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ROBUST ADAPTIVE NN CONTROL OF A CLASS OF SEMI-STRICT FEEDBACK NONLINEAR SYSTEMS

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Abstract: This paper presents a robust adaptive control approach for a class of semi-strict feedback nonlinear system with both unknown virtual control coefficients and unknown nonlinearities. It has been proven that the proposed robust adaptive scheme can guarantee the uniform ultimate boundedness of the closed-loop system signals. Simulation studies are included to illustrate the effectiveness of the proposed approach. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Recently a series of works have been focused on the robust adaptive control of a class of nonlinear systems whose uncertainties include nonlinearly appearing parametric uncertainty, uncertain nonlinearities as well as unmeasured input-to-state stable dynamics (Jiang and Hill, 1999)(Plycarpou and Ioannou, 1995)(Yao and Tomizuka, 1997). A robust adaptive nonlinear control design procedure was presented in (Plycarpou and Ioannou, 1995) for a class of nonlinear systems with both parametric uncertainty and unknown nonlinearities under the assumption that unknown functions satisfy a so-called *triangular bounds* condition. In (Jiang and Hill, 1999), the authors proposed a robust adaptive control scheme for perturbed strict feedback nonlinear systems subject to nonlinear parametric uncertainty, uncertain nonlinearity, and unmodeled dynamics. For a similar class of nonlinear system, (Yao and Tomizuka, 1997) also presented an adaptive robust control method by combining the backstepping adaptive control with conventional deterministic robust control.

In order to cope with the completely unknown nonlinear system functions, as an alternative, approximator-based adaptive control approaches have also been extensively studied in the past decade using Lyapunov stability theory (Ge *et al.*, 2001)(Polycarpou, 1996)(Yesildirek and Lewis, 1995)(Wang, 1994). In (Yesildirek and Lewis, 1995)(Ge *et al.*, 1999), stable adaptive NN controllers were proposed for nonlinear systems in a Brunovsky form. The same system was studied in (Wang, 1994)(Spooner and Passino, 1996) by using fuzzy systems as function approximator and different adaptive fuzzy controllers have been derived. Using the idea of adaptive backstepping, the developed approximator-based adaptive control approaches were recently extended to nonlinear systems without satisfying matching condition (Ge *et al.*, 2001)(Polycarpou, 1996). In (Polycarpou, 1996), a stable adaptive neural control method was presented for a second-order nonlinear system, where the unknown system function was parameterized by RBF neural networks, and unknown neural reconstruction error bound was also adaptively tuned on-line. The existing

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robust adaptive algorithms for triangular systems are developed with the virtual control coefficients equal to one (Jiang and Hill, 1999)(Plycarpou and Ioannou, 1995). The problem of adaptive control of nonlinear systems with unknown virtual control coefficients have recently received a lot of attention from many researchers (Krstic *et al.*, 1995)(Ye and Jiang, 1998)(Kaloust and Qu, 1995)(Ge *et al.*, 2002). In (Kaloust and Qu, 1995), a robust control design was developed for a class of uncertain nonlinear systems satisfying the so-called generalized matching conditions without *a priori* knowledge of control directions. In (Ye and Jiang, 1998), by using Nussbaum gain (Nussbaum, 1983), adaptive control algorithms were presented for parametric-strict-feedback systems with unknown virtual control coefficients. However, the discussed systems in the above mentioned works are mostly focused on the so-called strict feedback nonlinear systems under the assumption that the system uncertainties have been linearly parameterized.

In this paper, we present a robust adaptive control design procedure for a class of semi-strict feedback nonlinear systems with both unknown virtual control coefficients and unknown nonlinearities. The unknown nonlinearities comprise two types of nonlinear functions: one naturally satisfies the "triangularity condition" and will be approximated by linearly parameterized approximators; while the other is assumed to be partially known and consists of parametric uncertainties and known "bounding functions" which also satisfy the "triangularity condition". With the utilization of adaptive backstepping and tuning functions which are for the reduction of overparameterization, the proposed design method expands the class of nonlinear systems for which robust adaptive control approaches have been studied. It has been proven that the proposed robust adaptive scheme can guarantee the uniform ultimate boundedness of the closed-loop system signals.

2. PRELIMINARIES

Consider the control problem of a single-input-single-output (SISO) nonlinear uncertain system transformable into

$$\begin{aligned} \dot{x}_i &= g_i x_{i+1} + f_i(x_1, \cdots, x_i) + \Delta_i(t, x) \\ \dot{x}_n &= g_n \beta(x) u + f_n(x) + \Delta_n(t, x) \\ y &= x_1 \end{aligned} \quad (1)$$

where $i = 1, \cdots, n-1$, $x = [x_1, \cdots, x_n]^T \in R^n$ is the state vector, $u \in R$ is the control, $f_1, \cdots, f_n$ are unknown smooth nonlinear functions, $\beta(x) : R^n \to R$ is known smooth function and $\beta(x) \neq 0, \forall x \in R^n$, $g_i, i = 1, \cdots, n$ are unknown constants, and they are referred to as virtual control coefficients, in particular, g_n is referred to as the high-frequency gain, and Δ_i's are unknown Lipschitz continuous functions. Let $\bar{x}_i = [x_1, \cdots, x_i]^T$. The control objective is to construct a robust adaptive nonlinear control law so that the output y of the above system is driven to a small neighborhood of the origin, while keeping internal Lagrange stability.

The system described by (1) is in the so-called semi-strict feedback form (Yao and Tomizuka, 1997), which has two types of unknown nonlinear functions: one naturally satisfies the "triangularity condition" and can be directly approximated by linearly parameterized approximators; while the other, arises owing to $\Delta_i(t, x)$, is assumed to be partially known and consists of parametric uncertainties and known "bounding functions" which also satisfy the "triangularity condition". The unknown nonlinear functions $\Delta_i(t, x)$ could be due to many factors, such as measurement noise, modeling errors, external disturbances, modeling simplifications or changes due to time variations (Plycarpou and Ioannou, 1995).

Assumption 1. For $1 \leq i \leq n$, there exists unknown positive constant p_i^* such that $\forall (t, x) \in R_+ \times R^n$, $|\Delta_i(t, x)| \leq p_i^* \phi_i(x_1, \cdots, x_i)$, where ϕ_i is a known nonnegative smooth function.

Assumption 2. The signs of $g_i, i = 1, \cdots, n$ are known.

A linearly parameterized approximator shall be used to approximate the unknown nonlinearities $f_i(\cdot)$. Several function approximators can be applied for this purpose, e.g., radial basis function (RBF) neural networks (Ge *et al.*, 2001)(Sanner and Slotine, 1992), high-order neural networks and so on, which can be described as $\theta^T \psi(z)$ with input vector $z \in R^n$, weight vector $\theta \in R^l$, node number l, and basis function vector $\psi(z) \in R^l$. Universal approximation results indicate that, if l is chosen sufficiently large, then $\theta^T \psi(z)$ can approximate any continuous function to any desired accuracy over a compact set. In this paper, we use the following RBF NN to approximate a smooth function. For the unknown nonlinear functions $f_i(\bar{x}_i), i = 1, \cdots, n$ in (1), we have the following approximation over the compact sets Ω_i

$$f_i(\bar{x}_i) = \theta_i^{*T} \psi_i(\bar{x}_i) + \omega_i(\bar{x}_i), \quad \forall \bar{x}_i \in \Omega_i \subset R^i \quad (2)$$

where $\psi(\bar{x}_i)$ is the basis function vector, $\omega_i(\bar{x}_i)$ is the approximation error and θ_i^* is an unknown constant parameter vector.

Remark 1. The optimal weight vector θ_i^* in (2) is an "artificial" quantity required only for analytical purposes. Typically, θ^* is chosen as the value of θ that minimizes $\omega_i(\bar{x}_i)$ for all $\bar{x}_i \in \Omega_i$, where $\Omega_i \subset R^i$ is a compact set, i.e., $\theta_i^* := \arg\min_{\theta_i \in R^n}\{\sup_{\bar{x}_i \in \Omega_i} |f_i(\bar{x}_i) - \theta_i^T \psi(\bar{x}_i)|\}$.

Assumption 3. On a compact region $\Omega_i \in R^i$, $|\omega_i(\bar{x}_i)| \leq \delta_i^*, \forall \bar{x}_i \in \Omega_i$, $i = 1, \cdots, n$, where $\delta_i^* \geq 0$ is an unknown bound.

3. ROBUST ADAPTIVE CONTROL

3.1 *Control Design with $g_i = 1$*

Before we introduce the robust adaptive control algorithm for system with unknown g_i, let us first give the robust adaptive control design for the case $g_i = 1$. Using (2), equation (1) can be expressed as

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \theta_i^{*T}\psi_i(\bar{x}_i) + \omega_i(\bar{x}_i) + \Delta_i(t,x) \\ \dot{x}_n &= \beta(x)u + \theta_n^{*T}\psi_n(x) + \omega_n(x) + \Delta_n(t,x) \quad (3) \\ y &= x_1 \end{aligned}$$

The system described by (3) has three types of uncertainty: parametric uncertainty, which arises due to the unknown θ_i^*, the bounding uncertainty that arises due to the unknown bounds on Δ_i and ω_i, and unknown virtual control coefficient g_n. The unknown parameters $\theta_i^*, p_i^*, \delta_i^*$ and g_n will be estimated on-line. In order to avoid the possible singularity problem for g_n, we estimate $\frac{1}{g_n}$ instead of g_n. Our design consists of n steps. At each intermediate step i, we design stabilizing function α_i using an appropriate Lyapunov function V_i, and give the parameters update law $\dot{\hat{b}}_i$ for b_i^* which is the grouped unknown bound for p_i^* and δ_i^*. The tuning functions $\tau_{j,i}$ for $\hat{\theta}_j$ are also proposed, where $\hat{\theta}_j$ represents the estimate of unknown parameter θ_j^*.

Step 1: To start, consider the subsystem of (3):

$$\dot{x}_1 = x_2 + \theta_1^{*T}\psi_1(x_1) + \omega_1(x_1) + \Delta_1(t,x) \quad (4)$$

where x_2 is taken for a virtual control input. To design a stabilizing adaptive control law for system (4), consider a Lyapunov function candidate $W_1 = \frac{1}{2}x_1^2$. In light of Assumptions 1 and 3, the time derivative of W_1 along the solutions of (4) satisfies

$$\dot{W}_1 \leq x_1(x_2 + \theta_1^{*T}\psi_1(x_1)) + b_1^*|x_1|\hat{\phi}_1(x_1) \quad (5)$$

where $b_1^* = \max\{\delta_1^*, p_1^*\}$, $\hat{\phi}_1(x_1) = 1 + \phi_1(x_1)$. Consider the Lyapunov function candidate $V_1 = W_1 + \frac{1}{2}(\hat{\theta}_1 - \theta_1^*)^T\Gamma_1^{-1}(\hat{\theta}_1 - \theta_1^*) + \frac{1}{2\lambda_1}(\hat{b}_1 - b_1^*)^2$, where $\Gamma_1 = \Gamma_1^T > 0$, $\lambda_1 > 0$, and $\hat{\theta}_1$ and $\hat{b}_1$ are the parameters estimates to be determined later. The time derivative of V_1 along (5) is

$$\begin{aligned} \dot{V}_1 &\leq x_1(x_2 + \theta_1^{*T}\psi_1(x_1)) + b_1^*|x_1|\hat{\phi}_1(x_1) \\ &+(\hat{\theta}_1 - \theta_1^*)^T\Gamma_1^{-1}\dot{\hat{\theta}}_1 + \frac{1}{\lambda_1}(\hat{b}_1 - b_1^*)\dot{\hat{b}}_1 \quad (6) \end{aligned}$$

Consider the following change of coordinates $z_1 = x_1, z_2 = x_2 - \alpha_1(x_1, \hat{\theta}_1, \hat{b}_1)$ with

$$\alpha_1 = -k_1x_1 - \hat{\theta}_1^T\psi_1(x_1) - \hat{b}_1\hat{\phi}_1(x_1)\tanh[\frac{x_1\hat{\phi}_1(x_1)}{\epsilon_1}]$$

where ϵ_1 is a small positive constant and $k_1 > 0$. As in (Plycarpou and Ioannou, 1995), in order to prevent parameters drift, we present the following adaptive law incorporating a leakage term based on a variant of the σ-modification. Let

$$\tau_{11} = \Gamma_1 x_1\psi_1(x_1) - \Gamma_1\sigma_{\theta_1}(\hat{\theta}_1 - \theta_1^0) \quad (7)$$

$$\dot{\hat{b}}_1 = \lambda_1 x_1\hat{\phi}_1(x_1)\tanh[\frac{x_1\hat{\phi}_1(x_1)}{\epsilon_1}] - \lambda_1\sigma_{b_1}(\hat{b}_1 - b_1^0) \quad (8)$$

where $\sigma_{\theta_1} > 0, \sigma_{b_1} > 0$ and $\theta_1^0, b_1^0 > 0$ are design constants. Using α_1, (7) and (8), a direct substitution of $x_2 = z_2 + \alpha_1$ into (6) gives

$$\begin{aligned} \dot{V}_1 \leq &-k_1x_1^2 + x_1z_2 + (\hat{\theta}_1 - \theta_1^*)^T\Gamma_1^{-1}(\dot{\hat{\theta}}_1 - \tau_{11}) \\ &+b_1^*|x_1|\hat{\phi}_1(x_1) - b_1^*x_1\hat{\phi}_1(x_1)\tanh[\frac{x_1\hat{\phi}_1(x_1)}{\epsilon_1}] \\ &-\sigma_{\theta_1}(\hat{\theta}_1 - \theta_1^*)^T(\hat{\theta}_1 - \theta_1^0) - \sigma_{b_1}(\hat{b}_1 - b_1^*)(\hat{b}_1 - b_1^0) \end{aligned}$$

By completing the squares and using the following nice property with regard to function $\tanh(\cdot)$ (Plycarpou and Ioannou, 1995): $0 \leq |x| - x\tanh(\frac{x}{\epsilon}) \leq 0.2785\epsilon$, for $\epsilon > 0, x \in R$, we have,

$$\begin{aligned} \dot{V}_1 \leq &-k_1z_1^2 - \frac{1}{2}\sigma_{\theta_1}|\hat{\theta}_1 - \theta_1^*|^2 - \frac{1}{2}\sigma_{b_1}(\hat{b}_1 - b_1^*)^2 \\ &+(\hat{\theta}_1 - \theta_1^*)^T\Gamma_1^{-1}(\dot{\hat{\theta}}_1 - \tau_{11}) + z_1z_2 + b_1^*0.2785\epsilon_1 \\ &+\frac{1}{2}\sigma_{\theta_1}|\theta_1^* - \theta_1^0|^2 + \frac{1}{2}\sigma_{b_1}(b_1^* - b_1^0)^2 \end{aligned}$$

Step 2: Let $V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}(\hat{\theta}_2 - \theta_2^*)^T\Gamma_2^{-1}(\hat{\theta}_2 - \theta_2^*) + \frac{1}{2\lambda_2}(\hat{b}_2 - b_2^*)^2$, differentiating V_2 with respect to time gives

$$\begin{aligned} \dot{V}_2 = \dot{V}_1 + z_2\Big[&x_3 + \theta_2^{*T}\psi_2 + \omega_2 + \Delta_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 \\ &-\frac{\partial\alpha_1}{\partial x_1}(x_2 + \theta_1^{*T}\psi_1 + \omega_1 + \Delta_1) - \frac{\partial\alpha_1}{\partial\hat{b}_1}\dot{\hat{b}}_1\Big] \\ &+(\hat{\theta}_2 - \theta_2^*)^T\Gamma_2^{-1}\dot{\hat{\theta}}_2 + \frac{1}{\lambda_2}(\hat{b}_2 - b_2^*)\dot{\hat{b}}_2 \end{aligned}$$

In view of Assumptions 1 and 3, we have

$$\begin{aligned} &z_2\Big(\omega_2 + \Delta_2 - \frac{\partial\alpha_1}{\partial x_1}(\omega_1 + \Delta_1)\Big) \\ &\leq |z_2|\Big(p_2^*\phi_2 + \delta_2^* + |\frac{\partial\alpha_1}{\partial x_1}|(p_1^*\phi_1 + \delta_1^*)\Big) \leq b_2^*|z_2|\hat{\phi}_2 \end{aligned}$$

where $b_2^* = \max\{p_1^*, p_2^*, \delta_1^*, \delta_2^*\}$, $\hat{\phi}_2 \geq \phi_2 + 1 + |\frac{\partial\alpha_1}{\partial x_1}|(\phi_1 + 1)$ is a smooth positive function. Let $z_3 = x_3 - \alpha_2$, and we select tuning functions τ_{12}, τ_{22}, stabilizing function α_2, and adaptive law for $\hat{b}_2$ as follows

$$\tau_{12} = \tau_{11} + \Gamma_1 z_2 \frac{\partial \alpha_1}{\partial x_1}\psi_1$$
$$\tau_{22} = \Gamma_2 z_2 \psi_2 - \Gamma_2 \sigma_{\theta_2}(\hat{\theta}_2 - \theta_2^0)$$
$$\dot{\hat{b}}_2 = \lambda_2 z_2 \hat{\phi}_2 \tanh\left[\frac{z_2\hat{\phi}_2}{\epsilon_2}\right] - \lambda_2\sigma_{b_2}(\hat{b}_2 - b_2^0)$$
$$\alpha_2 = -k_2 z_2 - z_1 - \hat{b}_2\hat{\phi}_2 \tanh\left[\frac{z_2\hat{\phi}_2}{\epsilon_2}\right] -\hat{\theta}_2^T\psi_2 + \frac{\partial\alpha_1}{\partial x_1}x_2 + \frac{\partial\alpha_1}{\partial \hat{b}_1}\dot{\hat{b}}_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\tau_{12} + \hat{\theta}_1^T\frac{\partial\alpha_1}{\partial x_1}\psi_1$$

Using the derivation procedures as in **Step 1**, a straightforward calculation yields

$$\begin{aligned}\dot{V}_2 \le & -\sum_{j=1}^{2} k_j z_j^2 - \sum_{j=1}^{2}\frac{1}{2}\sigma_{\theta_j}|\hat{\theta}_j - \theta_j^*|^2 \\ & -\sum_{j=1}^{2}\frac{1}{2}\sigma_{b_j}(\hat{b}_j - b_j^*)^2 + z_2 z_3 \\ & + \Big((\hat{\theta}_1 - \theta_1^*)^T\Gamma_1^{-1} - z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\Big)(\dot{\hat{\theta}}_1 - \tau_{12}) \\ & + (\hat{\theta}_2 - \theta_2^*)^T\Gamma_2^{-1}(\dot{\hat{\theta}}_2 - \tau_{22}) + \sum_{j=1}^{2} b_j^* 0.2785\epsilon_j \\ & + \sum_{j=1}^{2}\frac{1}{2}\sigma_{\theta_j}|\theta_j^* - \theta_j^0|^2 + \sum_{j=1}^{2}\frac{1}{2}\sigma_{b_j}(b_j^* - b_j^0)^2\end{aligned}$$

Step i ($3 \le i \le n$): A similar procedure is employed recursively for each step $i = 3,\cdots,n-1$. Consider the Lyapunov function candidate $V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}(\hat{\theta}_i - \theta_i^*)^T\Gamma_i^{-1}(\hat{\theta}_i - \theta_i^*) + \frac{1}{2\lambda_i}(\hat{b}_i - b_i^*)^2$. Differentiating V_i with respect to time gives

$$\begin{aligned}\dot{V}_i = & \dot{V}_{i-1} + z_i\Big[x_{i+1} + \theta_i^{*T}\psi_i(\bar{x}_i) + \omega_i(\bar{x}_i) + \Delta_i(t,x) \\ & -\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1} + \theta_j^{*T}\psi_j(\bar{x}_j) + \omega_j(\bar{x}_j) \\ & + \Delta_j(t,x)) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j\Big] \\ & + (\hat{\theta}_i - \theta_i^*)^T\Gamma_i^{-1}\dot{\hat{\theta}}_i + \frac{1}{\lambda_i}(\hat{b}_i - b_i^*)\dot{\hat{b}}_i\end{aligned}$$

In view of Assumptions 1 and 3, we have

$$\begin{aligned} & z_i\Big(\Delta_i(t,x) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j(t,x) + \omega_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\omega_j\Big) \\ & \le |z_i|\Big[p_i^*\phi_i + \sum_{j=1}^{i-1}p_j^*\Big|\frac{\partial\alpha_{i-1}}{\partial x_j}\Big|\phi_j + \delta_i^* + \sum_{j=1}^{i-1}\Big|\frac{\partial\alpha_{i-1}}{\partial x_j}\Big|\delta_j^*\Big] \\ & \le b_i^*|z_i|\hat{\phi}_i(\bar{x}_i)\end{aligned}$$

where $b_i^* = \max\{p_1^*,\cdots,p_i^*,\delta_1^*,\cdots,\delta_i^*\}$, $\hat{\phi}_i(\bar{x}_i) \ge \phi_i + \sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|\phi_j + 1 + \sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|$ is a smooth positive function. By selecting $\tau_{m,i}(m = 1,\cdots,i-1), \tau_{i,i}, \alpha_i$ and adaptive law for $\hat{b}_i$ as follows

$$\tau_{m,i} = \tau_{m,i-1} + \Gamma_m z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j, \quad m = 1,\cdots,i-1$$
$$\tau_{i,i} = \Gamma_i z_i\psi_i - \Gamma_i\sigma_{\theta_i}(\hat{\theta}_i - \theta_i^0)$$
$$\dot{\hat{b}}_i = \lambda_i z_i\hat{\phi}_i\tanh\left[\frac{z_i\hat{\phi}_i}{\epsilon_i}\right] - \lambda_i\sigma_{b_i}(\hat{b}_i - b_i^0)$$
$$\begin{aligned}\alpha_i = & -k_i z_i - z_{i-1} - \hat{b}_i\hat{\phi}_i\tanh\left[\frac{z_i\hat{\phi}_i}{\epsilon_i}\right] - \hat{\theta}_i^T\psi_i \\ & + \sum_{j=1}^{i-1}\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1} + \frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j\Big) \\ & + \sum_{m=1}^{i-1}\Big(\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_m}\tau_{m,i} + \hat{\theta}_m^T\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\Big)\end{aligned}$$

and using the same techniques as done previously, we can obtain that

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i}k_j z_j^2 - \sum_{j=1}^{i}\frac{1}{2}\sigma_{\theta_j}|\hat{\theta}_j - \theta_j^*|^2 \\ & -\sum_{j=1}^{i}\frac{1}{2}\sigma_{b_j}(\hat{b}_j - b_j^*)^2 + z_i z_{i+1} \\ & + (\hat{\theta}_i - \theta_i^*)^T\Gamma_i^{-1}(\dot{\hat{\theta}}_i - \tau_{i,i}) + \sum_{j=1}^{i}b_j^* 0.2785\epsilon_j \\ & + \sum_{m=1}^{i-1}\Big((\hat{\theta}_m - \theta_m^*)^T\Gamma_m^{-1} - \sum_{j=m}^{i-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}_m}\Big)(\dot{\hat{\theta}}_m \\ & - \tau_{m,i}) + \sum_{j=1}^{i}\frac{1}{2}\sigma_{\theta_j}|\theta_j^* - \theta_j^0|^2 + \sum_{j=1}^{i}\frac{1}{2}\sigma_{b_j}(b_j^* - b_j^0)^2\end{aligned}$$

Step n: In this final step, the actual control u appears, and we finally present the update laws for $\hat{\theta}_i, i = 1,\cdots,n$.

Theorem 1. For semi-strict feedback nonlinear system (3), under Assumptions 1 and 3, if we apply the control design procedure in the above statement, the solutions of the resulting closed-loop system are uniformly ultimately bounded. Furthermore, given any $\mu^* > \sqrt{2\rho}$, there exists T such that, for all $t \ge T$, we have $|z(t)| \le \mu^*$. The compact set $\Omega_z = \{z \in R^n : |z(t)| \le \mu^*\}$ can be made as small as desired by an appropriate choice of the design constants. Correspondingly, the output $y(t)$ satisfies the following property:

$$|y(t)| \le \sqrt{2\rho + 2V_n(0)e^{-c_1 t}} \tag{9}$$

where $\rho := \frac{c_2}{c_1}$, and constants $c_1 > 0$ and $c_2 > 0$ are defined as $c_1 := \min\{2k_j, \frac{\sigma_{\theta_j}}{\lambda_{\min}(\Gamma_j^{-1})}, \sigma_{b_j}\lambda_j, j = 1,\cdots,n\}$, $c_2 := \sum_{j=1}^{n}b_j^* 0.2785\epsilon_j + \sum_{j=1}^{n}\frac{1}{2}\sigma_{\theta_j}|\theta_j^* - \theta_j^0|^2 + \sum_{j=1}^{n}\frac{1}{2}\sigma_{b_j}(b_j^* - b_j^0)^2$, with $k_j, \Gamma_j, \lambda_j, \sigma_{\theta_j}, \sigma_{b_j}, \epsilon_j$ being design parameters.

Proof: Based on the coordinate change $z_n = x_n - \alpha_{n-1}$, the time derivative of the overall Lyapunov function candidate $V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}(\hat{\theta}_n - \theta_n^*)^T\Gamma_n^{-1}(\hat{\theta}_n - \theta_n^*) + \frac{1}{2\lambda_n}(\hat{b}_n - b_n^*)^2$ satisfies

$$\dot{V}_n \leq \dot{V}_{n-1} + z_n\Big[\beta(x)u + \theta_n^{*T}\psi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(x_{j+1} + \theta_j^{*T}\psi_j) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j\Big] + b_n^*|z_n|\hat{\phi}_n + (\hat{\theta}_n - \theta_n^*)^T\Gamma_n^{-1}\dot{\hat{\theta}}_n + \frac{1}{\lambda_n}(\hat{b}_n - b_n^*)\dot{\hat{b}}_n$$

where $b_n^* = \max\{p_1^*,\cdots,p_n^*,\delta_1^*,\cdots,\delta_n^*\}$, $\hat{\phi}_n(\bar{x}_n) = \phi_n + \sum_{j=1}^{n-1}\left|\frac{\partial\alpha_{n-1}}{\partial x_j}\right|\phi_j + 1 + \sum_{j=1}^{n-1}\left|\frac{\partial\alpha_{n-1}}{\partial x_j}\right|$.

By selecting $\tau_{m,n}(m = 1,\cdots,n-1), \tau_{n,n}, \alpha_n$ and adaptive law for $\hat{b}_n$ as follows

$$\tau_{m,n} = \tau_{m,n-1} + \Gamma_m z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j, \quad m = 1,\cdots,n-1 \tag{10}$$

$$\tau_{n,n} = \Gamma_n z_n\psi_n - \Gamma_n\sigma_{\theta_n}(\hat{\theta}_n - \theta_n^0) \tag{11}$$

$$\dot{\hat{b}}_n = \lambda_n z_n\hat{\phi}_n\tanh\left[\frac{z_n\hat{\phi}_n}{\epsilon_n}\right] - \lambda_n\sigma_{b_n}(\hat{b}_n - b_n^0) \tag{12}$$

$$\alpha_n = -k_n z_n - z_{n-1} - \hat{b}_n\hat{\phi}_n\tanh\left[\frac{z_n\hat{\phi}_n}{\epsilon_n}\right] - \hat{\theta}_n^T\psi_n + \sum_{j=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1} + \frac{\partial\alpha_{n-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j) + \sum_{m=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_m}\tau_{m,n} + \hat{\theta}_m^T\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j) \tag{13}$$

and letting $\dot{\hat{\theta}}_m = \tau_{m,n}, m = 1,\cdots,n$, $u = \frac{\alpha_n}{\beta(x)}$. Similarly, straightforward calculation yields

$$\dot{V}_n \leq -\sum_{j=1}^{n}k_j z_j^2 - \sum_{j=1}^{n}\frac{1}{2}\sigma_{\theta_j}|\hat{\theta}_j - \theta_j^*|^2 - \sum_{j=1}^{n}\frac{1}{2}\sigma_{b_j}(\hat{b}_j - b_j^*)^2 + \sum_{j=1}^{n}\frac{1}{2}\sigma_{\theta_j}|\theta_j^* - \theta_j^0|^2 + \sum_{j=1}^{n}\frac{1}{2}\sigma_{b_j}(b_j^* - b_j^0)^2 + \sum_{j=1}^{n}b_j^* 0.2785\epsilon_j$$

This leads to $\dot{V}_n \leq -c_1V_n + c_2$, then $V_n(t)$ satisfies

$$0 \leq V_n(t) \leq \rho + [V_n(0) - \rho]e^{-c_1t} \tag{14}$$

Therefore $z(t), \hat{\theta}_i(t), \hat{b}_i(t)(i = 1,\cdots,n)$ and $x(t)$ are uniformly ultimately bounded. Since $y(t) = x_1(t) = z_1(t)$, from the definition of V_n and (14), the property (9) can be easily obtained. Thus, by appropriately choosing the design constants, we can achieve the regulation of the output $y(t)$ to any prescribed accuracy while keeping the boundedness of all the signals and states of the close-loop system. $\diamondsuit$

3.2 *Control Design with all g_i Unknown ($i = 1,\cdots,n$)*

In this subsection, we give the robust adaptive control design algorithms for system (1) where all virtual control coefficients $g_i, i = 1,\cdots,n$ are unknown. The complete design procedure is given by the following expressions (with $z_0 = 0, \alpha_0 = 0, \hat{g}_0 = 0$): Define the unknown bounds as: $b_i^* = \max\{p_1^*,\cdots,p_i^*,\delta_1^*,\cdots,\delta_i^*\}$, $\hat{\phi}_i(\bar{x}_i) \geq \phi_i + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\phi_j + 1 + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|$ is a smooth positive function. Coordinate transformation: $z_i = x_i - \alpha_{i-1}, \quad i = 1,\cdots,n$. Tuning functions for $\hat{\theta}_i$:

$$\tau_{i,i} = \Gamma_i z_i\psi_i - \Gamma_i\sigma_{\theta_i}(\hat{\theta}_i - \theta_i^0), i = 1,\cdots,n$$

$$\tau_{i,k} = \tau_{i,k-1} + \Gamma_i z_k\sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_j}\psi_j, \quad i = 1,\cdots,n-1,\ k = i+1,\cdots,n$$

Tuning functions for $\hat{g}_i,\ i = 1,\cdots,n-1$:

$$\pi_{i,i} = \lambda_{\varrho_i}z_i z_{i+1} - \lambda_{\varrho_i}\sigma_{g_i}(\hat{g}_i - g_i^0),$$

$$\pi_{i,k} = \pi_{i,k-1} - \lambda_{\varrho_i}\frac{\partial\alpha_i}{\partial x_i}x_{i+1}z_k, \quad k = i+1,\cdots,n$$

Stabilizing functions:

$$\alpha_i = \hat{\varrho}_i\bar{\alpha}_i, \quad i = 1,\cdots,n-1$$

$$\bar{\alpha}_i = -k_i z_i - \hat{g}_{i-1}z_{i-1} - \hat{b}_i\hat{\phi}_i\tanh\left[\frac{z_i\hat{\phi}_i}{\epsilon_i}\right] - \hat{\theta}_i^T\psi_i + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\hat{g}_j x_{j+1} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\varrho}_j}\dot{\hat{\varrho}}_j + \sum_{m=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_m}\tau_{m,i} + \hat{\theta}_m^T\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j), i = 1,\cdots,n-1$$

$$\alpha_n = -k_n z_n - z_{n-1} - \hat{b}_n\hat{\phi}_n\tanh\left[\frac{z_n\hat{\phi}_n}{\epsilon_n}\right] - \hat{\theta}_n^T\psi_n + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\hat{g}_j x_{j+1} + \sum_{j=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1} + \frac{\partial\alpha_{n-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j) + \sum_{m=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_m}\tau_{m,n} + \hat{\theta}_m^T\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j)$$

Adaptive control law: $u = \frac{\hat{\varrho}_n}{\beta(x)}\alpha_n$. Parameters update laws:

$$\dot{\hat{\theta}}_i = \tau_{i,n}, \quad i = 1,\cdots,n$$

$$\dot{\hat{b}}_i = \lambda_i z_i\hat{\phi}_i\tanh\left[\frac{z_i\hat{\phi}_i}{\epsilon_i}\right] - \lambda_i\sigma_{b_i}(\hat{b}_i - b_i^0),$$

$$\dot{\hat{g}}_i = \pi_{i,n}\ i = 1,\cdots,n-1$$

$$\dot{\hat{\varrho}}_i = -\lambda_{\varrho_i}\mathrm{sgn}(g_i)\alpha_i z_i + \lambda_{\varrho_i}\sigma_{\varrho_i}(\hat{\varrho}_i - \varrho_i^0).$$

4. SIMULATION

Consider the regulation of the second-order system $\dot{x}_1 = g_1 x_2 + f_1(x_1) + \Delta_1(t,x)$, $\dot{x}_2 = g_2 u + f_2(x) + \Delta_2(t,x)$, $y = x_1$, where $x = [x_1, x_2]^T$, g_1, g_2 are unknown virtual control coefficients, but with known signs. We assume that $g_1 > 0, g_2 > 0$. $f_1(x), f_2(x)$ are unknown system functions, and $\Delta_1(t,x), \Delta_2(t,x)$ are unknown bounded disturbances. For simulation purpose, we let $f_1(x_1) = 0.1x_1^2$, $f_2(x) = 0.2e^{-x_2} + x_1\sin(x_2)$, $\Delta_1(t,x) = 0.6\sin(x_2)$, $\Delta_2(t,x) = 0.5(x_1^2 + x_2^2)\sin^3 t$, and $g_1 = 1$ and $g_2 = 1$. The bounds on Δ_1 and Δ_2 are $|\Delta_1(x,t)| \le p_1^*\phi_1(x_1)$, $|\Delta_2(x,t)| \le p_2^*\phi_2(x)$, where $p_1^* := 0.6$, $p_2^* := 0.5$, $\phi_1(x_1) = 1$, and $\phi_2(x) = x_1^2 + x_2^2$. We use RBF NNs to approximate $f_1(x_1), f_2(x)$, i.e., $f_1(x_1) = \theta_1^{*T}\psi_1(x_1) + \omega_1(x_1), f_2(x) = \theta_2^{*T}\psi_2(x) + \omega_2(x)$, where $|\omega_1| \le \delta_1^*, |\omega_2| \le \delta_2^*$. $b_1^* = \max\{\delta_1^*, p_1^*\}, b_2^* = \max\{\delta_1^*, \delta_1^*, p_1^*, p_2^*\}$. For the design of robust adaptive controller, let $\hat{\theta}_1, \hat{\theta}_2, \hat{b}_1, \hat{b}_2, \hat{g}_1, \hat{\varrho}_1$, and $\hat{\varrho}_2$ be the estimates of unknown parameters $\theta_1^*, \theta_2^*, b_1^*, b_2^*, g_1, \varrho_1 = \frac{1}{g_1}$, and $\varrho_2 = \frac{1}{g_2}$, and $z_1 = x_1, z_2 = x_2 - \alpha_1$. The following initial conditions and controller design parameters are adopted in the simulation: $x(0) = [1,1]^T, \hat{\theta}_1(0) = 0, \hat{\theta}_2(0) = 0, \hat{b}_1(0) = 0, \hat{b}_2(0) = 0, \hat{g}_1(0) = 0, \hat{\varrho}_1(0) = 0$, $\hat{\varrho}_2(0) = 0$, and $k_1 = k_2 = 2$, $\Gamma_1 = \Gamma_2 = 10$, $\lambda_1 = \lambda_2 = \lambda_{\varrho_1} = \lambda_{\varrho_2} = 1$, $\sigma_{\theta_1} = \sigma_{\theta_2} = \sigma_{\varrho_1} = \sigma_{\varrho_2} = \sigma_{b_1} = \sigma_{b_1} = 0.1$, $\epsilon_1 = \epsilon_2 = 0.1$, $\theta_1^0 = \theta_2^0 = 0$, and $b_1^0 = b_2^0 = \varrho_1^0 = \varrho_2^0 = g_1^0 = 0.1$.

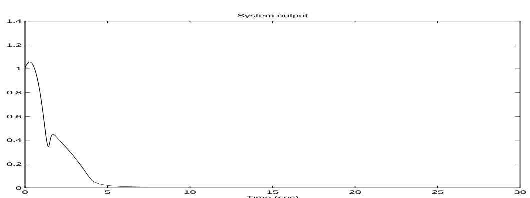

Fig. 1. Output y

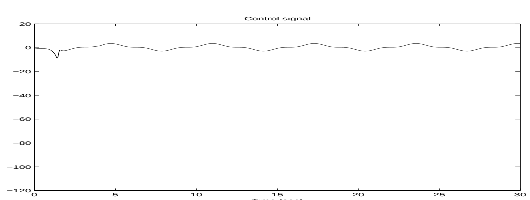

Fig. 2. Control input u

5. CONCLUSION

In this paper, a robust adaptive control approach for a class of uncertain semi-strict feedback nonlinear systems with unknown virtual control coefficients has been presented. Simulation results have shown the effectiveness of the proposed method.

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