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ESTIMATION OF THE WATER-MASS-FLOW THROUGH A CENTRAL HEATING BOILER

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Abstract: Conventional central heating systems in German buildings are hot-water heating systems. These systems consist of a boiler and radiators or heating coils in the floor. A heating characteristic maps the current outdoor temperature to the set-point of the boiler outlet-temperature. Conventional PI-controllers with fixed parameters adjust the boiler outlet-temperature to the set-point. Since the process of heating water in the boiler is nonlinearly dependent on the water-mass-flow, knowledge of the flow-rate could improve the quality of control. This paper presents two different observer-based schemes for the estimation of the water flow rate through a central heating boiler. One is a disturbance observer and the other is a parameter observer.

Keywords: observer, central heating boiler, mass flow, estimation

1. INTRODUCTION

Almost all of the residential buildings in Germany have central heating systems. The task of these heating systems is to heat the living quarters in order to achieve an adequate room temperature during the cold winter months (Recknagel *et al.*, 1999).

Almost 40 % of the primary energy consumption is due to building heating. The need for a decrease in the consumption of valuable fossil energy and exhaust emissions has sparked many efforts to optimize heating systems.

Most of the central heating systems use hot water in combination with radiators or floor heating coils to heat the rooms of the building. These so-called hot-water heating systems use boilers to heat water. Each building has a unique heating characteristic due to factors like e.g. heating insulation of the building. In order to reach and keep a desired room-temperature the heating system must make up for the heat losses of the building by heating the water in the boiler to a certain temperature.

The required water temperature mainly depends on the current outdoor temperature. The heating characteristic as part of the temperature controller of the boiler maps the outdoor temperature to the water temperature set-point of the boiler. In modern boilers a PI-controller then adjusts the outlet temperature according to the set-point. Fig. 1 illustrates the schematics of the described central heating system. The outlet temperature does not only depend on the amount of heat energy generated by the boiler. The mass-flow rate also influences the outlet temperature of the boiler. Since the parameters of the PI-controller are fixed this may lead to unsatisfactory quality of the temperature control. A better control can be achieved if the parameters of the feedback controller are adjusted according to the current mass-flow rate. Unfortunately measurements of the mass-flow rate are not available in conventional heating systems. This is due to the fact that mass-flow rate sensors are relatively expensive, have measurement uncertainties of approximately ±10%

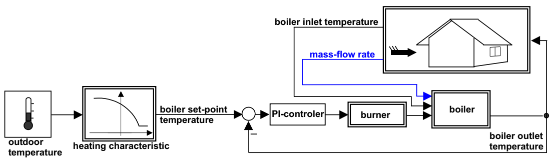

Fig. 1. Control of a central heating system

(Pfannstiel, 1991) and that corrosion influences the long time measurement quality.

The problem of certain quantities not being accessible to measurement because of restrictions to cost, quality etc. is not an uncommon problem in other fields of application, too. Several techniques have been developed to overcome this problem. These techniques use estimates of the unknown or unmeasurable quantities. Mathematical process models are the basis for different kinds of estimation schemes. (Tsai *et al.*, 2000), (Powell *et al.*, 1998), (Moskwa and Pan, 1995), (Umeno *et al.*, 2001) and (Medromi *et al.*, 1994) all use observers to calculate estimations of quantities difficult to measure and use these estimates to improve the control quality.

The purpose of this paper is to show how a simple mathematical model of the boiler may be used to estimate the mass-flow rate. The estimate can then be used to adjust the parameters of the feedback controller and therefore improve the control quality. The paper is organized as follows. First, section 2 describes the mathematical model of the heating boiler used for estimating the flow rate. In section 3 the two observer based estimation schemes are described and some simulation results are given. Finally section 4 compares the estimation schemes regarding the quality of the estimation and the possible implementation in existing microcontrollers for boiler control purposes.

2. MATHEMATICAL MODEL OF THE CENTRAL HEATING BOILER

2.1 *Description of the boiler*

The boiler under investigation is a standard low temperature central heating boiler from the *Viessmann* company, a big German manufacturer of building heating equipment. It has an output power of approximately 22 kW. Fig. 2 shows the schematics of the boiler's cross-section.

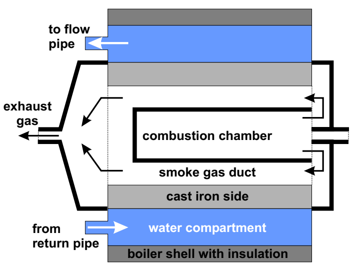

Fig. 2. Cross section of the boiler

A premix burner ignites a mixture of natural gas and air. The resulting smoke gas passes through the smoke gas duct and leaves the boiler as exhaust gas through the chimney. The design of the cast iron shell is such that a condensation of the smoke gas is prohibited and thus decreases the danger of corrosion of the iron wall. This improves the boiler's durability.

The hot smoke gas heats the cast iron shell of the boiler. The heat passes through the iron and is passed on to the water in the boiler's water compartment. Depending on the duration of the premix burner running time and the water flow rate, the water leaving the boiler reaches a certain temperature level.

This outlet temperature is regulated according to the outdoor temperature dependent set-point which is specified by the building's heating characteristic. For regulation purposes measurements of the outdoor temperature and the boiler outlet temperature are available. The regulator adjusts the premix burner running time according to the error signal. The regulator parameters are fixed and thus the regulator can not adjust to different water flow rates which influence the outlet temperature. The following section gives a brief description of the simple mathematical boiler model which is the basis for an estimation of the water flow rate.

2.2 *Simplified mathematical boiler-model*

This section describes a simple mathematical model of the boiler. The model describes the boiler as a pipe. Thermal energy from the premix burner passes through the pipe wall and heats water flowing in the pipe. For estimating the water flow rate $\dot{m}$ through the boiler this model is sufficient. Fig. 3 shows a schematic of the simplified boiler-model.

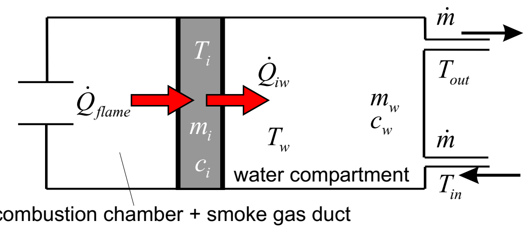

Fig. 3. Cross section of the simplified central heating furnace

The simplified boiler consists of two parts: a combustion chamber with a smoke gas duct and a water compartment. An iron cast shell surrounds the water compartment and separates it from the combustion chamber. The premix burner flame generates energy which results in a heat flow rate $\dot{Q}_{flame}$. The convection heat transfer from the hot gas to the iron cast shell of mass m_i, heat capacity c_i and surface area A_i is responsible for bringing the iron cast shell to the temperature T_i. Eq. 1 and Eq. 2 describe the dynamics of the iron cast wall.

$$m_i \cdot c_i \cdot \frac{dT_i}{dt} = \dot{Q}_{flame} - \dot{Q}_{iw} \tag{1}$$

$$\dot{Q}_{iw}(t) = A_i \cdot h_{iw} \cdot (T_i - T_w) \tag{2}$$

If the iron cast shell temperature T_i is higher than the water temperature T_w a convection heat transfer from the shell to the water starts. The convection heat transfer coefficient is h_{iw}.

The heat flow rate $\dot{Q}_{iw}$ is responsible for heating water of mass m_w and heat capacity c_w in the water compartment to the temperature T_w. The dynamic behavior of the water compartment is described by Eq. 3 in a similar way as that of the iron cast shell. The energy stored in the water is computed by balancing heat gains and heat losses. Heat gains are the heat flow from the wall to the water $\dot{Q}_{iw}$ and the heat flow from the returning water $\dot{Q}_{in}$. Heat losses consist of the heat flow out of the boiler to the building's rooms $\dot{Q}_{out}$. Additional losses due to bad insulation is neglected. Eq. 4 and Eq. 5 describe the heat transfer to and from the boiler's water compartment

$$m_w \cdot c_w \frac{T_w(t)}{dt} = \dot{Q}_{iw}(t) + \dot{Q}_{in}(t) - \dot{Q}_{out}(t) \quad (3)$$

$$\dot{Q}_{in} = \dot{m} \cdot c_w T_{in} \quad (4)$$

$$\dot{Q}_{out} = \dot{m} \cdot c_w T_{out} \quad (5)$$

Calculation of the Laplace transform of Eq. 3, solving for $\dot{Q}_{iw}$ and substituting the result in Eq. 1 we get the transfer function given in Eq. 6 for the outlet temperature T_{out}. It is assumed that the water temperature T_w is equal to the outlet temperature T_{out}.

$$T_{out} = G_1(s) \cdot \dot{Q}_{flame} + G_2(s) \cdot T_{in} \quad (6)$$

with

$$G_1(s) = \frac{\frac{1}{\dot{m}c_w}}{\frac{m_w m_i c_i}{\dot{m} h_{iw} A_i} s^2 + \left(\frac{m_w}{\dot{m}} + \frac{m_i c_i}{\dot{m} c_w} + \frac{m_i c_i}{h_{iw} A_i}\right) s + 1} \quad (7)$$

and

$$G_2(s) = \frac{\frac{m_i c_i}{h_{iw} A_i} s + 1}{\frac{m_w m_i c_i}{\dot{m} h_{iw} A_i} s^2 + \left(\frac{m_w}{\dot{m}} + \frac{m_i c_i}{\dot{m} c_w} + \frac{m_i c_i}{h_{iw} A_i}\right) s + 1} \quad (8)$$

For observing the water mass flow $\dot{m}$ a state space representation is more suitable.

$$\underline{\dot{x}} = \mathbf{A} \cdot \underline{x} + \mathbf{B} \cdot \underline{u} \quad (9)$$

$$T_{out} = \underline{c}^T \cdot \underline{x} \quad (10)$$

with

$$\underline{x}^T = \begin{pmatrix} T_{out} & T_i \end{pmatrix} \quad (11)$$

$$\underline{u}^T = \begin{pmatrix} \dot{Q}_{flame} & T_{in} \end{pmatrix} \quad (12)$$

$$\mathbf{A} = \begin{pmatrix} -\frac{h_{iw}A_i}{m_w c_w} - \frac{\dot{m}}{m_w} & \frac{h_{iw}A_i}{m_w c_w} \\ \frac{h_{iw}A_i}{m_i c_i} & -\frac{h_{iw}A_i}{m_i c_i} \end{pmatrix} \quad (13)$$

$$\mathbf{B} = \begin{pmatrix} 0 & \frac{\dot{m}}{m_w} \\ \frac{1}{m_i c_i} & 0 \end{pmatrix} \quad (14)$$

$$\underline{c}^T = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (15)$$

Both representations of the boiler model show that T_{out} depends not only on the inlet temperature T_{in} and the heat flow of the flame $\dot{Q}_{flame}$ but also on the water mass flow $\dot{m}$. Furthermore, it is clear that it is a non-linear dependency. For controlling or predicting T_{out} an estimate of $\dot{m}$ is necessary. Since this quantity is not being measured for reasons mentioned in section 1 estimating it is the simplest was of accessing this important quantity. The following section will describe the observer-based estimation schemes investigated.

3. MODEL-BASED ESTIMATION OF THE WATER-MASS-FLOW

This section investigates two different approaches to observing the water flow rate through the boiler. The observer schemes are based on the simple model described in section 2.2 and make use of the special design of the system matrix $\mathbf{A}$. One possible approach is the design of a disturbance observer which is described in section 3.1. This observer is based on rewriting the equations of the state space model. The water flow rate can then be seen as a disturbance and hence a disturbance observer like the one described in section 3.1 seems to be appropriate. The second observer applied to the problem is based on the fact that the unknown water flow rate $\dot{m}$ influences T_{out}. The prediction error is fed back and leads to a correction factor representing $\dot{m}$. This observer schemes is described in section 3.2.

3.1 *Disturbance observer*

Looking at the system matrix $\mathbf{A}$ and the input matrix $\mathbf{B}$ in Eq. 13 and Eq. 14 one finds that the unknown water flow rate $\dot{m}$ only appears in the elements $a_{11} = -\frac{h_{iw}A_i}{m_w c_w} - \frac{\dot{m}}{m_w}$ and $b_{12} = \frac{\dot{m}}{m_w}$. Separation of each of these two matrices into two submatrices $\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2$ and $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$ leads to the state space Eq. 16. The matrix $\mathbf{E}$ and the disturbance z are derived from Eq. 9 by simplifying the expression $\mathbf{A}_2 \cdot \underline{x} + \mathbf{B}_2 \cdot \underline{u}$ and combining the elements of $\underline{x}$ and $\underline{u}$ into z.

$$\underline{\dot{x}} = \mathbf{A}_1 \cdot \underline{x} + \mathbf{B}_1 \cdot \underline{u} + \mathbf{E} \cdot z \quad (16)$$

with

$$\mathbf{A}_1 = \begin{pmatrix} -\frac{h_{iw}A_i}{m_w c_w} & \frac{h_{iw}A_i}{m_w c_w} \\ \frac{h_{iw}A_i}{m_i c_i} & -\frac{h_{iw}A_i}{m_i c_i} \end{pmatrix}, \quad (17)$$

$$\mathbf{B}_1 = \begin{pmatrix} 0 & 0 \\ \frac{1}{m_i c_i} & 0 \end{pmatrix}, \quad (18)$$

$$\mathbf{E} = \begin{pmatrix} -\frac{1}{m_w} \\ 0 \end{pmatrix} \quad (19)$$

and

$$z = (T_{out} - T_{in}) \cdot \dot{m} = \Delta T \cdot \dot{m} \quad (20)$$

This new formulation of the boiler's model can be interpreted as a state space model with an additional disturbance z. (White *et al.*, 2000) and (Iwasaki *et al.*, 1999) e.g. use disturbance observers for improving track following in magnetic disk drives and nonlinear friction compensation respectively. In both cases the disturbance observer computes estimates for nonmeasurable quantities which are necessary for improving control quality. Therefore, a disturbance observer seems to be appropriate for the task of estimating $\dot{m}$ of the boiler. The design of the disturbance observer is done according to (Föllinger, 1994) and (Johnson, 1971).

Since the disturbance z is a nonmeasurable quantity one has to define a model of the disturbance signal. It is important to mention that this model is not a physical one. It only describes the behavior of the disturbance signal with respect to time. In our case it is reasonable to model the disturbance as a homogenous differential equation for a first order linear system (PT_1) with a time constant T of approximately 7 s.

$$T \cdot \dot{z} + z = 0 \tag{21}$$

or in state space form

$$\begin{aligned} \underline{\dot{x}}_s &= \mathbf{A}_s \underline{x}_s = -\frac{1}{T} x_s \\ z &= \mathbf{C}_s \underline{x}_s = x_s \end{aligned} \tag{22}$$

Combining the process model and the disturbance model leads to an extended process model:

$$\begin{bmatrix} \underline{\dot{x}} \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{E} \cdot \mathbf{C}_s \\ 0 & \mathbf{A}_s \end{bmatrix} \cdot \begin{bmatrix} \underline{x} \\ x_s \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ 0 \end{bmatrix} \cdot \underline{u}, \tag{23}$$

$$T_{out} = \begin{bmatrix} \underline{c}^T 0 \end{bmatrix} \cdot \begin{bmatrix} \underline{x} \\ x_s \end{bmatrix}. \tag{24}$$

The design of an observer for this extended model leads to two separate observers: a Luenberger observer for $\underline{x}$ and a disturbance observer for z. Since z does not directly represent $\dot{m}$ some additional computation is needed. Extraxting $\dot{m}$ can either be done by rewriting Eq. 20 ($\dot{m} = \frac{z}{\Delta T}$) or by using one of state equations.

$$\dot{m} = m_w \frac{a_{11}\hat{x}_1 + a_{12}\hat{x}_2 - \dot{\hat{x}}_1}{\Delta T} \tag{25}$$

Although the disturbance z is no longer present in the above equation it is nevertheless necessary for computing $\dot{m}$ because $\underline{\hat{x}}$ would otherwise not be correctly computed.

3.2 *Parameter observer*

This section deals with another kind of observer scheme for estimating $\dot{m}$. In contrast to section 3.1 the unknown water flow rate is not treated as a state of the (extended) model. This approach uses the prediction error $e = T_{out} - \hat{T}_{out}$ to compute an estimate of $\dot{m}$.

A change in the outlet temperature is due to changes of

- the flame's heat flow rate,
- the inlet temperature or
- the water flow rate.

Since the first two quantities are known it is possible to model their effect on the outlet temperature T_{out} as shown in section 2. Therefore, a false estimate for T_{out} must originate from a change in $\dot{m}$. The outlet temperature prediction error $e = T_{out} - \hat{T}_{out}$ is a measure for the unknown water flow rate $\dot{m}$. By feeding this error back to the model as shown in Fig. 4 and using it to enhance the prediction quality it is possible to compute an estimate for $\dot{m}$. The equations for the boiler model are now:

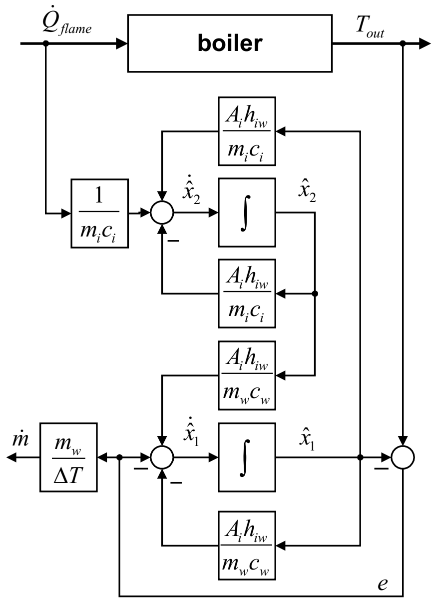

Fig. 4. Estimating $\dot{m}$ with a parameter observer.

$$\dot{x}_1 = -\frac{h_{iw}A_i}{m_w c_w} \cdot x_1 + \frac{h_{iw}A_i}{m_w c_w} \cdot x_2 - e \tag{26}$$

$$\dot{x}_2 = \frac{h_{iw}A_i}{m_i c_i} \cdot x_1 - \frac{h_{iw}A_i}{m_i c_i} \cdot x_2 + \frac{1}{m_i c_i} \cdot \dot{Q}_{flame} \tag{27}$$

with the error signal

$$e = \frac{\dot{m}}{m_w} \cdot \left(x_1 - u_2\right) = \frac{\dot{m}}{m_w} \cdot \Delta T \tag{28}$$

Taking a look at the above equation it is clear that the prediction error e is related to the water flow rate $\dot{m}$. By adapting the model's estimate for the outlet temperature to the measured one it is possible to get an estimate for the unknown variable $\dot{m}$.

4. COMPARISON OF THE ESTIMATION SCHEMES

The results of the comparison are based on work done by (Rückbrodt, 2000). In this work the boiler model and the observers were implemented in a *Matlab/Simulink*© simulation environment. Fig. 5 shows this *Simulink* simulation environment. Since a test stand with the required measurement equipment was

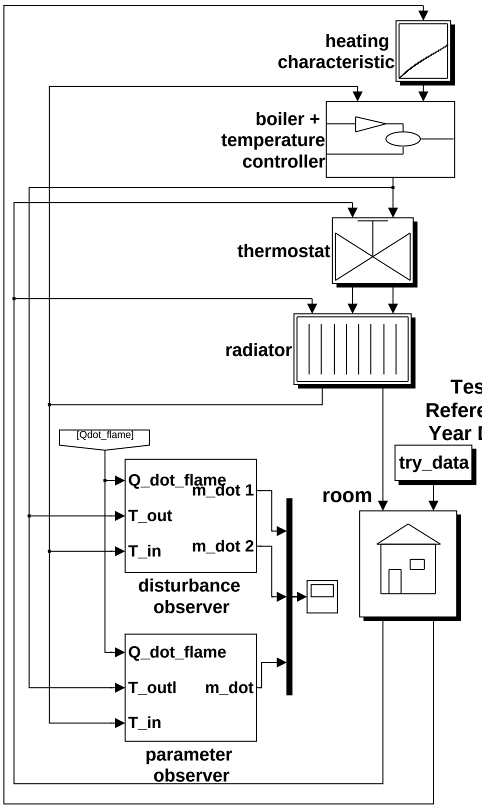

Fig. 5. Simulink model of the $\dot{m}$-observer

not available during the time the validation and comparison of the observer schemes was done by simulations. The simulation environment contains models of a single room house, a radiator with thermostat-valve, the boiler and a temperature controller. These models were previously verified with experimental data. Data from a *test reference year* contain environmental influences. The technical data of the boiler necessary for the simulations was taken from a data sheet provided by the boiler manufacturer. Tab. 1 shows the technical data together with some material properties of water and the iron cast shell. Fig. 6 and Fig. 7 show that both observer schemes are capable of estimating the water flow rate $\dot{m}$. The estimates of the disturbance observer obtained with Eq. 25 are slightly better than those obtained with Eq. 20. Both estimates correctly reproduce the dynamic behavior of $\dot{m}$ with an offset-error of about 8% for the estimation with Eq. 20. A step-like change in the water flow rate results in a sharp increase of the estimation errors but it quickly returns to zero and 8% in both cases. A similar result is obtained with the parameter observer. The difference between the disturbance observer's estimation of $\dot{m}$ and that of the parameter observer lies in the work

Table 1. Parameters of the simulation

BOILER TECHNICAL DATA			
$\dot{Q}_{flame}$	m_w	A_i	m_i
22 kW	88 kg	2.5 m^2	152 kg
MATERIAL PROPERTIES			
c_w	h_{iw}	c_i	ρ_w
4180 $\frac{J}{kgK}$	900 $\frac{W}{m^2K}$	477 $\frac{J}{kgK}$	1000 $\frac{kg}{m^3}$

necessary for designing the two observers. The design of the parameter observer is much simpler.

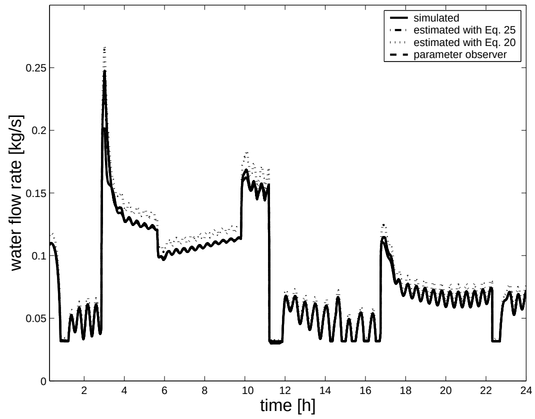

Fig. 6. Comparison of the results for the observer-based mass flow estimation-schemes

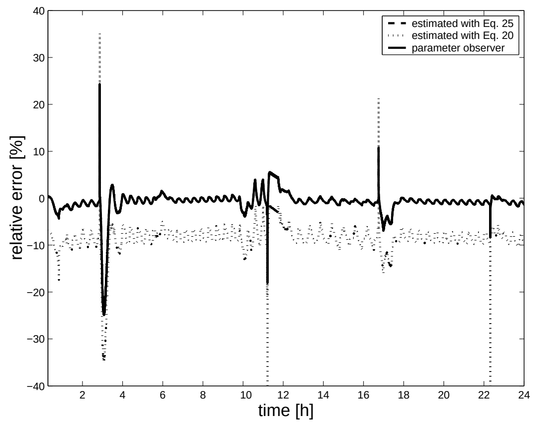

Fig. 7. Relative estimation-errors of the the mass flow observers

5. CONCLUDING REMARKS

This paper has shown that a simple mathematical model of a boiler can be used to estimate an important yet nonmeasurable variable of a central heating system. The rate at which water flows through the boiler has a significant influence on the boiler's capability to transfer thermal energy on to the water. Estimating the water flow rate with an observer structure can improve the temperature-control-quality since the parameters of the controller can now be adapted according to the water flow rate. Simulations have shown that both the disturbance and the parameter observer are capable to give fairly good estimates of the water flow rate. The next step will be the practical implementation of the observers and to show their capabilities in a test stand which is being built at the moment.

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