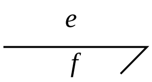
$$y^{(\beta+1)} = v \qquad (3)$$

where v is the new input vector, by means of an endogenous feedback (Rotella and Carillo, 1999).

To date, there exists however no systematic method to determine the flatness of a system.

2.2 Bond graph modelling

Bond graph is a modelling tool which yields a lumped parameters graphical description of energy exchanges in dynamic systems. Each elementary energy transfer is represented by means of a bond with an half array indicating its conventional direction, as depicted in figure 1. Two variables are associated with each bond, namely the effort e and the flow f, the product of which gives the power transferred. See (Karnopp, *et al.*, 1990; Borne, *et al.*, 1992) for an in-depth description.



$$\wp = e\, f$$

Fig.1. Bond graph representation of an elementary power transfer.

This tool has proved to be particularly convenient to deal with multidisciplinary systems, since it is characterized by a unique and reduced formalism whatever the physical field of interest may be. Thus any model can be put in the generic form of figure 2.

Given such a bond graph, a structural analysis can be performed, as well as a generation of its symbolic equations, thanks to the essential notion of causality which provides physical models with a computational input-output structure (Sueur and Dauphin-Tanguy, 1989; Sueur and Dauphin-Tanguy, 1991). Standard causality is based on the principle that an effort imposed at one end of a bond necessarily implies a flow imposed at the other end. By convention, a causal stroke is put at the end of the bond where effort is imposed, as depicted in figure 3. Systematic procedures exist in order to achieve causality assignment for a whole model in a consistent manner.

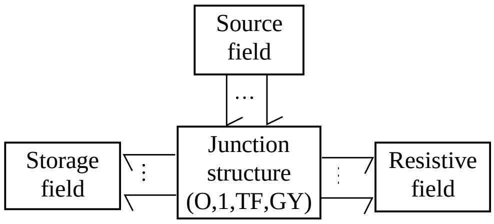


Fig.2. Bond graph generic model.

$$\begin{cases} e_2 := e_1 \\ f_1 := f_2 \end{cases} \qquad \begin{cases} e_1 := e_2 \\ f_2 := f_1 \end{cases}$$

Fig.3. Bond graph notation for standard causality.

Using the causal orientation of bonds, state space representations can be systematically derived from possibly nonlinear bond graph models. The resulting equations generally have the following form:

$$\dot{x} = f(x) + g(x)u \qquad (4)$$

which appears as a particular case of (1).

This property explains why bond graph can be thought of as a suitable tool to analyse flatness.

2.3 Bicausality

The notion of bicausality was introduced by Gawthrop as an extension of conventional bond graph theory, in order to handle systems with non standard input-output patterns (Gawthrop, 1995). In conventional bond graph models, bonds are unicausal in the sense that a single causal (full) stroke is attached to each of them. In the context of bicausality instead, bonds are provided with two causal half strokes, which results in decoupling the effort and flow respective causalities. More precisely, a causal half stroke put on the flow side of a bond (i.e. on the half arrow side) means a flow imposed on the variable associated with the far end of this bond, whereas a causal half stroke put on the effort side of a bond means an effort imposed on the variable associated with the near end of this bond. With such a convention, unicausal bonds appear as particular cases of bicausal ones where both causal half strokes coincide. For a single bond, two different extra configurations can be found (besides the ones of figure 3), which appear to be specifically bicausal. They are shown in figure 4, with the corresponding assignment statements.

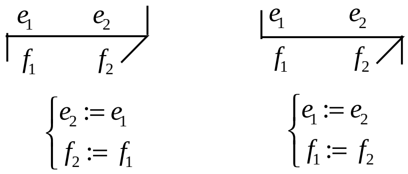


Fig.4. Bicausal configurations for a single bond.

Bicausality can be used to derive properties about inverse dynamics, state and parameter estimation. The inverse of a dynamic system such as addressed here is defined as the new system which, given the initial system output as its input, will exactly reproduce the system input as its output. It is therefore a question of (possibly partial) inversion with respect to input-output pairs. Now it has been demonstrated by Gawthrop that the inverse of a system modelled by a standard bond graph is best represented by a bicausal one, since only the latter permits all the equations of the inverse dynamics to be directly represented (Gawthrop, 1998). It is particularly true for systems with non-collocated input-output pairs. The same author has also introduced new bond graph components, namely source-sensor components denoted by SS, in order to help define inverse systems (Gawthrop and Smith, 1992). In the context of bicausal bond graphs, these components provide a more convenient representation of the input and output ports of systems than standard effort and flow sources, since the causality on their bond is not irrevocably fixed. Note that one can conventionally distinguish between input and output SS components, by using the adequate direction for power (Fotsu-Ngwompo, et al., 1997).

By another way, it is obvious from (2) that the proof for flatness has to do with system inversion. Moreover, it will be seen in the following that parameter estimation can be necessary in the process of inverting a nonlinear system modelled by bond graphs. As a result, bicausal bond graphs associated with the use of SS components seem to provide a particularly adequate framework to study flat systems.

3. BASIC EXAMPLE

For the sake of both clarity and simplicity, the proposed analysis method will first be illustrated using an example instead of being exposed theoretically. Let take the case of the two dimensional crane displayed in figure 5, where the trolley travels horizontally while its load, which behaves like a variable length pendulum, remains in the fixed vertical plane of the figure. The trolley position *D* and the pendulum length *R* are control inputs.

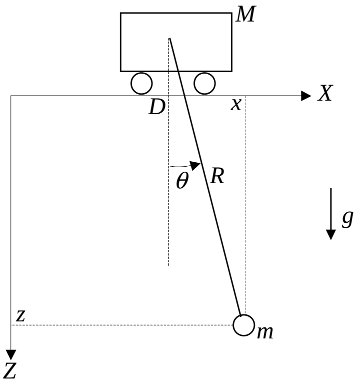


Fig.5. 2-D crane.

The corresponding basic bond graph model is shown in figure 6. Some geometrical constraints which cannot be directly represented on that bond graph must be joined.

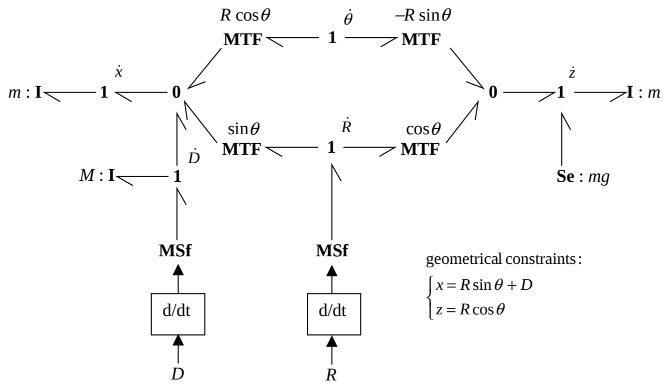


Fig.6. Basic bond graph.

According to the literature, couple (x, z) is a flat output for the system (Fliess, *et al.*, 1995). Its two constitutive variables, however, do not appear as natural outputs of the bond graph, since they cannot be measured. On the other hand, their derivatives do, because each of them corresponds to the flow variable on a particular 1 junction. In order to measure them without disturbing the dynamic behaviour, source-sensor components *injecting a null effort* are added to the model. Each original modulated flow source is also replaced by a source-sensor component, in order to properly represent the inputs of the system with a view to its coming inversion. As a last point, internal variable θ is made implicit. The equivalent model resulting from these transformations is shown in figure 7. A preferred causality has been assigned to this model, revealing an apparent order of 1, due to the presence of one storage element with integral causality. In order to determine the actual order of the model, the number of integrations, if any, which are necessary to compute all the MTF moduli must be taken into account. Actually, D and R being given as control inputs, the only knowledge of x and z is needed. Besides, it just requires one integration, since both variables are related by a geometrical constraint. Thus the actual order of the model is 2.

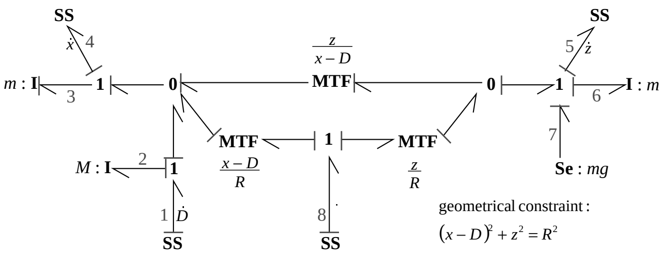


Fig.7. Equivalent causal bond graph with SS components.

Let p_3 and x be the state variables. In order to verify the flatness property, it must be shown that the latter as well as the inputs can be derived from the flat outputs without integrating. As for the state variables, it is obvious since one has $p_3 = m\,\dot{x}$ and $x = x$. As for the inputs, the causality of both SS elements corresponding to the flat outputs is modified so that they now inject the flow variable they used to measure, as well as continuing to provide a null effort. Thus the corresponding bonds become bicausal ones. Then propagation rules are used as defined in (Gawthrop, 1995). The result is the partly inverted bond graph of figure 8.

Knowing x and z, hence $\dot{x}$ and $\dot{z}$, the unknown MTF modulus $z/_{x-D}$ can be obtained from the partial causality assignment, as the ratio of the efforts which are imposed to the MTF component on both of its bonds. This situation illustrates the ability of bicausal bond graphs to allow parameter estimation, not only for one-port components as described in (Gawthrop, 1995), but also for multi-port ones.

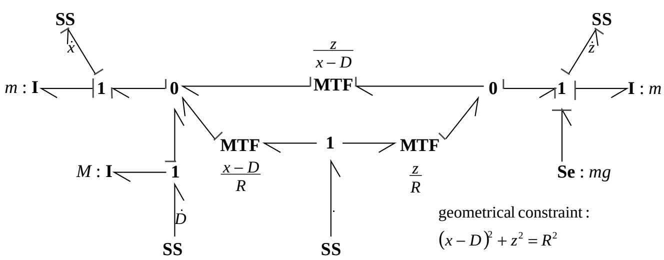


Fig.8. Partial inverse bicausal bond graph.

The expression of D is straightforwardly deduced:

$$D = x - z\frac{\ddot{x}}{\ddot{z} - g} \tag{5}$$

Then the geometrical constraint yields that of R:

$$R = \sqrt{\left(z\frac{\ddot{x}}{\ddot{z} - g}\right)^2 + z^2} \tag{6}$$

At this stage, the flatness property has been demonstrated. It is possible however to go further into the analysis. Indeed, using the previous pieces of information, the non standard causality assignment can be completed in order to define the whole inverse model, as depicted in figure 9.

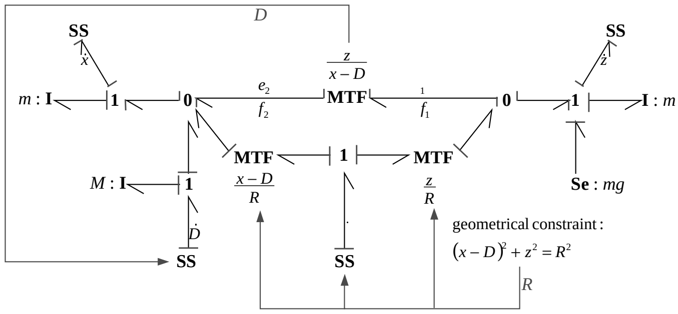


Fig.9. Complete inverse bicausal bond graph.

It can be seen that a double determination of the MTF modulus primarily used to find D as a function of x and z results from the causal assignment achieved. The consistency of such a situation must be verified. Actually, the expressions found for D and R yield:

$$\frac{e_1}{e_2} = \frac{\dot{x} - \dot{D} - \left(\frac{x-D}{R}\right)\dot{R}}{\frac{z}{R}\dot{R} - \dot{z}} = \frac{\ddot{z} - g}{\ddot{x}} = \frac{f_2}{f_1} \tag{7}$$

4. MORE REFINED EXAMPLE

Now the previous system is augmented with the traversing and hoisting dynamics, which implies taking new control inputs, namely force F and torque Γ, according to the bond graph of figure 10. r is the radius of the hoist pulley and J its inertia momentum. This time, the apparent order of the system is 3 and the actual one 6 (taking the geometrical constraint into account). The natural state variables (in a bond graph sense) are p_2, p_3 and p_9; x, D and R (for instance) must be added to them.

It has already been shown in other contributions that this augmented system is still flat, with unchanged flat outputs (D'Andréa-Novel and Levine, 1990; Siguerdidjane, 1998). The point here is to verify this property directly from the bond graph model using bicausality, as done just before for the basic system. In order to invert the new system dynamics, it can be proceeded the same way as previously, modifying the causality of both SS elements associated with the candidate flat outputs and propagating through the junction structure.

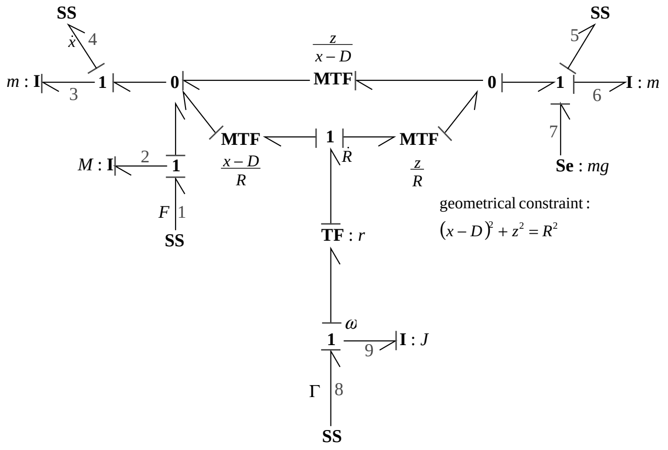
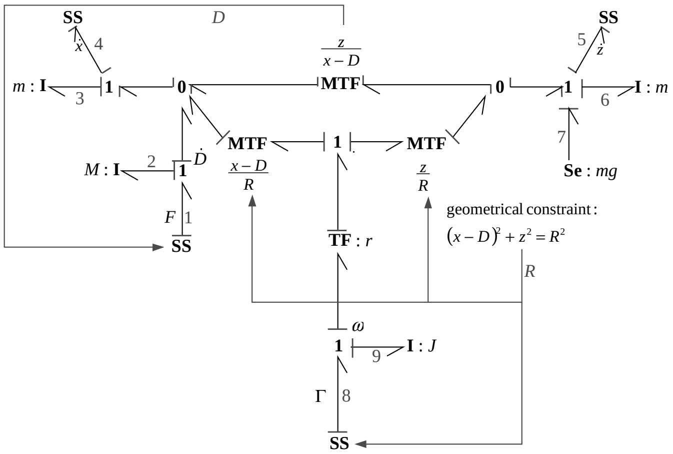


Fig.10. Unicausal bond graph of the augmented system.



F ig.11. Bicausal bond graph of the inverted augmented system.

D and R are identically determined, and thus given by the same formulas (5) and (6). Then, it can be verified that the (bi)causal assignment is such that F and Γ can be deduced from the graph. Moreover, since all the storage elements have been imposed a derivative causality, the order of the inverse system is zero and no integration will be needed for their computation.

Thus one gets:

$$\begin{cases} F = M\,\ddot{D} + m\,\ddot{x} \\ \Gamma = \dfrac{J}{r}\,\ddot{R} + \dfrac{m\,r}{R}\left[(x-D)\ddot{x} + z\,(\ddot{z}-g)\right] \end{cases} \tag{8}$$

Besides, the state variables are given by:

$$\begin{cases} x = x \\ D = x - z\,\dfrac{\ddot{x}}{\ddot{z}-g} \\ R = \sqrt{\left(z\,\dfrac{\ddot{x}}{\ddot{z}-g}\right)^2 + z^2} \\ p_2 = M\,\dot{D} \\ p_3 = m\,\dot{x} \\ p_9 = \dfrac{J}{r}\,\dot{R} \end{cases} \tag{9}$$

which demonstrates the flatness of the system with respect to the same couple of linearizing outputs (x, z).

Once again, a double determination of the MTF modulus is encountered, whose consistency straightforwardly results from the verification made in the previous section.

5. GENERALIZATION

From the previous examples, a general methodology can be deduced in order to analyse the flatness of a system. Given a standard bond graph model, a sequential procedure including 3 stages is proposed:

- First assign conventional causality to the model, using SCAP or any equivalent method based upon preferred integral causality. Identify the resulting state variables (p on I components, q on C components in integral causality, plus possibly extra variables in the case of modulated components such as MTF for instance).

- Coming back to the acausal model, replace the sources by input SS components and attribute output SS components to the candidate flat outputs.

- Assign the flow-source/effort-source causality to the output SS components and propagate as far as possible. Whenever the bicausal information allows estimating a modulated component parameter, use the resulting piece of knowledge to propagate again (by determining other modulated components parameters or partly imposing the causality of input SS components for instance). Also use any additional physical constraint possibly not included in the bond graph.

If at the end of this step:

- the whole model has been causally completed (which means that any bond has its two causal half strokes set),

- no storage element has been imposed an integral causality,

- the causality of the input SS components is such that every original input of the system now appears as an output of the bicausal junction structure, then the original system is flat with respect to the chosen outputs.

In such a case indeed, the inverse system exists and its order is zero, which implies that any variable, including the original system inputs and state variables, can be calculated without integrating given its inputs (i.e. the original system outputs). In order to find the laws corresponding to (2), one just has to write the equations of the bond graph in the form dictated by causality.

On the other hand, if one of the previous conditions is not satisfied, then it can just be concluded that the considered outputs are not linearizing ones for the system.

6. CONCLUSION

This contribution shows how bicausal bond graphs can be used to verify the flatness of a system with respect to identified linearizing outputs, as well as to derive the resulting open-loop control laws. First, applying a standard causality assignment to the bond graph model straightforwardly leads to a minimal state space representation. Then, respectively attributing SS components to the input and output ports, and giving an adequate bicausal assignment to the latter, one can build the inverse model, the order of which must be zero for the original system to be flat. Further research will consist in investigating how to seek for flat outputs if any, given some system.

REFERENCES

D'Andréa-Novel, B., Levine, J. (1990). Modelling and Nonlinear control of an overhead crane. In: *Robust Control of Linear and Nonlinear Systems, Mathematical Theory of Networks and Systems* (MTNS'89), Vol. 2, pp. 523-529.

Borne, P., Dauphin-Tanguy, G., Richard, J.P., Rotella, F., Zamvettakis, I. (1992). *Modélisation et Identification des processus*, Tome 2, Technip.

Fliess, M., Levine, J., Martin, P., Rouchon, P. (1995). Flatness and Defect of Nonlinear Systems: Introductory Theory and Examples. In: *International Journal of Control*, Vol. 61, No. 6, pp.1327-1361.

Fotsu-Ngwompo, R., Scavarda, S., Thomasset, D. (1997). Physical Interpretation of Zero Dynamics for Linear SISO Control Systems. In: *IFAC-IFIP-IMACS Conference,* Belfort, pp. 232-236.

Gawthrop, P.J., Smith, L. (1992). Causal Augmentation of Bond Graphs. In: *Journal of the Franklin Institute*, 329(2), pp. 291-303.

Gawthrop, P.J. (1995). Bicausal Bond Graphs. In: *Proceedings of International Conference on Bond Graph Modeling and Simulation* (ICBGM'95), Las Vegas, pp. 83-88.

Gawthrop, P.J. (1998). Physical Interpretation of Inverse Dynamics using Bond Graphs. In: *The Bond Graph Digest*, Vol. 2.

Gil, J.C., Pedraza A., Delgado, M. (1997). Flatness and Passivity from a Bond Graph. In: IEEE.

Karnopp, D.C., Margolis, D.L., Rosenberg, R.C. (1990). *System Dynamics : a Unified Approach*. Wiley Interscience, second edition.

Rotella, F., Carillo, F.J. (1999). Flatness Approach for the Numerical Control of a Tuning Process. In: *Les systèmes plats : aspects théoriques et pratiques, mise en œuvre, Journée thématique PRC-GDR Automatique*, pp.143-149.

Rudolph, J. (1999). Flatness Based Control of Chemical Reactors. In: Les systèmes plats : aspects théoriques et pratiques, mise en œuvre, Journée thématique PRC-GDR Automatique, pp.179-219.

Siguerdidjane, H.B. (1998). Divers schémas de Commande Non Linéaire et Applications. In : *Habilitation Degree from University of Paris XI.*

Sueur, C., Dauphin-Tanguy, G. (1989). Structural Controllability/Observability of Linear Systems Represented by Bond Graphs. In: *Journal of the Franklin Institute*, Vol. 326, No. 6, pp. 869-883.

Sueur, C., Dauphin-Tanguy, G. (1991). Bond Graph Approach for Structural Analysis of MIMO Linear Systems. In: *Journal of the Franklin Institute*, Vol. 328, No. 1, pp. 55-70.

Wu, S.T., Youcef-Toumi, K. (1993). On Relative Degrees and Zero Dynamics from System Configuration. In: *Proceedings of the American Control* Conference, San Fransisco, pp. 1025-1029.