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# ROBUST DECENTRALIZED RELIABLE CONTROL FOR UNCERTAIN INTERCONNECTED DELAYED SYSTEMS

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Abstract: The problem of robust decentralized reliable control for a class of uncertain interconnected delayed systems through state feedback and dynamic output feedback is investigated in this paper. Based on Lypunov stability theory, a decentralized observer and a linear memoryless state feedback decentralized controller are designed such that for all admissible uncertainties as well as actuator failures occurring among a preassigned subset of actuators, the closed loop system is asymptotically stable. The conclusions extend and improve some results in the literature. The emulation is given to illustrate the validity of the obtained results. *Copyright © 2002 IFAC*

Keywords: Uncertain interconnected delayed system; Dynamic output feedback; Decentralized reliable control

## 1. INTRODUCTION [1]

Recently, the problem of robust reliable control for the uncertain delayed systems was studied in many empirical papers (see, for example, Wang, *et al.*, 1999, Zhao, and Jiang, 1998, Wang, *et al.,* 1996, Seo, and kim, 1996). Most of these papers discussed the questions of centralized control. In allusion to the interconnected systems, Hu, *et al.,* (1996) gave us a method to design a decentralized controller when all of the actuators of one subsystem failure. In Xu (2000), the problem of state feedback and dynamic output feedback decentralized stabilization was discussed for partly actuator failures occurring among a preassigned subset of actuators, but the input matrix of the system had no uncertainties.

In this paper, a class of interconnected systems is discussed, which not only has state delayed and input delayed term but also has uncertainties in the state matrix and input matrix. Based on the Riccati equation method, a sufficient condition of state feedback and dynamic output feedback is obtained for reliable decentralized stabilization. This method is more flexible than Su, *et al.* (1999) in the way of adjusting and choosing parameters. At the same time, the model of the system in this paper is more general. In section 2, it is shown the description of the system, some necessary assumption, and the questions that will be resolved in the paper. In section 3 and section 4, the state feedback controllers and the dynamic feedback controllers are designed. An emulation is given in section 5 and in section 6 the final conclusions are drawn.

## 2. SYSTEM DESCRIPTION AND PROBLEMS

Consider the following nonlinear uncertain interconnected delayed system:

$$\begin{aligned}\dot{x}_i(t) &= (A_i + \Delta A_i(t))x_i(t) \\ &\quad + (A_{id} + \Delta A_{id}(t))x_i(t - d_{i1}) \\ &\quad + (B_i + \Delta B_i(t))u_i(t) \\ &\quad + (B_{id} + \Delta B_{id}(t))u_i(t - d_{i2}) \\ &\quad + f_i(x_i, t) + \sum_{j=1, j\neq i}^{m} A_{ij} x_j(t - h_{ij}) \qquad (1) \\ y_i(t) &= C_i x_i(t)\end{aligned}$$

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$$x_i(t)=\phi_i(t) \quad t\in[-d_i,0] \quad i=1,2,\cdots,m$$

Where $x_i(t)\in R^{n_i}, u_i(t)\in R^{q_i}, y_i(t)\in R^{q_i}$ are the state vector, control input vector and output vector of the ith subsystems at time t. $A_i, A_{id}, B_i, B_{id}$ and $A_{ij}$ are the known delayed constant matrixes which have proper dimensions. $\Delta A_i, \Delta A_{id}, \Delta B_i, \Delta B_{id}$ are the time-varying uncertain matrixes of the ith subsystem. $f_i(x_i,t)$ is uncertain vector space, it shows the structure uncertainty of the ith subsystem. $d_{i1}, d_{i2}, h_{ij}(>0)$ are delayed constants. $d=\max\{d_{i1}, d_{i2}, h_{ij}\}$. $\phi_i(t)$ is the continual state vector initial function.

Assumption 1 The parameter uncertainties in system (1) have following form:

$$\Delta B_i(t)=H_iF_i(t)E_i, \Delta B_{id}(t)=H_{id}F_{id}(t)E_{id}$$
$$\Delta A_i(t)=M_iF_i(t)N_i, \Delta A_{id}(t)=M_{id}F_{id}(t)N_{id} \quad (2)$$

Where $H_i, H_{id}, M_i, M_{id}, E_i, E_{id}, N_i, N_{id}$, are constant matrixes with proper dimensions. $F_i(t), F_{id}(t)$ are unknown function matrixes which satisfy the inequalities:

$$F_i^T(t)F_i(t)\le I, F_{id}^T(t)F_{id}(t)\le I \quad i=1,2,\cdots,m \quad (3)$$

Assumption 2 The nonlinear uncertainty $f_i(x_i,t)$ in system (1) satisfies the condition

$$\|f_i(x_i,t)\|\le\|G_ix_i\| \quad i=1,2,\cdots,m \quad (4)$$

$G_i$ is constant matrix with proper dimension.

Here, suppose the set of the admissible failure actuators in the ith subsystem can be denoted as $\Omega_i\subseteq\{1,2,\cdots,q_i\}$, it means the actuators in $\Omega_i$ are redundant for the system stability, but they can improve the properties of the system.

Problem:

(1) For system (1)-(4), design decentralized linear memoryless state feedback controllers:

$$u_i=K_ix_i \quad i=1,2,\cdots,m \quad (5)$$

which make the close loop system of system (1) asymptotically stable for all admissible uncertainties as well as actuator failures occurring among $\Omega_i$.

(2) Design decentralized dynamic output feedback controllers:

$$u_i(t)=-K_i\hat{x}_i(t) \quad (6)$$

$$\dot{\hat{x}}_i(t)=A_i\hat{x}_i(t)+A_{id}\hat{x}_i(t-d_{i1}) + B_iu_i(t)+B_{id}u_i(t-d_{i2}) + L_i(y_i(t)-C_i\hat{x}_i(t))+\sum_{j\ne i}^{m}A_{ij}\hat{x}_j(t-h_{ij}) \quad (7)$$

which make the close loop system of system (1) asymptotically stable for all admissible uncertainties as well as actuator failures occurring among $\Omega_i$.

## 3. DESIGNING OF DECENTRALIZED LINEAR MEMORYLESS STATE FEEDBACK RELEABLE CONTROLLERS

(1) Consider that there are no actuator failures ($\Omega_i=\Phi$)

Theorem 1 If there are positive constants $\varepsilon_{i1}, \varepsilon_{i2},, \varepsilon_{i3}, \varepsilon_{i4}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}$ and matrix $K_i$, such that the following inequality has a positive definite solution matrix $P_i$, then the controller (5) can make the close loop system of system (1) asymptotically stable.

$$P_i(A_i+B_iK_i)+(A_i+B_iK_i)^TP_i + P_iR_iP_i+Q_i<0 \quad i=1,2,\cdots,m \quad (8)$$

In above inequality the parameters are as following:

$$R_i=\varepsilon_{i1}M_iM_i^T+\gamma_{i1}H_iH_i^T+\varepsilon_{i2}A_{id}A_{id}^T + \varepsilon_{i3}M_{id}M_{id}^T+\gamma_{i2}B_{id}B_{id}^T+\gamma_{i3}H_{id}H_{id}^T + \varepsilon_{i4}I+\sum_{j\ne i}^{m}A_{ij}A_{ij}^T \quad (9)$$

$$Q_i=\varepsilon_{i1}^{-1}N_i^TN_i+(\varepsilon_{i2}^{-1}+m-1)I + \varepsilon_{i3}^{-1}N_{id}^TN_{id}+\gamma_{i1}^{-1}K_i^TE_i^TE_iK_i + \varepsilon_{i4}^{-1}G^TG+\gamma_{i2}^{-1}K_i^TK_i+\gamma_{i3}^{-1}K_i^TE_{id}^TE_{id}K_i \quad (10)$$

Proof: It similar to the proof of theorem 5.1.1 in Xu (2000).

The distinguishing lies in: denotes

$$S_{i1}=\varepsilon_{i2}^{-1}I+\varepsilon_{i3}^{-1}N_{id}^TN_{id}$$
$$S_{i2}=\gamma_{i2}^{-1}K_i^TK_i+\gamma_{i3}^{-1}K_i^TE_{id}^TE_{id}K_i \quad (11)$$

In $\dot{V}(x)$, there are questions how to deal with $\Delta B_i, \Delta B_{id}$.

$$2x_i^T(t)P_i\Delta B_iK_ix_i(t) = 2x_i^T(t)P_iH_iF_i(t)E_iK_ix_i(t) \le \gamma_{i1}x_i^T(t)P_iH_iH_i^TP_ix_i(t) + \gamma_{i1}^{-1}x_i^T(t)K_i^TE_i^TE_iK_ix_i(t) \quad (12)$$

$$2x_i^T(t)P_i\Delta B_{id}K_ix_i(t-d_{i2}) = 2x_i^T(t)P_iH_{id}F_{id}(t)E_{id}K_ix_i(t-d_{i2}) \le \gamma_{i3}x_i^T(t)P_iH_{id}H_{id}^TP_ix_i(t) + \gamma_{i3}^{-1}x_i^T(t-d_{i2})K_i^TE_{id}^TE_{id}K_ix_i(t-d_{i2}) \quad (13)$$

Thus the conclusion can be drawn by using (8), (9), (10).

Corollary 1 Design linear memoryless state feedback decentralized controller for system (1) as following:

$$u_i(t) = K_i x_i(t)$$

$$K_i = -[\gamma_{i1}^{-1} E_i^T E_i + \gamma_{i2}^{-1} I + \gamma_{i3}^{-1} E_{id}^T E_{id}]^{-1} B_i^T P_i \quad i = 1,2,\cdots,m \tag{14}$$

Proof: Conclusion of theorem 1 can be made good use of to prove this corollary.

Remark: corollary 1 gives us the method to design the controllers.

(2) Consider that partly of the actuators fail ($\Omega_i \neq \Phi$)

Suppose the corresponding input is zero while the actuator failures, then when $W_i \subseteq \Omega_i$ is satisfied, the affection of the actuator failures to the input can be shown as a switched matrix $E_{w_i} = diag(e_{i1}, e_{i2}, \cdots, e_{iq_i})$, which will be put between the input matrix $B_i$ and the input gain matrix $K_i$, where

$$e_{ik} = 1, i = 1,2,\cdots,m$$

The kth actuator of the ith subsystem doesn't fail

$$e_{ik} = 0, k = 1,2,\cdots,q$$

The kth actuator of the ith subsystem fails

(15)

The fault close loop system can be rewritten as:

$$\begin{aligned}\dot{x}_i(t) = {} & [A_i + M_i F_i(t) N_i + B_i E_{w_i} K_i \\ & + H_i F_i(t) E_i E_{w_i} K_i] x_i(t) \\ & + [A_{id} + M_{id} F_{id}(t) N_{id}] x_i(t - d_{i1}) \\ & + [B_{id} E_{w_i} K_i + H_{id} F_{id}(t) E_{id} \\ & \quad E_{w_i} K_i] x_i(t - d_{i2}) \\ & + \sum_{j \neq i}^{m} A_{ij} x_j(t - h_{ij}) \qquad i = 1,2,\cdots,m\end{aligned} \tag{16}$$

Theorem 2 If there exist positive constants $\varepsilon_{i1}, \varepsilon_{i2}, , \varepsilon_{i3}, \varepsilon_{i4}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}$ ,such that the following Riccati inequality has symmetric positive definite solution matrix $\widetilde{P}_i$,

$$\widetilde{P}_i A_i + A_i^T \widetilde{P}_i + \widetilde{P}_i \widetilde{R}_i \widetilde{P}_i + \widetilde{Q}_i < 0 \tag{17}$$

$$\begin{aligned}\widetilde{R}_i = {} & \varepsilon_{i1} M_i M_i^T + \gamma_{i1} H_i H_i^T \\ & + \varepsilon_{i2} A_{id} A_{id}^T + \varepsilon_{i3} M_{id} M_{id}^T \\ & + \gamma_{i2} B_{id} B_{id}^T + \gamma_{i3} H_{id} H_{id}^T \\ & + \varepsilon_{i4} I + \sum_{j \neq i}^{m} A_{ij} A_{ij}^T \\ & - B_i [\gamma_{i1}^{-1} E_i^T E_i + \gamma_{i2}^{-1} I \\ & + \gamma_{i3}^{-1} E_{id}^T E_{id}]^{-1} E_{\Omega_i} B_i^T\end{aligned} \tag{18}$$

$$\begin{aligned}\widetilde{Q}_i = {} & \varepsilon_{i1}^{-1} N_i^T N_i + (\varepsilon_{i2}^{-1} + m - 1) I \\ & + \varepsilon_{i3}^{-1} N_{id}^T N_{id} + \varepsilon_{i4}^{-1} G^T G\end{aligned} \tag{19}$$

Then, the fault close loop system (15) is asymptotically stable for arbitrary $W_i \subseteq \Omega_i$ and parameter uncertainties by the linear memeoryless state feedback decentralized controller:

$$u_i(t) = K_i x_i(t)$$

$$K_i = -[\gamma_{i1}^{-1} E_i^T E_i + \gamma_{i2}^{-1} I + \gamma_{i3}^{-1} E_{id}^T E_{id}]^{-1} B_i^T \widetilde{P}_i \tag{20}$$

Proof: It is similar to theorem 1 and corollary 1, here,

$$\begin{aligned}s_{i2} = {} & \gamma_{i2}^{-1} \widetilde{P}_i B_i [\gamma_{i1}^{-1} E_i^T E_i + \gamma_{i2}^{-1} I \\ & + \gamma_{i3}^{-1} E_{id}^T E_{id}]^{-1} E_{\Omega_i} B_i \widetilde{P}_i \\ & + \gamma_{i3}^{-1} E_{id}^T \widetilde{P}_i B_i [\gamma_{i1}^{-1} E_i^T E_i + \gamma_{i2}^{-1} I \\ & + \gamma_{i3}^{-1} E_{id}^T E_{id}]^{-1} E_{\Omega_i} B_i \widetilde{P}_i E_{id}\end{aligned} \tag{21}$$

and $E_{W_i}^T E_{W_i} = E_{W_i}$, $E_{W_i} - E_{\Omega_i}$ are semi-positive definite matrixes.

## 4. DESIGNING OF DECENTRALIZED DYNAMIC OUTPUT FEEDBACK CONTROLLERS

(1) $\Omega_i = \Phi$

Denotes $e_i(t) = x_i(t) - \hat{x}_i(t)$, the error equation can be transformed into the following form:

$$\begin{aligned}\dot{e}_i(t) = {} & (A_i - L_i C_i) e_i(t) \\ & + A_{id} e_i(t - d_{i1}) + \Delta A_i x_i(t) \\ & + \Delta A_{id} x_i(t - d_{i1}) + \Delta B_i u_i(t) \\ & + \Delta B_{id} u_i(t - d_{i2}) + f_i(x_i, t) \\ & + \sum_{j \neq i}^{m} A_{ij} x_j(t - h_{ij})\end{aligned} \tag{22}$$

Theorem 3 If there exist $\varepsilon_{ik} (k = 1,2,3,4)$, $\gamma_{is} (s = 1,2,\cdots,6)$ and matrixes $L_i, K_i$, such that the following Riccati inequalities have symmetric answer matrixes $P_{ic}, P_{i0}$, where

$$\begin{aligned}P_{ic}(A_i - B_i K_i) + (A_i - B_i K_i)^T P_{ic} \\ + P_{ic} R_{ic} P_{ic} + Q_{ic} < 0\end{aligned} \tag{23}$$

$$\begin{aligned}P_{i0}(A_i - L_i C_i) + (A_i - L_i C_i)^T P_{i0} \\ + P_{i0} R_{i0} P_{i0} + Q_{i0} < 0\end{aligned} \tag{24}$$

$$\begin{aligned}R_{ic} = {} & \varepsilon_{i1} M_i M_i^T + \varepsilon_{i2} A_{id} A_{id}^T \\ & + \varepsilon_{i3} M_{id} M_{id}^T + \varepsilon_{i4} I + B_i B_i^T \\ & + \sum_{j \neq i}^{m} A_{ij} A_{ij}^T + (\gamma_{i1} + \gamma_{i2}) H_i H_i^T \\ & + (\gamma_{i3} + \gamma_{i4}) B_{id} B_{id}^T + (\gamma_{i5} + \gamma_{i6}) H_{id} H_{id}^T\end{aligned} \tag{25}$$

$$Q_{ic} = (1+\varepsilon_{i1}^{-1})N_i^T N_i + (\varepsilon_{i2}^{-1} + m - 1)I + (1+\varepsilon_{i3}^{-1})N_{id}^T N_{id} + (1+\varepsilon_{i4}^{-1})G^T G + (1+\gamma_{i1}^{-1})K_i^T E_i^T E_i K_i + \gamma_{i3}^{-1} K_i^T K_i + (1+\gamma_{i4}^{-1})K_i^T E_{id}^T E_{id} K_i \quad (26)$$

$$R_{i0} = M_i M_i^T + A_{id} A_{id}^T + M_{id} M_{id}^T + I + \sum_{j\neq i}^{m} A_{ij} A_{ij}^T + 2H_i H_i^T + 2H_{id} H_{id}^T \quad (27)$$

$$Q_{i0} = (1+\gamma_{i2}^{-1})K_i^T E_i^T E_i K_i + \gamma_{i4}^{-1} K_i^T K_i + (1+\gamma_{i6}^{-1})K_i^T E_{id}^T E_{id} K_i + mI \quad (28)$$

Then the dynamic output feedback controller (6) (7) can make the close loop system asymptotically stable.

Corollary 2 If there exist positive constants $\varepsilon_{ik}(k=1,2,3,4), \gamma_{is}(s=1,2,\cdots,6)$, such that the Riccati matrix inequalities (29) and (30) have symmetric positive definite matrixes $\overline{P}_{ic}, \overline{P}_{i0}$, where

$$\overline{P}_{ic} A_i + A_i^T \overline{P}_{ic} + \overline{P}_{ic}\overline{R}_{ic}\overline{P}_{ic} + \overline{Q}_{ic} < 0 \quad (29)$$

$$\overline{P}_{i0} A_i + A_i^T \overline{P}_{i0} + \overline{P}_{i0}\overline{R}_{i0}\overline{P}_{i0} + \overline{Q}_{i0} < 0 \qquad i = 1,2,\cdots,m \quad (30)$$

$$\overline{R}_{ic} = \varepsilon_{i1} M_i M_i^T + \varepsilon_{i2} A_{id} A_{id}^T + \varepsilon_{i3} M_{id} M_{id}^T + \varepsilon_{i4} I + (1-\alpha) B_i B_i^T + \sum_{j\neq i}^{m} A_{ij} A_{ij}^T + (\gamma_{i1}+\gamma_{i2}) H_i H_i^T + (\gamma_{i3}+\gamma_{i4}) B_{id} B_{id}^T + (\gamma_{i5}+\gamma_{i6}) H_{id} H_{id}^T \quad (31)$$

$$\overline{Q}_{ic} = (1+\varepsilon_{i1}^{-1})N_i^T N_i + (\varepsilon_{i2}^{-1} + m - 1)I + (1+\varepsilon_{i3}^{-1})N_{id}^T N_{id} + (1+\varepsilon_{i4}^{-1})G^T G \quad (32)$$

$$\alpha = [(1+\gamma_{i1}^{-1})E_i^T E_i + \gamma_{i3}^{-1} I + (1+\gamma_{i4}^{-1})E_{id}^T E_{id}]^{-1} \quad (33)$$

$$\overline{Q}_{i0} = -2\beta C_i^T C_i + \alpha^2 \delta \overline{P}_{ic}^T B_i B_i^T \overline{P}_{ic} + mI \quad (34)$$

$$\overline{R}_{i0} = R_{i0} \quad (35)$$

$$\delta = [(1+\gamma_{i2}^{-1})E_i^T E_i + (1+\gamma_{i4}^{-1})I + (1+\gamma_{i6}^{-1})E_{id}^T E_{id}]^{-1} \quad (36)$$

then, system (1) can be stabilized by dynamic output feedback controller (6) (7) with

$$K_i = \alpha B_i^T \overline{P}_{ic}, \ L_i = \beta \overline{P}_{i0}^{-1} C_i^T \quad (37)$$

(2) $\Omega_i \neq \Phi$

Similarly to theorem 2, the fault close loop system and error equations can be written as (16) and (38).

$$\dot{e}_i(t) = (A_i - L_i C_i)e_i(t) + A_{id} e_i(t-d_{i1}) + \Delta A_i x_i(t) + \Delta A_{id} x_i(t-d_{i1}) + \Delta B_i E_{w_i} K_i(t)[x_i(t) - e_i(t)] + \Delta B_{id} K_i [x_i(t) - e_i(t)] + f_i(x_i, t) + \sum_{j\neq i}^{m} A_{ij} x_j(t-h_{ij}) \quad (38)$$

Theorem 4 If there exist positive constants $\varepsilon_{ik}(k=1,2,3,4), \gamma_{is}(s=1,2,\cdots,6)$ such that Riccati matrix inequalities (39) (40) have symmetric positive solve matrixes $\overline{P}_{ic}, \overline{P}_{i0}$, where

$$\overline{P}_{ic} A_i + A_i^T \overline{P}_{ic} + \overline{P}_{ic}\overline{R}_{ic}\overline{P}_{ic} + \overline{Q}_{ic} < 0 \quad (39)$$

$$\overline{P}_{i0} A_i + A_i^T \overline{P}_{i0} + \overline{P}_{i0}\overline{R}_{i0}\overline{P}_{i0} + \overline{Q}_{i0} < 0 \qquad i = 1,2,\cdots,m \quad (40)$$

$$\overline{R}_{ic} = \varepsilon_{i1} M_i M_i^T + \varepsilon_{i2} A_{id} A_{id}^T + \varepsilon_{i3} M_{id} M_{id}^T + \varepsilon_{i4} I + (1-\alpha) B_i E_{\Omega_i} B_i^T + \sum_{j\neq i}^{m} A_{ij} A_{ij}^T + (\gamma_{i1}+\gamma_{i2}) H_i H_i^T + (\gamma_{i3}+\gamma_{i4}) B_{id} B_{id}^T + (\gamma_{i5}+\gamma_{i6}) H_{id} H_{id}^T \quad (41)$$

$$\overline{Q}_{ic} = (1+\varepsilon_{i1}^{-1})N_i^T N_i + (\varepsilon_{i2}^{-1} + m - 1)I + (1+\varepsilon_{i3}^{-1})N_{id}^T N_{id} + (1+\varepsilon_{i4}^{-1})G^T G \quad (42)$$

$$\alpha = [(1+\gamma_{i1}^{-1})E_i^T E_{\Omega_i} E_i + \gamma_{i3}^{-1} I + (1+\gamma_{i4}^{-1})E_{id}^T E_{\Omega_i} E_{id}]^{-1} \quad (43)$$

$$\overline{Q}_{i0} = -2\beta C_i^T C_i + mI + \alpha^2 \delta \overline{P}_{ic}^T B_i E_{\Omega_i} B_i^T \overline{P}_{ic} \quad (44)$$

$$\overline{R}_{i0} = R_{i0} \quad (45)$$

$$\delta = [(1+\gamma_{i2}^{-1})E_i^T E_{\Omega_i} E_i + (1+\gamma_{i4}^{-1})I + (1+\gamma_{i6}^{-1})E_{id}^T E_{\Omega_i} E_{id}]^{-1} \quad (46)$$

then, for arbitrary $W_i \subseteq \Omega_i$ and admissible uncertainties ,system (1) can be robust stabilized by the dynamic output feedback controller (6) (7) with

$$K_i = \alpha B_i^T \overline{P}_{ic}, \ L_i = \beta \overline{P}_{i0}^{-1} C_i^T \quad (47)$$

Remark: In this paper, the resolving of Riccati inequalities can be changed into the solving of LMI.

## 5. EMULATION

Consider a system with $N = 2$. The special data are as following:

$$A_1 = \begin{bmatrix} -5.5 & 1 \\ -1 & -6 \end{bmatrix}, \ A_{1d} = \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.5 \end{bmatrix}$$

$$B_1 = B_{1d} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

$$F_i(\cdot) = F_{id}(\cdot) = \sin t$$

$$M_1 = M_{1d} = H_1 = H_{1d} = \begin{bmatrix} 0.3 \\ 0.15 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 0.1 \end{bmatrix}, \qquad N_{1d} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}$$

$$E_1 = E_{1d} = 0.2, \qquad N_1 = \begin{bmatrix} 0 & 0.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -50 & 0.5 \\ -0.3 & -80 \end{bmatrix},\ A_{2d} = \begin{bmatrix} 0.12 & -0.07 \\ 0.04 & -0.01 \end{bmatrix}$$

$$M_2 = M_{2d} = H_2 = H_{2d} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$

$$N_2 = [0 \quad 0.5],\ N_{2d} = [0.5 \quad 0]$$

$$E_2 = E_{2d} = 0.5,\ A_{21} = \begin{bmatrix} 0.2 & -0.2 \\ 0.5 & -0.3 \end{bmatrix}$$

$$d_{11} = 0.2,\ d_{22} = 0.3,\ d_{12} = 0.3,\ d_{21} = 0.3$$

Suppose $\Omega_1 = \{1\}$, $\Omega_2 = \{2\}$, it means $E_{\Omega_1} = diag\{0,1\}$ and $E_{\Omega_2} = diag\{1,0\}$

By using theorem 2, the follows can be obtained

$$\tilde{R}_1 = \begin{bmatrix} 0.54 & 0.39 \\ 0.39 & 0.58 \end{bmatrix},\ \tilde{Q}_1 = \begin{bmatrix} 22.25 & 0.25 \\ 0.25 & 22.25 \end{bmatrix}$$

$$\tilde{P}_1 = \begin{bmatrix} 2.3051 & 0.2626 \\ 0.2626 & 2.1644 \end{bmatrix}$$

$$u_1 = K_1 x_1(t) = -0.0498 * [0 \quad 1] * \tilde{P}_1 = [-0.0131 \quad -0.1078]$$

$$\tilde{R}_2 = \begin{bmatrix} 0.3105 & 0.2467 \\ 0.2467 & 0.4329 \end{bmatrix},\ \tilde{Q}_2 = \begin{bmatrix} 22.25 & 0.25 \\ 0.25 & 22.25 \end{bmatrix}$$

$$\tilde{P}_2 = \begin{bmatrix} 0.2251 & 0.0025 \\ 0.0025 & 0.1391 \end{bmatrix}$$

$$u_2 = K_2 x_2(t) = -0.0488 * [1 \quad 0] * \tilde{P}_2 = [-0.011 \quad -0.0001]$$

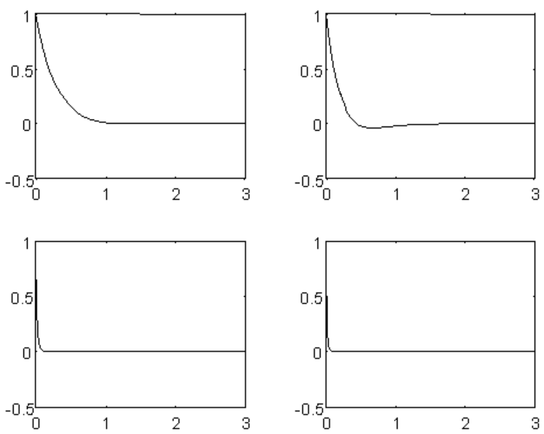


Fig. 1. Curves of the states of the system

## 6. CONCLUSION

In this paper the problem of reliable control for the uncertain delayed composite system with state delay and control delay is studied. And the sufficient condition for the memeoryless state feedback and dynamic output feedback robust reliable stabilization is obtained. The controllers can be designed by resolving two Riccati inequalities (or LMI). Comparing with the existing literature, it has three merits:

(1) There are uncertainties in the input matrixes of the system such that the conclusion can be used in a more extensive range.

(2) In the process of designing the controllers, the adjusting and choosing parameters are more agility.

(3) For the delayed term, there's no need for the boundary of it's differential.

## REFERENCE


Wang Z., Huang, B. and Unbehauen, H. (1999) Robust reliable control for a class of uncertain nonlinear state-delayed systems. *Automatica*, **35(9)**: 955-963

Zhao, Q., Jiang, J. (1998) Reliable state feedback control system design against actuator failures. *Automatica*, **34(10)**: 1267-1272

Wang, Z., Sun, X., Sun, J., et al, (1996) Robust fault-tolerant control and design for uncertain linear systems. *Acta Aeronaut. Astronaut*, **17**:112-115

Seo, C. J., Kim, B. K. (1996). Robust and $H_\infty$ control for linear systems with parameter uncertainty and actuator failure. *Automatica*, **32**: 465-467

Hu, S. S., Cao, J., Wang, Y. (1996). The decentralized strong stability control for large-scale interconnected systems. *Control Theory and Applications*, **13(3)**: 282-286

Xu, Z. D. (2000). Study of robust decentralized control of uncertain interconnected systems. [D], *China, Northeastern university*

Su, H. Y., Wang, J. C., Zhou, K. (1999). Robust output feedback control for linear time-varying uncertain time-delay systems. *ACTA AUTOMATICA SINICA*, **25(4)**: 513-517