15th Triennial World Congress, Barcelona, Spain

ROBUST HYBRID CONTROL OF UNCERTAIN CONSTRAINED MECHANICAL SYSTEMS

Zhuping Wang, Shuzhi S. Ge and T. H. Lee

Department of Electrical and Computer Engineering
National University of Singapore
Singapore 117576
elegesz@nus.edu.sg

Abstract: Robust hybrid controller design is presented in this paper for motion/force control of mechanical systems subjected to a set of holonomic or classical nonholonomic constraints and in the presence of uncertainties about plant parameters. A unified and systematic procedure is employed to derive the controllers for both holonomic and nonholonomic constrained mechanical systems, respectively. The proposed controllers can guarantee the system motion asymptotically converges to the desired manifold, and the force tracking errors to be bounded. Numerical simulation has been done to show the effectiveness of the proposed controllers.

Keywords: Robust control, Constrained mechanical systems, Uncertainty

1. INTRODUCTION

In recent years, much attention has been devoted to the problem of controlling mechanical systems with holonomic or nonholonomic constraints. The constrained robot is the most typical holonomic control system. In many industrial tasks such as writing, scribing, and grinding, the robot's end-effector is required to keep contact with its environment. During the execution of such tasks, contact forces are included between the end-effector and environmental constraint surfaces. Hence, a hybrid motion/force control design for robot manipulators under this kind of constrained motion is necessary.

Both trajectory tracking and force control are manageable with a constrained robot if the exact robot dynamic model is available to the controller. Different advanced control techniques have been successfully applied to solve the motion/force control problem, such as hybrid schemes for both position tracking and force control, nonlinear decoupling method, descriptor method and computed-torque controller. All these methods depend on the exact cancellation of the robot dynamics to achieve the two control objectives (Raibert and Craig, 1981; Mills and Goldenberg, 1989; McClamroch and Wang, 1988). In real applications, however, perfect cancellation of the robot dynamics is rarely possible. Thanks to the researches in (Carelli and Kelly, 1991; Young, 1988), the motion control part can be reduced to a problem similar to the free-motion control of a robot with less degrees of freedom. Force control, however, remains a difficult problem. The use of force feedback was considered in (Su *et al.*, 1992; Grabbe and Bridges, 1994) to improve the force control performance.

On the other hand, considerable attention has been paid to the motion control of nonholonomic constrained mechanical systems during last few years. It is well known that in rolling or cutting motions, the kinematic constraint equations are classical nonholonomic and the dynamics of such systems is also well understood. As considerable research works are concentrated on motion control of classical nonholonomic mechanical systems (I. and H., 1995; Campion *et al.*, 1991; Sarkar *et al.*, 1994), the motion/force control of these systems represents another important class of control problems (Su and Stepanenko, 1994; Su and Stepanenko, 1995). In both researches, however, the control law requires the knowledge of the regressor matrix. Implementation of this approach requires a precise knowledge of the structure of the entire dynamic model. Hence, development of an alternative approach to treat the motion/force control of holonomic and nonholonomic constrained mechanical systems with plant uncertainties and external disturbances is highly desirable. In this paper, robust and adaptive robust control algorithms for motion/force control of uncertain constrained mechanical systems are considered. The controllers are non-regressor based and require no information on the system dynamics.

2. SYSTEM DESCRIPTIONS

According to the Euler-Lagrangian formulation, the joint-space dynamics of an n-dimensional constrained mechanical system can be described as

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{d} = \mathbf{B}(\mathbf{q})\tau + \mathbf{f} \quad (1)$$

where $\mathbf{q} = [q_1, \ \ldots, \ q_n]^T \in R^n$ denotes the vector of generalized coordinates; $\mathbf{D}(\mathbf{q}) \in R^{n\times n}$ is the symmetric bounded positive definite inertia matrix; $\mathbf{C}(\dot{\mathbf{q}},\mathbf{q})\dot{\mathbf{q}} \in R^n$ denotes the Centripetal and Coriolis torques; $\mathbf{G}(\mathbf{q}) \in R^n$ is the gravitational torque vector; $\mathbf{B} \in R^{n\times r}$ is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system, $\tau \in R^r$ is the vector of generalized control input force with $r \geq n-m$; $\mathbf{d}$ denotes the external disturbances and $\mathbf{f} = \mathbf{J}^T(\mathbf{q})\lambda$ denotes the constraint force due to the reaction of the following two cases.

Property 1. : There exist some constants $c_i > 0$ $(1 \leq i \leq 4)$ and $c_5 \geq 0$ such that $\forall \mathbf{q} \in R^n$, $\forall \dot{\mathbf{q}} \in R^n$ $||\mathbf{D}(\mathbf{q})|| \leq c_1$, $||\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})|| \leq c_2 + c_3||\dot{\mathbf{q}}||$, $||\mathbf{G}(\mathbf{q})|| \leq c_4$, and $\sup_{t\geq 0}||\mathbf{d}(t)|| \leq c_5$.

2.1 *Holonomic constrained mechanical systems*

Considering m independent frictionless constraints expressed as

$$\mathbf{\Phi}(\mathbf{q}) = 0 \in R^m \quad (2)$$

Defining

$$\mathbf{J}(\mathbf{q}) = \frac{\partial \mathbf{\Phi}}{\partial \mathbf{q}} \quad (3)$$

It is a common assumption that the constraints are holonomic and frictionless, and $\mathbf{\Phi}(\mathbf{q})$ is twice continuously differentiable. Hence, the holonomic constraint on the robot's end effector can be viewed as restricting only the dynamics on the constraint manifold Ω_h defined by

$$\Omega_h = \{(\mathbf{q},\dot{\mathbf{q}}) | \mathbf{\Phi}(\mathbf{q}) = 0, \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = 0\} \quad (4)$$

The vector $\mathbf{q} \in R^n$ can always be properly rearranged and partitioned into the form $\mathbf{q} = [{\mathbf{q}^1}^T \ {\mathbf{q}^2}^T]^T$, $\mathbf{q}^1 = [q_1^1 \ \ldots \ q_{n-m}^1]^T \in R^{n-m}$ describes the constrained motion of the manipulator and $\mathbf{q}^2 = [q_1^2 \ \ldots \ q_m^2]^T \in R^m$ denotes the remaining joint variables. Moreover, there is a unique function $v : R^{n-m} \to R^m$ such that the constraint condition can always be expressed as $\mathbf{q}^2 = v(\mathbf{q}^1)$ (McClamroch and Wang, 1988). By defining $\mathbf{L}(\mathbf{q}^1) = [\mathbf{I}_{n-m}^T \ \frac{\partial v(\mathbf{q}^1)}{\partial \mathbf{q}^1}^T]^T$, $\mathbf{I}_{n-m} \in R^{(n-m)\times(n-m)}$ is an identity matrix, it can be obtained

$$\dot{\mathbf{q}} = \mathbf{L}(\mathbf{q}^1)\dot{\mathbf{q}}^1 \quad (5)$$

The dynamic model (1) of robots, when restricted to the constraint surface, can be expressed in the reduced form as

$$\begin{aligned}&\mathbf{D}(\mathbf{q}^1)\mathbf{L}(\mathbf{q}^1)\ddot{\mathbf{q}}^1 + \mathbf{C}_1(\mathbf{q}^1,\dot{\mathbf{q}}^1)\dot{\mathbf{q}}^1 + \mathbf{G}(\mathbf{q}^1) + \mathbf{d} \\ &= \mathbf{B}\tau + \mathbf{J}^T(\mathbf{q}^1)\lambda \end{aligned} \quad (6)$$

where $\mathbf{C}_1 = \mathbf{D}(\mathbf{q}^1)\dot{\mathbf{L}}(\mathbf{q}^1) + \mathbf{C}(\mathbf{q}^1,\dot{\mathbf{q}}^1)\mathbf{L}(\mathbf{q}^1)$.

Property 2. : Matrix $\mathbf{D}_L = \mathbf{L}^T\mathbf{D}\mathbf{L}$ is symmetric and positive definite.

Property 3. : Matrix $\dot{\mathbf{D}}_L - 2\mathbf{C}_L$ is skew-symmetric, $\mathbf{C}_L = \mathbf{L}^T\mathbf{C}_1$.

Property 4. : $\mathbf{L}^T(\mathbf{q}^1)\mathbf{J}^T(\mathbf{q}^1) = \mathbf{J}(\mathbf{q}^1)\mathbf{L}(\mathbf{q}^1) = 0$.

2.2 *Nonholonomic constrained mechanical systems*

When the system is subjected with nonholonomic constraint, the m nonintegrable and independent velocity constraints can be expressed as

$$\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = 0 \quad (7)$$

where $\mathbf{J} : R^n \to R^{m\times n}$ is the kinematic constraint matrix which is assume to have full rank m. The constraint (7) is referred to the classical nonholonomic constraint when it is not integrable. The effect of the constraints can be viewed as restricting the dynamics on the manifold Ω_{nh} as

$$\Omega_{nh} = \{(\mathbf{q},\dot{\mathbf{q}}) | \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = 0\} \quad (8)$$

It is noted that since the nonholonomic constraint (7) is nonintegrable, there is no explicit restriction on the values of the configuration variables.

Let $r_1(\mathbf{q}),\ \ldots,\ r_{n-m}(\mathbf{q})$ be a set of smooth and linearly independent vector fields in the null space of $\mathbf{J}(\mathbf{q})$. Then, the following relations are satisfied in local coordinates

$$\mathbf{J}(\mathbf{q})\mathbf{R}(\mathbf{q}) = 0 \tag{9}$$

where $\mathbf{R}(\mathbf{q}) = [r_1(\mathbf{q}),\ \ldots,\ r_{n-m}(\mathbf{q})] \in R^{n\times(n-m)}$. Constraints (7) and (9) imply the existence of vector $\dot{\mathbf{z}} \in R^{n-m}$, such that

$$\dot{\mathbf{q}} = \mathbf{R}(\mathbf{q})\dot{\mathbf{z}} \tag{10}$$

The dynamic equation (1), which satisfies the nonholonomic constraint (7), can be rewritten in terms of the internal state variable $\dot{\mathbf{z}}$ as (Su and Stepanenko, 1994; Chang and Chen, 2000)

$$\begin{aligned}&\mathbf{D}(\mathbf{q})\mathbf{R}(\mathbf{q})\ddot{\mathbf{z}} + \mathbf{C}_2(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{z}} + \mathbf{G}(\mathbf{q}) \\ &= \mathbf{B}(\mathbf{q})\tau + \mathbf{J}^T(\mathbf{q})\lambda + \mathbf{d}\end{aligned} \tag{11}$$

where $\mathbf{C}_2(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{D}(\mathbf{q})\dot{\mathbf{R}}(\mathbf{q}) + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{R}(\mathbf{q})$.

Property 5. : Matrix $\mathbf{D}_R = \mathbf{R}^T\mathbf{D}\mathbf{R}$ is symmetric and positive-definite.

Property 6. : Matrix $\mathbf{N}_R = \dot{\mathbf{D}}_R - 2\mathbf{C}_R$ is skew-symmetric, $\mathbf{C}_R = \mathbf{R}^T\mathbf{C}_2$.

Property 7. : $\mathbf{R}^T(\mathbf{q})\mathbf{J}^T(\mathbf{q}) = 0$.

3. ROBUST CONTROL DESIGN

In this section, robust controller design for holonomic mechanical systems as well as nonholonomic mechanical systems with plant uncertainties and external disturbances is considered.

3.1 *Robust control of holonomic mechanical systems*

The control objective is specified as: given a desired joint trajectory $\mathbf{q}_d(t)$ and a desired constraint force $\mathbf{f}_d(t)$, or, equivalently, a desired multiplier $\lambda_d(t)$, determine a control law such that for any $(\mathbf{q}(0),\dot{\mathbf{q}}(0)) \in \Omega_h$, $\mathbf{q}^1, \dot{\mathbf{q}}$ and λ asymptotically converge to a manifold Ω_{hd} specified as

$$\Omega_{hd} = \{(\mathbf{q},\dot{\mathbf{q}},\lambda)|\mathbf{q}^1 = \mathbf{q}_d^1, \dot{\mathbf{q}} = \mathbf{L}(\mathbf{q})\dot{\mathbf{q}}_d^1, \lambda = \lambda_d\}$$

Assumption 1. : The desired reference trajectory $\mathbf{q}_d(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. The desired Lagrangian multiplier $\lambda_d(t)$ is also assumed to be bounded and uniformly continuous.

In the following, define $\mathbf{e}_{q^1} = \mathbf{q}^1 - \mathbf{q}_d^1$, $\mathbf{e}_\lambda = \lambda - \lambda_d$, $\dot{\mathbf{q}}_r^1 = \dot{\mathbf{q}}_d^1 - \rho_1\mathbf{e}_{q^1}$ and $\mathbf{s} = \dot{\mathbf{e}}_{q^1} + \rho_1\mathbf{e}_{q^1}$. Apparently

$$\dot{\mathbf{q}}^1 = \dot{\mathbf{q}}_r^1 + \mathbf{s} \tag{12}$$

Consider the control law as

$$\mathbf{B}\tau = -\mathbf{K}\mathbf{L}\mathbf{s} - \frac{\mathbf{L}^+\mathbf{s}\Phi^2}{||\mathbf{s}||\Phi + \delta} - \mathbf{J}^T\lambda_c \tag{13}$$

where $\mathbf{K}$ is positive definite, $\mathbf{L}^+$ is the left inverse of $\mathbf{L}^T$ defined as $\mathbf{L}^+ = \mathbf{L}(\mathbf{L}^TL)^{-1}$, $\delta(t) > 0$ such that $\int_0^t \delta(\omega)d\omega = a < \infty$. There are many choices for $\delta(t)$. For example, $\delta(t)$ may be $(1+t)^{-l_1}(l_1 > 1)$ or $e^{-l_2 t}(l_2 > 0)$. Φ is given by

$$\begin{aligned}\Phi = {} & ||\mathbf{L}(\mathbf{q})||\{c_1||\frac{d}{dt}[\mathbf{L}(\mathbf{q})\dot{\mathbf{q}}_r^1]|| + (c_2 + c_3||\dot{\mathbf{q}}||) \\ & ||\mathbf{L}(\mathbf{q})\dot{\mathbf{q}}_r^1|| + c_4 + c_5\}\end{aligned} \tag{14}$$

and the force term λ_c is defined as $\lambda_c = \lambda_d - \mathbf{K}_\lambda\mathbf{e}_\lambda$, $\mathbf{K}_\lambda$ is a constant matrix of force control feedback gains.

Theorem 1. : Consider the mechanical system described by (6), using the control law (13), the following holds for any $(\mathbf{q}(0),\dot{\mathbf{q}}(0)) \in \Omega_h$:

(i). $\mathbf{e}_q$ and $\dot{\mathbf{e}}_q \to 0$ as $t \to \infty$;

(ii). $\mathbf{e}_\lambda$ is uniformly ultimately bounded and inversely proportional to the norm of the matrix $\mathbf{K}_\lambda + \mathbf{I}$.

Proof: According to Property 4, and substituting (13) into (6), the closed-loop dynamic equation is obtained

$$\mathbf{D}_L\dot{\mathbf{s}} = -\mathbf{L}^T\mathbf{K}\mathbf{L}\mathbf{s} - \frac{\mathbf{s}\Phi^2}{||\mathbf{s}||\Phi + \delta} - \xi - \mathbf{C}_L\mathbf{s} \tag{15}$$

where $\xi = \mathbf{D}_L\ddot{\mathbf{q}}_r^1 + \mathbf{C}_L\dot{\mathbf{q}}_r^1 + \mathbf{G}_L + \mathbf{d}_L$.

Consider the Lyapunov candidate function

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{D}_L\mathbf{s} \tag{16}$$

Its time derivative along the trajectory of (15) is

$$\begin{aligned}\dot{V} &= -\mathbf{s}^T\mathbf{L}^TK\mathbf{L}\mathbf{s} - \mathbf{s}^T\xi - \frac{||\mathbf{s}||^2\Phi^2}{||\mathbf{s}||\Phi + \delta} \\ &\le -\mathbf{s}^T\mathbf{L}^T\mathbf{K}\mathbf{L}\mathbf{s} + ||\mathbf{s}||||\xi|| - \frac{||\mathbf{s}||^2\Phi^2}{||\mathbf{s}||\Phi + \delta} \\ &\le -\mathbf{s}^T\mathbf{L}^TK\mathbf{L}\mathbf{s} + ||\mathbf{s}||\Phi - \frac{||\mathbf{s}||^2\Phi^2}{||\mathbf{s}||\Phi + \delta} \\ &\le -\mathbf{s}^TL^TK\mathbf{L}\mathbf{s} + \delta\end{aligned} \tag{17}$$

Integrating both sides of (17) gives

$$V(t) - V(0) = -\int_0^t \mathbf{s}^T\mathbf{L}^TK\mathbf{L}\mathbf{s}ds + \int_0^t \delta ds$$

$$\leq a \tag{18}$$

Thus, V is bounded, which implies that $\mathbf{s} \in L_\infty^{n-m}$.

From (18), we have

$$\int_0^t \mathbf{s}^T \mathbf{L}^T K \mathbf{L} \mathbf{s} ds \leq V(0) - V(t) + a \tag{19}$$

Hence, $\mathbf{s} \in L_2^{n-m}$ can be obtained.

Since $\mathbf{s} = \dot{\mathbf{e}}_{q^1} + \rho_1 \mathbf{e}_{q^1}$, it can be concludee $\mathbf{e}_{q^1}, \dot{\mathbf{e}}_{q^1} \in L_\infty^{n-m}$.

Since $\mathbf{e}_{q^1}, \dot{\mathbf{e}}_{q^1} \in L_\infty$, it can be concluded that $\mathbf{q}^1(t), \dot{\mathbf{q}}^1(t), \dot{\mathbf{q}}_r^1(t), \ddot{\mathbf{q}}_r^1(t) \in L_\infty^{n-m}$ and $\dot{\mathbf{q}} \in L_\infty^n$.

Therefore, all the signals on the right hand side of (15) are bounded, $\dot{\mathbf{s}}$ and therefore $\ddot{\mathbf{q}}^1$ are bounded. Hence, $\mathbf{s} \to 0$ as $t \to \infty$. Therefore, $\mathbf{e}_{q^1} \to 0, \dot{\mathbf{e}}_{q^1} \to 0$ as $t \to \infty$. It follows that $\mathbf{e}_q, \dot{\mathbf{e}}_q \to 0$ as $t \to \infty$.

Substituting the control (13) into the reduced order dynamic system model (6) yields

$$\mathbf{J}^T(\lambda - \lambda_c) = \alpha(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}^1, \ddot{\mathbf{q}}^1, \dot{\mathbf{q}}_r^1, \ddot{\mathbf{q}}_r^1) \tag{20}$$

where α is a bounded function. Thus, $\mathbf{J}^T e_\lambda = (\mathbf{K}_\lambda + \mathbf{I})^{-1}\alpha$, and therefore the force tracking error $(\mathbf{f} - \mathbf{f}_d)$ is bounded and can be adjusted by changing the feedback gain $\mathbf{K}_\lambda$. □

3.2 *Robust control of nonholonomic mechanical systems*

Consider the constrained dynamic equation (1) together with m independent nonholonomic constraints (7).

Assumption 2. : The matrix $\mathbf{R}^T(\mathbf{q})\mathbf{B}(\mathbf{q})$ is of full rank, which guarantees all $n-m$ degrees of freedom can be (independently) actuated.

The above assumption always holds for a large class of nonholonomic mechanical systems such as nonholonomic Caplygin systems (which include a vertical wheel rolling without slipping on a plane surface, a mobile wheeled robot moving on a horizontal plane, and a knife edge moving in point contact on a plane surface, etc.).

By appropriate selecting a set of $(n-m)$ vector of variables $\mathbf{z}(\mathbf{q})$ and $\dot{\mathbf{z}}(\mathbf{q})$, the control objective can be specified as: given a desired $\mathbf{z}_d$, $\dot{\mathbf{z}}_d$, and desired constraint λ_d, determine a control law such that for any $(\mathbf{q}(0), \dot{\mathbf{q}}(0)) \in \Omega$ then $\mathbf{z}(\mathbf{q}), \dot{\mathbf{q}}$ and λ asymptotically converge to a manifold Ω_d specified as

$$\Omega_{nhd} = \{(\mathbf{q}, \dot{\mathbf{q}}, \lambda) | \mathbf{z}(\mathbf{q}) = \mathbf{z}_d, \dot{\mathbf{q}} = \mathbf{R}(\mathbf{q})\dot{\mathbf{z}}_d, \lambda = \lambda_d\}$$

Assumption 3. : The desired reference trajectory $\mathbf{z}_d(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. The desired $\lambda_d(t)$ is also assumed to be bounded and uniformly continuous.

Defining $\mathbf{e}_z = \mathbf{z} - \mathbf{z}_d$, $\mathbf{e}_\lambda = \lambda - \lambda_d$, $\dot{\mathbf{z}}_r = \dot{\mathbf{z}}_d - \rho_1 \mathbf{e}_z$ and $\mathbf{s} = \dot{\mathbf{e}}_z + \rho_1 \mathbf{e}_z$, considering the control law as

$$\mathbf{B}\tau = -\mathbf{K}\mathbf{R}\mathbf{s} - \frac{\mathbf{R}^\ddagger \mathbf{s} \Phi^2}{||\mathbf{s}||\Phi + \delta} - \mathbf{J}^T \lambda_c \tag{21}$$

where $\mathbf{K}$ is positive definite, $\mathbf{R}^\ddagger$ is the left inverse of $\mathbf{R}^T$ defined as $\mathbf{R}^\ddagger = \mathbf{R}(\mathbf{R}^T\mathbf{R})^{-1}$, $\lambda_c = \lambda_d - \mathbf{K}_\lambda \mathbf{e}_\lambda$, $\delta(t) > 0$ such that $\int_0^t \delta(\omega) d\omega = a < \infty$ and Φ is given by

$$\begin{aligned}\Phi = \; & ||\mathbf{R}(\mathbf{q})||\{c_1||\frac{d}{dt}[\mathbf{R}(\mathbf{q})\dot{\mathbf{z}}_r]|| + (c_2 + c_3||\dot{\mathbf{q}}||) \\ & ||\mathbf{R}(\mathbf{q})\dot{\mathbf{z}}_r|| + c_4 + c_5\}\end{aligned} \tag{22}$$

Theorem 2. : Consider the mechanical system described by (11), using the control law (21), the following holds for any $(\mathbf{q}(0), \dot{\mathbf{q}}(0)) \in \Omega_{nh}$:

(i). $\mathbf{e}_z$ and $\dot{\mathbf{e}}_z \to 0$ as $t \to \infty$;

(ii). $\mathbf{e}_\lambda$ is uniformly ultimately bounded and inversely proportional to the norm of the matrix $\mathbf{K}_\lambda + \mathbf{I}$.

Proof: The proof is similar to that of Theorem 1. □

4. ADAPTIVE ROBUST CONTROL DESIGN

In developing control laws (13) and (21), $c_i, 1 \leq i \leq 5$ are supposed to be known. However, in reality, these constants cannot be obtained easily. Although any fixed large c_i can guarantee good performance, however, it is not recommended in practice. Therefore, it is necessary to develop a control law which does not require the knowledge of $c_i, 1 \leq i \leq 5$.

4.1 *Adaptive robust control of holonomic mechanical systems*

Consider the control law as

$$\mathbf{B}\tau = -\mathbf{K}\mathbf{L}\mathbf{s} - \sum_{i=1}^{5} \frac{\mathbf{L}^+ \mathbf{s} \hat{c}_i \Phi_i^2}{||\mathbf{s}||\Phi_i + \delta_i} - \mathbf{J}^T \lambda_c \tag{23}$$

and the adaptation law as

$$\dot{\hat{c}}_i = -\sigma_i \hat{c}_i + \frac{\gamma_i \Phi_i^2 ||\mathbf{s}||^2}{\Phi_i ||\mathbf{s}|| + \delta_i} \qquad i = 1, \ldots, 5 \tag{24}$$

where

$$\Phi_1 = ||\mathbf{L}(\mathbf{q})||.||\frac{d}{dt}[\mathbf{L}(\mathbf{q})\dot{\mathbf{q}}_r^1]|| \quad (25)$$

$$\Phi_2 = ||\mathbf{L}(\mathbf{q})||.||\mathbf{L}(\mathbf{q})\dot{\mathbf{q}}_r^1|| \quad (26)$$

$$\Phi_3 = ||\mathbf{L}(\mathbf{q})||.||\dot{\mathbf{q}}||.||\mathbf{L}(\mathbf{q})\dot{\mathbf{q}}_r^1|| \quad (27)$$

$$\Phi_4 = \Phi_5 = ||\mathbf{L}(\mathbf{q})|| \quad (28)$$

$\mathbf{K}$ is positive definite, $\mathbf{L}^+ = \mathbf{L}(\mathbf{L}^T\mathbf{L})^{-1}$, $\lambda_c = \lambda_d - \mathbf{K}_\lambda \mathbf{e}_\lambda$, $\gamma_i > 0$, $\delta_i(t) > 0$ and $\sigma_i(t) > 0$ such that $\int_0^t \delta_i(\omega)d\omega = a_i < \infty$ and $\int_0^t \sigma_i(\omega)d\omega = b_i < \infty$.

Theorem 3. : Consider the mechanical system described by (6), using the control law (23) and adaptation law (24), the following holds for any $(\mathbf{q}(0), \dot{\mathbf{q}}(0)) \in \Omega_h$:

(i). $\mathbf{e}_q$ and $\dot{\mathbf{e}}_q \to 0$ as $t \to \infty$;

(ii). $\mathbf{e}_\lambda$ is uniformly ultimately bounded and inversely proportional to the norm of the matrix $\mathbf{K}_\lambda + \mathbf{I}$.

Proof: Consider the Lyapunov candidate function

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{D}_L\mathbf{s} + \sum_{i=1}^{5}\frac{1}{2\gamma_i}\tilde{c}_i^2 \quad (29)$$

The proof is then similar to that of Theorem 1. Following the similar analysis as in Proof of Theorem 1, it is easy to obtain that $\mathbf{e}_q, \dot{\mathbf{e}}_q \to 0$ as $t \to \infty$, and the force tracking error $(\mathbf{f} - \mathbf{f}_d)$ is bounded and inversely proportional to the norm of the matrix $\mathbf{K}_\lambda + \mathbf{I}$. □

4.2 *Adaptive robust control for nonholonomic mechanical systems*

Consider the control law as

$$\mathbf{B}\tau = -\mathbf{KRs} - \sum_{i=1}^{7}\frac{\mathbf{R}^\ddagger \mathbf{s}\hat{c}_i\Phi_i^2}{||\mathbf{s}||\Phi_i + \delta_i} - \mathbf{J}^T\lambda_c \quad (30)$$

and the adaptation law as

$$\dot{\hat{c}}_i = -\sigma_i\hat{c}_i + \frac{\gamma_i\Phi_i^2||\mathbf{s}||^2}{\Phi_i||\mathbf{s}|| + \delta_i} \qquad i = 1, \ldots, 7 \quad (31)$$

where

$$\Phi_1 = ||\mathbf{R}(\mathbf{q})||.||\frac{d}{dt}[\mathbf{R}(\mathbf{q})\dot{\mathbf{z}}_r]|| \quad (32)$$

$$\Phi_2 = ||\mathbf{R}(\mathbf{q})||.||\mathbf{R}(\mathbf{q})\dot{\mathbf{z}}_r|| \quad (33)$$

$$\Phi_3 = ||\mathbf{R}(\mathbf{q})||.||\dot{\mathbf{q}}||.||\mathbf{R}(\mathbf{q})\dot{\mathbf{z}}_r|| \quad (34)$$

$$\Phi_4 = \Phi_5 = ||\mathbf{R}(\mathbf{q})|| \quad (35)$$

$\mathbf{K}$ is positive definite, $\mathbf{R}^\ddagger = \mathbf{R}(\mathbf{R}^T\mathbf{R})^{-1}$, $\lambda_c = \lambda_d - \mathbf{K}_\lambda\mathbf{e}_\lambda$, $\gamma_i > 0$, $\delta_i(t) > 0$ and $\sigma_i(t) > 0$ such that $\int_0^t \delta_i(\omega)d\omega = a_i < \infty$ and $\int_0^t \sigma_i(\omega)d\omega = b_i < \infty$.

Theorem 4. : Consider the mechanical system described by (11), using the control law (30) and adaptation law (31), then the following holds for any $(\mathbf{q}(0), \dot{\mathbf{q}}(0)) \in \Omega_{nh}$:

(i). $\mathbf{e}_z$ and $\dot{\mathbf{e}}_z \to 0$ as $t \to \infty$;

(ii). $\mathbf{e}_\lambda$ is uniformly ultimately bounded and inversely proportional to the norm of the matrix $\mathbf{K}_\lambda + \mathbf{I}$.

Proof: The proof is similar to that of Theorem 3. □

5. SIMULATION RESULTS

A two-link robotic manipulator with a circular path constraint is simulated to verify the proposed controller. The constrained dynamic equation in the form of (1) can be written as

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -C_{12}\dot{q}_2 & -C_{12}(\dot{q}_1 + \dot{q}_2) \\ C_{12}\dot{q}_1 & 0 \end{bmatrix}$$
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad (36)$$

where $D_{11} = (m_1 + m_2)l_1^2 + m_2l_2^2 + 2m_2l_1l_2\cos q_2$, $D_{12} = m_2l_2^2 + m_2l_1l_2\cos q_2$, $D_{22} = m_2l_2^2$, $C_{12} = m_2l_1l_2\sin q_2$, $G_1 = (m_1 + m_2l_1g\cos q_1) + m_2l_2\cos(q_1 + q_2)$, and $G_2 = m_2l_2g\cos(q_1 + q_2)$. The constraint is a circle in the work space (the (x, y) plane) whose center coincides with axis of rotation of the first link. The constraint, when expressed in terms of joint space, is $\Phi(q) = l_1^2 + l_2^2 + 2l_1l_2\cos q_2 - r^2 = 0$, which has a unique constant solution for $q_2 = \cos^{-1}[\frac{r^2 - (l_1^2 + l_2^2)}{2l_1l_2}] = q_2^*$. Hence, the Jacobian matrix is $\mathbf{J}(q) = [0 \quad -2l_1l_2\sin q_2]$ and the matrix $\mathbf{L}(q^1) = [1 \;\; 0]^T$. For the convenience of simulation, the nominal parameters of the robot system are taken as $m_1 = 1$Kg, $m_2 = 2$Kg, $l_1 = l_2 = 1$m, $r = \sqrt{2}$m, and $g = 9.8\text{m}/s^2$, the initial conditions are taken as $q(0) = [1.0, \;\; 0]^T$, $\dot{q}(0) = [0, \;\; 0]^T$, and the desired manifold Ω_{hd} is chosen as $\Omega_{hd} = \{(\mathbf{q}, \dot{\mathbf{q}}, \lambda)|q^1 = 0, \dot{\mathbf{q}} = 0, \lambda = 10\}$.

Using the controller (23) and adaptation law (24), the control gain $\mathbf{K}$ and force control gain K_λ are selected as $\mathbf{K} = \text{diag}(1, \; 1)$, $K_\lambda = 1$, and ρ_1 is chosen as $\rho_1 = 5$. The adaptation gain in adaptation law (24) is chosen as $\gamma_i = 0.5$ and $\sigma_i = \delta_i = \frac{1}{(1+t)^2}$. The results of the simulation are shown in Figs. 1-3. Fig. 1 shows the system responses, including q_1, q_2, $\dot{q}_1$ and $\dot{q}_2$, of the simulated costrained robot. Fig. 2 shows the force tacking error. It can be seen that motion tracking error converges to zero as desired and force tracking error is bounded. The torques exerted at the constrained robot are given by Fig. 3. It can be seen that all signals in closed-loop are bounded. These results verify the validity of the proposed algorithm.

6. CONCLUSION

In this paper, the problem of motion/force control for both holonomic and nonholonomic mechanical systems with uncertain dynamics is considered. Both robust control algorithm and adaptive robust control algorithm have been designed to drive the system motion converge to the desired manifold and at the same time guarantee the boundedness of the force tracking error. The proposed controllers are non-regressor based and require no information on the system dynamics. Simulation results have shown that the effectiveness of the proposed controllers.

7. REFERENCES

Campion, G., B. d'Andrea Novel and G. Bastin (1991). Controllability and state feedback stability of nonholonomic mechanical systems. *Advanced Robot Control* pp. 106–124.

Carelli, R. and R. Kelly (1991). An adaptive impedence/force controller for robot manipulators. *IEEE Trans. Automat. Contr.* **36**, 967–971.

Chang, Y. C. and B. S. Chen (2000). Robust tracking designs for both holonomic and nonholonomic constrained mechanical systems: adaptive fuzzy approach. *IEEE Trans. Fuzzy Systems* **8**(1), 46–66.

Grabbe, M. T. and M. M. Bridges (1994). Comments on 'force/motion control of constrained robots using sliding mode'. *IEEE Trans. Automat. Contr.* **39**, 179.

I., Kolmanovsky and McClamroch N. H. (1995). Developments in nonholonomic control problems. *IEEE Contr. Sys. Mag.* **15**(6), 20–36.

McClamroch, N. H. and D. Wang (1988). Feedback stabilization and tracking of constrained robots. *IEEE Trans. Automat. Contr.* **33**, 419–426.

Mills, J. K. and A. Goldenberg (1989). Force and position control of manipulators during constrained motion tasks. *IEEE Trans. Robotics Automat.* **5**, 30–46.

Raibert, M. H. and J. J. Craig (1981). Hybrid position/force control of manipulators. *ASME J. Dynam. Syst., Meas. Contr.* **102**, 126–133.

Sarkar, N., X. Yun and V. Kumar (1994). Control of mechanical systems with rolling constraints: Applications to dynamic control of mobile robots. *Int. J. Robot. Res.* **13**, 55–69.

Su, C. Y. and Y. Stepanenko (1994). Robust motion/force control of mechanical systems with classical nonholonimic constraints. *IEEE Trans. Automatic Contr.* **39**(3), 609–614.

Su, C. Y. and Y. Stepanenko (1995). Adaptive control of mechanical systems with classical nonholonimic constraints. In: *Proc. IEEE Int. Conf. Systems, Man and Cybernetics*. pp. 4143–4148.

Su, C. Y., T. P. Leung and Q. J. Zhou (1992). Force/motion control of constrained robots using sliding mode. *IEEE Trans. Automat. Contr.* **37**, 668–672.

Young, K. D. (1988). Applications of sliding mode to constrained robot motion control. In: *Proc. Amer. Contr. Conf.*. pp. 912–916.

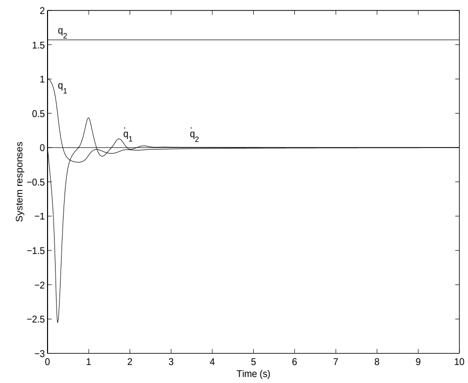

Fig. 1. Simulated system's responses

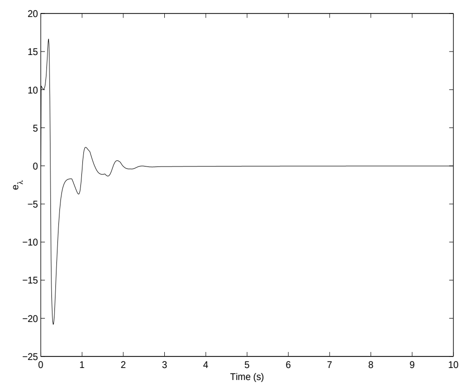

Fig. 2. e_λ of the simulated system

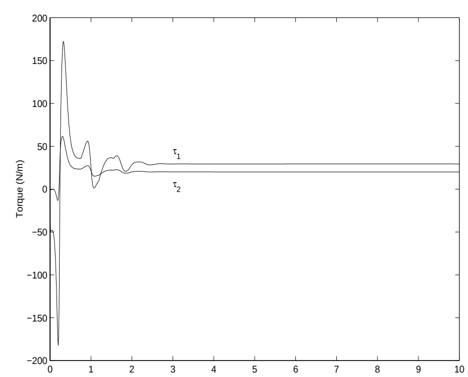

Fig. 3. Control torque of the simulated system