15th Triennial World Congress, Barcelona, Spain

ROBUST LEARNING CONTROL FOR A CLASS OF UNCERTAIN NONLINEAR SYSTEMS

Yu-Ping Tian*, Xinghuo Yu**

*Department of Automatic Control, Southeast University, Nanjing, 210096, P.R.China
E-mail: yptian@seu.edu.cn

**Faculty of Informatics and Communication, Central Queensland University, Rockhampton QLD 4702, Australia.
E-mail: x.yu@cqu.edu.au

Abstract: This paper addresses the robust learning control problem for a class of nonlinear systems with structured periodic and unstructured aperiodic uncertainties. A recursive technique is proposed which extends the currently popular backstepping idea to the robust repetitive learning control systems. An learning evaluation function instead of a Lyapunov function is formulated as a guideline for derivation of the control strategy which guarantees the asymptotic stability of the tracking system. The proposed method is validated by simulation of tracking control of two systems with periodic uncertainties, one of which is the well-known van der Pol chaotic oscillator.

Keywords: Iterative learning; Robust control; Nonlinear systems, Time-varying uncertainties, Control synthesis

1. Introduction

Recently some nonlinear learning control schemes were proposed by Park *et al.* (1996), Xu and Qu (1998). It was shown that nonlinear feedback can be incorporated into iterative learning control to achieve asymptotic convergence of tracking control for a class of nonlinear systems described by an canonical integrator chained form. In Xu *et al.* (2000) it was further shown that this nonlinear learning control scheme can be applied to systems with both periodic and aperiodic uncertainties by introducing a sliding mode into the learning control systems. Although these methods are effective, they are quite restrictive due to the requirement of specific system models, and all uncertainties appears only in the last equation, that is, uncertainties are matched with control input.

As it is well known, nonlinear systems with constant unknown parameters have been extensively studied along the direction of adaptive control strategy mainly based on Lyapunov method. In recent years, a constructive design methodology utilizing the so-called backstepping technique has been developed and attracted much attention (Kokotović and Arcak 2001, Kristć *et al.* 1995). A distinguishing advantage of this method is it overcomes the relative degree one restriction and can be used to treat the unmatched uncertainties. Furthermore, this method is based on the concept of adaptive control Lyapunov function and, therefore, enables rigorous analysis of asymptotic stability and convergence. However, a hypothesis of zero derivative of uncertain parameters is required, which makes it difficult for application in the systems with time-varying uncertainties.

In this paper we try to combine together the backstepping technique and the learning control mechanism for developing a constructive control strategy to cope with nonlinear systems with both structured periodic and unstructured aperiodic uncertainties. This, on the one hand, will broaden the domain of the applicability of both two useful design tools, and on the other hand, will give a new method for solving the tracking control problem for a large class of nonlinear systems. For this purpose a multiple learning mechanism is proposed for the first time, which allows one to overcome the unmatched uncertainty difficulty. Unlike other learning control schemes, a moving average value of the estimation (not the estimation itself) is used in our learning controller. This makes it possible to design the controller by a recursive procedure.

2. Problem formulation

Consider the following nonlinear uncertain system:

$$\begin{array}{rcl}\dot{x}_1 & = & x_2 + w_1(x_1,t) + \phi_1^T(x_1)\gamma(t) \\ \dot{x}_2 & = & x_3 + w_2(x_1,x_2,t) + \phi_2^T(x_1,x_2)\gamma(t) \\ & \vdots & \\ \dot{x}_{n-1} & = & x_n + w_{n-1}(x_1,\cdots,x_{n-1},t) \\ & & \quad +\phi_{n-1}^T(x_1,\cdots,x_{n-1})\gamma(t) \\ \dot{x}_n & = & u + w_n(x_1,\cdots,x_n,t) \\ & & \quad +\phi_n^T(x_1,\cdots,x_n)\gamma(t) \\ y & = & x_1 \end{array} \tag{1}$$

where $x(t) = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the system state, $u \in R$ is the control input to be determined, $\phi_i(x), i = 1, \ldots, n$ are known smooth vector functions, $\gamma(t) \in R^p$ represents some structured time-varying uncertainty with τ-periodic property, i.e., $\gamma(t) = \gamma(t+\tau)$, and $w_1(x_1,t), \cdots, w_n(x_1,\cdots,x_n,t)$ are some unstructured uncertainties bounded by known functions:

$$\|w_i(x_1,\cdots,x_i,t)\| \le \Delta_i(x_1,\cdots,x_i), i = 1,\cdots,n. \tag{2}$$

For functions $\Delta_i(x_1,\cdots,x_n)$ we make the following assumptions:

Assumption 1: It is assumed that $\Delta_i(0) = 0$, and derivative of $\Delta_i(x)$ exists and is zero at $x = 0$ for all $i = 1, \cdots, n$.

Assumption 1 is made for establishing asymptotic stability with respect to $x = 0$. When it is not the case, a slight modification of the proposed procedure in the next section will achieve boundedness and convergence to a compact set around $x = 0$.

The reference signal is generated by a known asymptotically stable system:

$$y_r = G_d(s)r(s) = \frac{k_d}{s^n + d_{n-1}s^{n-1} + \cdots + d_0} r(s). \tag{3}$$

where $s^n + d_{n-1}s^{n-1} + \cdots + d_0$ is Hurwitzian, $k_d > 0$, and $r(t)$ is bounded and piecewise continuous. In repetitive control $r(t)$ is a τ-periodic although it may not be required in general in our problem. The control objective is to design an appropriate control input $u(t) \in R$ for the uncertain system (1) such that the system output $y(t)$ tracks $y_r(t)$ with a predescribed accuracy, i.e., for a given small tolerance $\epsilon > 0$, there exists a $t_1 > 0$ such that

$$\|y(t) - y_r(t)\| \le \epsilon, \forall t > t_1 \tag{4}$$

where, and throughout the paper, $\|\cdot\|$ denotes Euclidean norm.

It is worth noting that both unstructured and structured periodic uncertainties are "unmatched" with the control input. This is the main obstacle against applying current leaning control mechanism to such systems. Note that system (1) is subject to the so-called parametric strict-feedback structure which has been extensively studied in references by using the backstepping-based robust adaptive control methodology technique (see, e.g., Kokotović and Arcak 2001, Kristć *et al.* 1995). It utilizes the concept of adaptive control Lyapunov function. A hypothesis of zero derivative of the uncertain parameters is thus needed for the method. However, in our problem $\gamma(t)$ can be time-varying rather than constant and its derivative is unknown or even does not exist. This makes the existing robust adaptive design procedure unapplicable either. The main goal of this paper is to extend the backstepping technique to the robust learning control mechanism and develop a new constructive control strategy to cope with nonlinear systems with both structured periodic and unstructured aperiodic uncertainties.

In learning control design, because of the periodicity of the operation and uncertainties, the time axis $[0, \infty)$ is segmented into a series of time intervals of the form $[(i-1)\tau, i\tau]$, $i = 1, 2, \cdots$, each of which is call a learning trial (Xu *et al.*, 2000). For convenience, we denote all variables in the i-th learning trial with the help of a superscript, for example, $x^i(t) = x((i-1)\tau + t), t \in [0, \tau]$, and $F^i(x) = F(x^i(t))$.

3. Robust learning control design

3.1. Robust learning control algorithm

The underlying idea of robust learning control is to learn and approximate the unknown periodic functions by using a repetitive learning mechanism and suppress any unstructured uncertainties by using a robust control technique.

The robust learning controller consists of three parts:

$$u^i = -(F_n^i)^T\bar{v}^i + \alpha_n^i(x, x_d) + \beta_n^i(x, x_d) \tag{5}$$

where $(F_n^i)^T\bar{v}^i$ is the learning function which approximates nonlinear dynamics with periodic uncertainties, $\alpha_n^i(x, x_d)$ is the learning control part which online compensates leaning and tracking errors, $\beta_n^i(x, x_d)$ is the robust control part suppressing unstructured uncertainties. In (5) $\bar{v}^i$ is the moving average estimation of uncertainty $\gamma(t)$ in the

$(i-1)$-th learning trial, i.e.,

$$\bar{v}^i(t) = \frac{1}{\tau}\left[\int_t^{\tau} v^{i-1}(s)\mathrm{d}s + \int_0^t v^i(s)\mathrm{d}s\right], \quad (6)$$

and the approximation for structured periodic uncertainty $\gamma(t)$ is updated via the following learning mechanism:

$$\begin{aligned} v^i &= v^{i-1} + k_1 F_1^i(x, x_d) z_1^i(x, x_d) + \cdots \\ &\quad + k_{n-1} F_{n-1}^i(x, x_d) z_{n-1}^i(x, x_d) \\ &\quad + k_n F_n^i(x, x_d) sgn(z_n^i(x, x_d)) \end{aligned} \quad (7)$$

with the initial value as a constant

$$v^1(t) = v_0, \ t \in [0, \tau], \quad (8)$$

where $k_1, k_2, \cdots, k_n > 0$ are some constant feed-forward gains, $F_j^i, z_i^j, j = 1, \cdots, n$ are some learning structure functions and error functions in the control process, which are to be specified in the subsequent subsections, $sgn(\cdot)$ is the sign function defined as

$$sgn(x) = \begin{cases} 1, \text{ if } x > 0; \\ 0, \text{ if } x = 0; \\ -1, \text{ if } x < 0. \end{cases} \quad (9)$$

3.2. Evaluation function for learning control

We introduce the following evaluation function as a guideline for derivation of the robust learning control law:

$$E^i(t) = \int_0^t \|\gamma(s) - v^i(s)\|^2 \mathrm{d}s. \quad (10)$$

The difference of evaluation function between two successive trials is

$$\Delta E^i(t) = E^i - E^{i-1}$$

$$= \int_0^t [(v^i(s) - v^{i-1}(s))^T (v^i(s) + v^{i-1}(s) - 2\gamma(s))]\mathrm{d}s.$$

The key idea of derivation of the learning controller is to ensure the decay of the evaluation function and hence ensure the convergence of the learning algorithm.

3.3. The recursive design procedure

We start by controlling the first equation of (1) considering x_2 to be a control input. The second step is a typical one and crucial for understanding the general design procedure. The robust learning controller and estimation update law are designed in the final step. The details of the design procedure and stability analysis of the closed-loop system are omitted due to page limitation. However, two design examples are given below to illustrate the method.

4. Design Examples

The first example shows the method is applicable to systems with nondifferentiable periodic uncertainties, and the second example shows it can be used for controlling chaotic systems with unknown forcing signals and parameters as well.

Example 1: Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 + x_1\gamma_1(t) \\ \dot{x}_2 = u + x_2^2\gamma_2(t) \end{cases} \quad (11)$$
$$y = x_1$$

where $\gamma_1(t)$ and $\gamma_2(t)$ are unknown periodic uncertainties, period of which is known as $\tau = 10$. In simulation they are taken as triangular and sinusoid functions shown in Figures 1. Obviously, these uncertainties can not be directly handled by traditional adaptive control mechanism.

The reference signal is generated by the following linear system:

$$\begin{cases} \dot{x}_{d,1} = x_{d,2} \\ \dot{x}_{d,2} = -5x_{d,1} - 4x_{d,2} + A\sin(\omega t + \frac{\pi}{3}) \end{cases} \quad (12)$$
$$y_r = x_{d,1}$$

In simulation we take $A = 5.5, \omega = \frac{\pi}{5}$. Note that, in general, the period of the target signal can be different from that of the uncertainties.

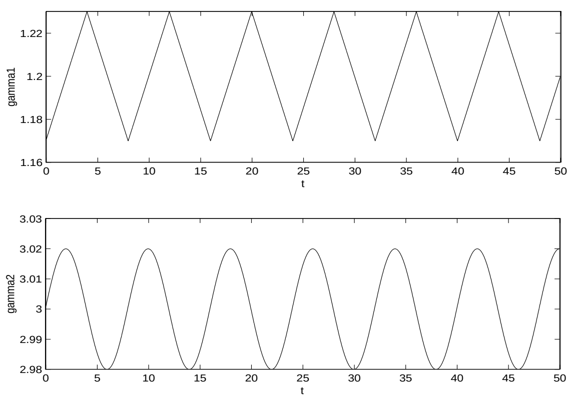

Figure 1: Periodic time-varying uncertainties

The learning control mechanism consists of the following two parts:

Learning control

$$u^i = -(F_2^i)^T \bar{v}^i + \alpha_2^i; \tag{13}$$

Update law

$$v^i = v^{i-1} + k_1 F_1^i z_1^i + k_2 F_2^i sgn(z_2^i), \tag{14}$$

where $k_1, k_2 > 0$.

Comparing system (11) with (1) shows that $w_1 = w_2 = 0$ and

$$\phi_1 = [x_1,\ 0]^T, \quad \phi_2 = [0,\ x_2^2]^T.$$

According to the design procedure sketched in Section 3.3, we let

$$z_1^i(t) = x_1^i(t) - x_{d,1}^i(t).$$

Recall that

$$x_1^i(t) = x_1((i-1)\tau + t), t \in [0, \tau].$$

Similar notations are applied to $z_1^i(t)$, $x_{d,i}^i(t)$ and other variables to be introduced in sequence.

The virtual control for x_2^i is

$$\mu_2^i = -(F_1^i)^T \bar{v}^i + \alpha_1^i,$$

where

$$\begin{aligned} \bar{v}^i &= \tfrac{1}{\tau}\left[\int_t^\tau v^{i-1}(s)\mathrm{d}s + \int_0^t v^i(s)\mathrm{d}s\right], \\ F_1^i &= \phi_1^i = [x_1^i,\ 0]^T, \end{aligned}$$

and

$$\alpha_1^i = -\frac{k_{f1}}{2}(x_1^i - x_{d,1}^i) + x_{d,2}^i - \frac{k_1}{2}(x_1^i)^2(x_1^i - x_{d,1}^i).$$

Therefore,

$$\begin{aligned} z_2^i = x_2^i - \mu_2^i = x_2^i + x_1^i \bar{v}^i + (\tfrac{k_{f1}}{2} \\ +(F_1^i)^T(v^i - v^{i-1}) + \tfrac{k_1}{2}(x_1^i)^2)(x_1^i - x_{d,1}^i) - x_{d,2}^i, \end{aligned}$$

$$\begin{aligned} F_2^i &= \phi_2^i - \frac{\partial \mu_2^i}{\partial x_1^i}\phi_1^i \\ &= [(\bar{v}_1^i + \frac{k_{f1}}{2} + \frac{3}{2}k_1(x_1^i)^2 - k_1 x_1^i x_{d,1}^i)x_1^i,\ (x_2^i)^2]^T. \end{aligned}$$

From the recursive design procedure we obtain

$$\begin{aligned} \alpha_2^i &= -\frac{k_{f2}}{2}z_2^i - \frac{k_2}{2}(F_2^i)^T F_2^i sgn(z_2^i) \\ &\quad -k_1(F_1^i)^T F_2^i z_1^i - \frac{k_1}{k_2}z_1^i + \frac{\partial \mu_2^i}{\partial x_1^i}x_2^i + \\ &\quad \frac{1}{2}(k_{f1} + k_1(x_1^i)^2)x_{d,2}^i \\ &\quad +(-5x_{d,1}^i - 4x_{d,2}^i + A\sin(\omega t + \frac{\pi}{3})) \\ &- \left((F_1^i)^T + \frac{1}{k_2}sgn(z_2^i)(v^{i-1} - \bar{v}^i)^T\right)(v^i - v^{i-1}). \end{aligned}$$

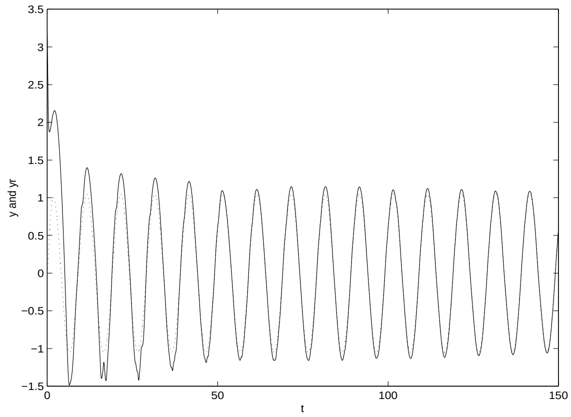

Figure 2: System output (solid) and reference signal (dashed).

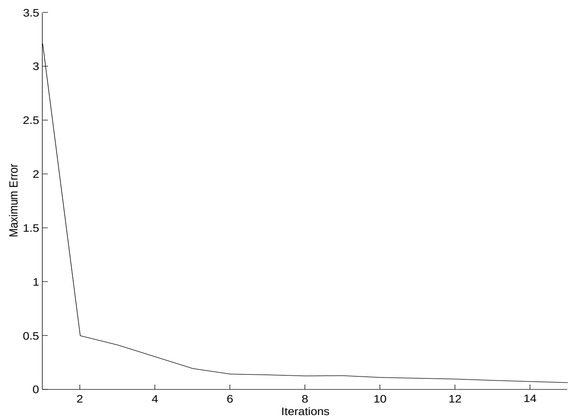

Figure 3: Error versus iteration.

We have thus determined the learning control (13) and update law (14). Taking the control parameters as $k_1 = 0.2$, $k_2 = 0.1$, $k_{f1} = k_{f2} = 6$ we have conducted the simulation. Figure 2 shows the system output tracks the reference signal after a few learning trials. Figure 3 depicts the maximum tracking error versus iteration.

Example 2: In this example we consider the well-known van der Pol oscillator

$$\begin{aligned} \dot{x}_1 &= x_1 - \tfrac{1}{3}x_1^3 - x_2 + p + F(t) \\ \dot{x}_2 &= 0.1(x_1 + a - bx_2) + u \end{aligned} \tag{15}$$

where $F(t) = qcos(wt)$ is a periodic exciting signal, u is a control input. Van der Pol's equation provides an example of an oscillator with nonlinear damping, energy being dissipated at large amplitudes and generated at low amplitudes. Such systems typically possess limit cycles, sustained even

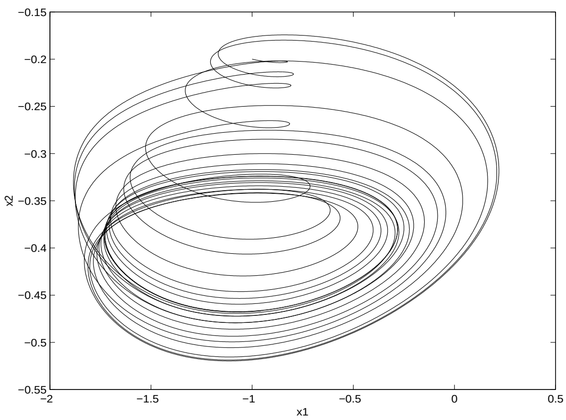

Figure 4: Von der Pol chaotic attractor.

chaotic oscillations around a state at which generation and dissipation balance, and they arise in many physical problems. A typical chaotic behavior of van der Pol oscillator without control (i.e., $u = 0$) is presented in Figure 4 (parameters are chosen as $\omega = 1$, $a = 0.7$, $b = 0.8$, $p = 0$ and $q = 0.74$).

In our problem not only the parameters a, b, p, q but also the forcing signal $F(t)$ are assumed unknown. Our task is to design a control input to force the system to track a periodic reference generated by the following system

$$\left\{ \begin{array}{l} \dot{x}_{d,1} = x_{d,2} \\ \dot{x}_{d,2} = -5x_{d,1} - 4x_{d,2} + r(t) \end{array} \right. \\ y_r = x_{d,1} \tag{16}$$

where $r(t) = -1 + 5.5\sin(\omega t + \frac{\pi}{2})$.

We do not try to suppress $F(t)$ by a robust controller, because, on the one hand, it is not a state-dependent perturbation or a minor disturbance but a significant forcing signal to the system; on the other hand, a robust controller does not give any performance improvement in presence of such a periodic uncertainty. So we treat $F(t)$ as a part of structured periodic uncertainties to be learnt. Now rewrite the van der Pol's equation in the canonical form (1):

$$\begin{array}{l} \dot{x}_1 = -x_2 + x_1 - \frac{1}{3}x_1^3 + \phi_1^T\gamma(t) \\ \dot{x}_2 = u + 0.1x_1 + \phi_2^T\gamma(t) \end{array} \tag{17}$$

where

$$\begin{array}{rcl} \phi_1 & = & [1,\ 1,\ 0,\ 0]^T, \\ \phi_2 & = & [0,\ 0,\ 1,\ -x_2]^T, \end{array}$$

and $\gamma(t) = [\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4]^T$ represents the periodic (or constant) uncertainties $p, F(t), a, b$.

There are some known terms in the right-hand side of equation (15). They may be considered as a trivial case of w_i in equation (1).

According to the design procedure sketched in Section 3.3, we let

$$z_1^i(t) = x_1^i(t) - x_{d,1}^i(t).$$

Recall that

$$x_1^i(t) = x_1((i-1)\tau + t), t \in [0, \tau].$$

The virtual control for $-x_2^i$ is

$$\mu_2^i = -(F_1^i)^T\bar{v}^i + \alpha_1^i + \beta_1^i,$$

where

$$F_1^i = \phi_1^i = [1,\ 1,\ 0,\ 0]^T,$$

$$\beta_1^i = -x_1^i + \frac{1}{3}(x_1^i)^3,$$

and

$$\begin{array}{rcl} \alpha_1^i & = & -\dfrac{k_{f1}}{2}(x_1^i - x_{d,1}^i) + x_{d,2}^i \\ & & -\dfrac{k_1}{2}(\phi_1^i)^T\phi_1^i(x_1^i - x_{d,1}^i) \\ & = & -(\dfrac{k_{f1}}{2} + k_1)(x_1^i - x_{d,1}^i) + x_{d,2}^i. \end{array}$$

Therefore,

$$\begin{array}{rcl} z_2^i & = & x_2^i - (-\mu_2^i) \\ & = & x_2^i - (\phi_1^i)^T\bar{v}^i - x_1^i + \dfrac{1}{3}(x_1^i)^3 \end{array}$$
$$-(F_1^i)^T(v^i - v^{i-1}) - (\frac{k_{f1}}{2} + k_1)(x_1^i - x_{d,1}^i) + x_{d,2}^i,$$

$$\begin{array}{rcl} F_2^i & = & \phi_2^i - \dfrac{\partial\mu_2^i}{\partial x_1^i}\phi_1^i \end{array}$$
$$[-1 + (x_1^i)^2 - \frac{k_{f1}}{2} - k_1,\ -1 + (x_1^i)^2 - \frac{k_{f1}}{2} - k_1,\ 1,\ -x_2^i]^T.$$

From the recursive design procedure we get

$$\begin{array}{rl} \alpha_2^i & = -\dfrac{k_{f2}}{2}z_2^i - \dfrac{k_2}{2}(F_2^i)^T F_2^i sgn(z_2^i) \\ & -k_1(F_1^i)^T F_2^i z_1^i - \dfrac{k_1}{k_2}z_1^i + \dfrac{\partial\mu_2^i}{\partial x_1^i}x_2^i \\ & -(\dfrac{k_{f1}}{2} + k_1)x_{d,2}^i - (-5x_{d,1}^i - 4x_{d,2}^i - 1 \\ & +5.5\sin(\omega t + \dfrac{\pi}{2})) \\ & + \left((F_1^i)^T - \dfrac{1}{k_2}sgn(z_2^i)(v^{i-1} - \bar{v}^i)^T\right)(v^i - v^{i-1}). \end{array}$$

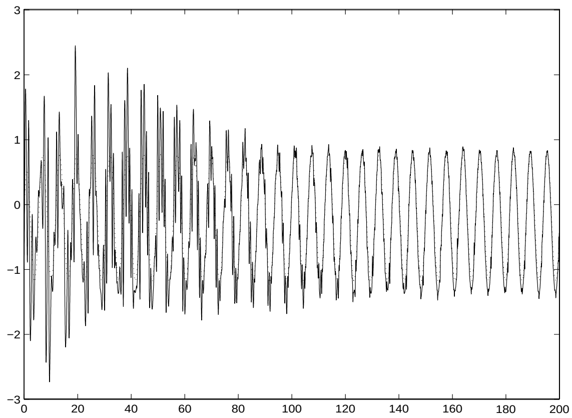

Figure 5: Controlled output (solid) and reference signal (dashed).

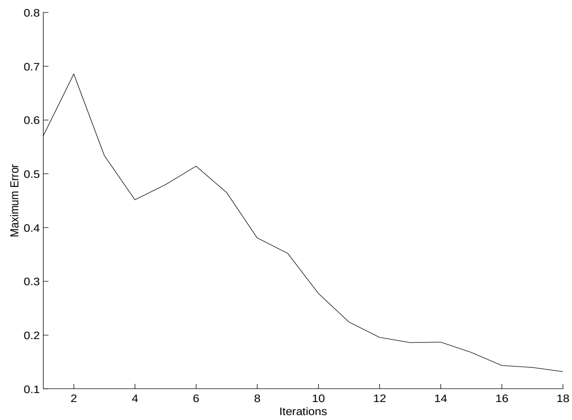

Figure 6: Error versus iteration.

We thus obtained the learning control and update law

$$u^i = -(F_2^i)^T \bar{v}^i - 0.1x_1^i + \alpha_2^i; \tag{18}$$

and

$$v^i = v^{i-1} + k_1 F_1^i z_1^i + k_2 F_2^i sgn(z_2^i). \tag{19}$$

Taking the control parameters as $k_1 = 0.2$, $k_2 = 0.1$, $k_{f1} = k_{f2} = 6$, we have conducted the simulation. Figure 5 shows the system output tracks the reference signal after several learning trials. Figure 6 depicts the maximum tracking error versus iteration. The figures also show that the learning time is longer than in the first example. This may be explained by the chaotic nature of the system.

5. Conclusion

A constructive method has been proposed for designing robust learning controllers for a class of nonlinear systems with periodic structured uncertainties and aperiodic unstructured uncertainties. The recursive technique proposed in this paper can be regarded as an extension of the currently popular backstepping idea to the robust learning control systems. However, the control law is not derived by the guidance of any (adaptive) Lyapunov function. Instead, a repetitive learning evaluation function was formulated for establishing the asymptotic stability of the tracking system. The main advantage of this method is it does not need the derivative information of the uncertainties. Therefore it makes it possible to remove the hypothesis of zero derivative of uncertain parameters in the case when the time-varying uncertainty is periodic. The proposed method is validated by simulation of tracking control of two systems with periodic uncertainties, one of which is the well-known van der Pol chaotic oscillator.

Acknowledgements

This work was supported by the National Key Project and National Natural Science Foundation of China, Fork Ying Tung Education Foundation, and partly by the Australian Research Council.

References

Kokotović, P., & Arcak, M. (2001). Constructive nonlinear control: a historical perspective. *Automatica*, 37, 637-662.

Kristić, M., Kanellakopoulos, I., & Kokotović, P. (1995). *Nonlinear and Adaptive Control Design.* New York: John Wiley & Sons, Inc.

Park, B. H., Kuc, T.-Y., & Lee, J.S. (1996). Adaptive learning control of uncertain robotic systems. *International Journal of Control*, 65, 725-744.

Xu, J.-X., & Qu, Z. (1998). Robust iterative learning control for a class of nonlinear systems. *Automatica*, 34, 983-988.

Xu, J.-X., Viswanathan, B., & Qu, Z. (2000). Robust learning control for robotic manipulators with an extension to a class of nonlinear system. *Int. J. Control*, 73, 955-967.