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SLIDING MODE OBSERVERS FOR ROBUST FAULT DETECTION & RECONSTRUCTION

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Abstract: This paper describes a method for designing sliding mode observers for detection and reconstruction of faults, that is robust against system uncertainty. The method uses $\mathcal{H}_\infty$ concepts to design the observer, minimising the effect of the uncertainty on the reconstruction of the faults. A VTOL aircraft example taken from the fault detection literature is used to demonstrate the method and its effectiveness.

Keywords: Fault detection, Sliding mode observer, Robustness

1. INTRODUCTION

The fundamental purpose of a Fault Detection & Isolation (FDI) scheme is to generate an alarm when a fault occurs and also to identify the nature and location of the fault. Many FDI methods are observer based: the plant output is compared with the output of an observer designed from a model of the system, and the discrepancy is used to form a residual. Using this residual, a decision is made as to whether a fault is present. However, the model of the system about which the observer is designed will possess uncertainties. These uncertainties could cause the FDI scheme to trigger a false alarm when there are no faults, or even worse, mask the effect of a fault, which may go undetected. Hence, there is a need for robust FDI schemes which are robust to model uncertainties.

Much has been done in the area of robust FDI. Examples of schemes using linear observers appear in (Chen and Zhang, 1991; Patton and Chen, 1992; Hou and Patton, 1996). FDI schemes have also been developed using nonlinear approaches, in particular, sliding mode observers. Yang and Saif (1995) and Hermans and Zarrop (1996) used sliding mode observers to generate residuals. In this paper, rather than generate residuals, a nonlinear sliding mode observer based strategy will be explored which seeks to *reconstruct* the fault signals (Edwards *et al.*, 2000). A sliding mode observer under consideration feeds back the output error through a discontinuous switched term which is intended to induce a sliding motion in the state estimation error space. It was argued by Edwards *et al.* (2000) that by appropriate processing of the so-called *equivalent output error injection* signal required to maintain sliding, information about the faults could be obtained. This paper builds on the work of Edwards *et al.* (2000) by introducing explicitly an uncertainty representation into the design framework. A new design method is presented for the observer gains which seeks to minimise the $\mathcal{L}_2$ gain between the uncertainty and the fault reconstruction signal. The efficacy of this method will be demonstrated with a VTOL aircraft taken from the FDI literature.

2. PRELIMINARIES

This section introduces the preliminaries and background ideas necessary for the work presented in this paper. Consider the uncertain dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t) + Ff_i(t,u) + M\xi(t,y,u) \quad (1)$$
$$y(t) = Cx(t) \quad (2)$$

where $A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^{n\times m}, C \in \mathcal{R}^{p\times n}$, $F \in \mathcal{R}^{n\times q}$ and $M \in \mathcal{R}^{n\times k}$ where $n > p \geq q$. Assume that the matrices C and F are full rank and the function $f_i : \mathcal{R}_+ \times \mathcal{R}^m \rightarrow \mathcal{R}^q$ is unknown but bounded so that $\|f_i(t,u)\| \leq \alpha(t,u)$ where $\alpha : \mathcal{R}_+ \times \mathcal{R}^m \rightarrow \mathcal{R}_+$ is a known function. The signal $f_i(t,u)$ represents an actuator fault. The map $\xi : \mathcal{R}_+ \times \mathcal{R}^p \times \mathcal{R}^m \rightarrow \mathcal{R}^k$ encapsulates the uncertainty present. It is assumed to be unknown but bounded subject to $\|\xi(t,y,u)\| < \beta$ where the positive scalar β is known.

Edwards and Spurgeon (1994) have proven that if $p > q$, $rank(CF) = q$ and any invariant zeros of (A, F, C) lie in the left half plane, then there exists a change of co-ordinates in which the system triple (A, F, C) has the following structure :

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, F = \begin{bmatrix} 0 \\ F_o \end{bmatrix}, C = \begin{bmatrix} 0 & T \end{bmatrix} \quad (3)$$

where $A_{11} \in \mathcal{R}^{(n-p)\times(n-p)}, F_o \in \mathcal{R}^{q\times q}$ is non-singular and $T \in \mathcal{R}^{p\times p}$ is orthogonal. Define A_{211} as the top $p-q$ rows of A_{21}. By construction, the pair (A_{11}, A_{211}) is detectable and the unobservable modes of (A_{11}, A_{211}) are the invariant zeros of (A, F, C) (Edwards and Spurgeon, 1994). Also for convenience, define $F_2 \in \mathcal{R}^{p\times q}$ as the bottom p rows of F (which therefore includes F_o). In this co-ordinate system, the matrix M has the general structure

$$M = \begin{bmatrix} M_1^T & M_2^T \end{bmatrix}^T$$

where $M_1 \in \mathcal{R}^{(n-p)\times k}$.

Edwards and Spurgeon (1994) propose a state observer of the form

$$\dot{z}(t) = Az(t) + Bu(t) - G_l e_y(t) + G_n \nu \quad (4)$$

The discontinuous output error injection vector ν is defined by

$$\nu = \begin{cases} -\rho(t,y,u)\frac{P_o e_y}{\|P_o e_y\|} & \text{if } e_y \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where $e_y := Cz - y$ and $P_o \in \mathcal{R}^{p\times p}$ is a symmetric positive definite (s.p.d.) matrix which will be defined later. The function $\rho : \mathcal{R}_+ \times \mathcal{R}^p \times \mathcal{R}^m \rightarrow \mathcal{R}_+$ will also be described formally later but it must represent an upper bound on the magnitude of the fault signal plus the uncertainty. The nonlinear gain G_n is assumed to have the structure

$$G_n = \begin{bmatrix} -LT^T \\ T^T \end{bmatrix} \quad (6)$$

where $L = \begin{bmatrix} L^o & 0 \end{bmatrix}$ with $L^o \in \mathcal{R}^{(n-p)\times(p-q)}$. The matrices L^o, P_o and G_l are to be determined. In (Tan and Edwards, 2000), the system associated with the state estimation error $e := z - x$ is analysed when $\xi = 0$ and $\rho = \|CF\|\alpha(t,u) + \eta_o$ where η_o is a positive scalar. The following result was proved:

Proposition 1. Suppose there exists a s.p.d. matrix P, with the structure

$$P = \begin{bmatrix} P_1 & P_1 L \\ L^T P_1 & P_2 + L^T P_1 L \end{bmatrix} > 0 \quad (7)$$

where $P_1 \in \mathcal{R}^{(n-p)\times(n-p)}$, $P_2 \in \mathcal{R}^{p\times p}$ are s.p.d., that satisfies $P(A-G_lC)+(A-G_lC)^TP < 0$, then if $P_o := TP_2T^T$, the error $e(t)$ is quadratically stable. Furthermore, a sliding motion occurs in finite time on $S = \{e : Ce = 0\}$ governed by the system matrix $A_{11} + L^o A_{211}$. □

This result will now be generalised to the case of uncertain systems as given in (1), using ideas similar to those by Koshkouei and Zinober (2000). From (1) and (4), and defining $A_o = A - G_l C$, the state estimation error satisfies

$$\dot{e}(t) = A_o e(t) + G_n \nu - F f_i(t,u) - M\xi(t,y,u) \quad (8)$$

Suppose there exists a s.p.d. matrix P which satisfies the requirements of Proposition 1. Define $\mu_0 = -\lambda_{max}(PA_o + A_o^T P)$ and $\mu_1 = \|PM\|$. If the scalar gain function in (5) satisfies $\rho \geq \|CF\|\alpha(t,u) + \eta_o$, where $\eta_o > 0$ then the following holds:

Lemma 1. The state estimation error $e(t)$ in (8) is ultimately bounded with respect to the set

$$\Omega_\epsilon = \{e : \|e\| < 2\mu_1\beta/\mu_0 + \epsilon\}$$

where $\epsilon > 0$ is an arbitrarily small positive scalar.

Proof : Ultimate boundedness with respect to Ω_ϵ can be shown using $V(e) = e^T Pe$ and employing arguments similar to those by Koshkouei and Zinober (2000). □

Lemma 1 will now be used to prove the main result of this section; that for an appropriate choice of ρ, a sliding motion can be induced on $\mathcal{S} = \{e : Ce = 0\}$. First introduce a co-ordinate change as in (Edwards and Spurgeon, 1994). Let $T_L : e \mapsto e_L$ where

$$T_L := \begin{bmatrix} I_{n-p} & L \\ 0 & T \end{bmatrix} \quad (9)$$

In this new co-ordinate system, the triple (A, F, C) will be in the form

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix}, \mathcal{F} = \begin{bmatrix} 0 \\ \mathcal{F}_2 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 0 & I_p \end{bmatrix} \quad (10)$$

where $\mathcal{A}_{11} = A_{11} + L^o A_{211}$ and $\mathcal{F}_2 = TF_2$. The matrix M will be transformed to be

$$\mathcal{M} = \begin{bmatrix} \mathcal{M}_1^T & \mathcal{M}_2^T \end{bmatrix}^T$$

where $\mathcal{M}_2 = TM_2$. The Lyapunov matrix will be

$$\mathcal{P} = (T_L{}^{-1})^T P (T_L{}^{-1}) = diag\{P_1, P_o\} \quad (11)$$

and the nonlinear gain will be

$$\mathcal{G}_n = \begin{bmatrix} 0 & I_p \end{bmatrix}^T \quad (12)$$

Edwards and Spurgeon (1994) argue that since $\mathcal{P}$ is a block diagonal Lyapunov matrix for $\mathcal{A} - \mathcal{G}_l\mathcal{C}$, it implies that $\mathcal{A}_{11}$ is stable and hence the sliding motion is stable. In the new co-ordinates

$$\dot{e}_L(t) = \mathcal{A}_o e_L(t) + \mathcal{G}_n \nu - \mathcal{F} f_i(t,u) - \mathcal{M}\xi \quad (13)$$

where $\mathcal{A}_o = \mathcal{A} - \mathcal{G}_l\mathcal{C}$. Partitioning e_L conformably with (10)

$$\dot{e}_1(t) = \mathcal{A}_{11}e_1(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1})e_y(t) - \mathcal{M}_1\xi \quad (14)$$

$$\dot{e}_y(t) = \mathcal{A}_{21}e_1(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})e_y(t) - \mathcal{F}_2 f_i(t,u) - \mathcal{M}_2\xi + \nu \quad (15)$$

where $\mathcal{G}_{l,1}$ and $\mathcal{G}_{l,2}$ represent appropriate partitions of $\mathcal{G}_l$.

Proposition 2. An ideal sliding motion takes place on $\mathcal{S} = \{e : Ce = 0\}$ in finite time if the function from (5) satisfies (for a positive scalar η_o)

$$\rho \geq 2\|\mathcal{A}_{21}\|\mu_1\beta/\mu_o + \|\mathcal{M}_2\|\beta + \|\mathcal{F}_2\|\alpha + \eta_o \quad (16)$$

Proof : Consider a candidate Lyapunov function $V_s = e_y^T P_o e_y$. Because $\mathcal{P}$ from (11) is a block diagonal Lyapunov matrix for $(\mathcal{A} - \mathcal{G}_l\mathcal{C})$, it follows that

$$(P_o(\mathcal{A}_{22} - \mathcal{G}_{l,2}) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})^T P_o) < 0$$

and hence

$$\dot{V}_s \leq 2e_y^T P_o(\mathcal{A}_{21}e_1 - \mathcal{F}_2 f_i - \mathcal{M}_2\xi) - 2\rho\|P_o e_y\|$$
$$\leq -2\|P_o e_y\|(\rho - \|\mathcal{A}_{21}e_1\| - \|\mathcal{F}_2 f_i\| - \|\mathcal{M}_2\xi\|)$$

From Lemma 1, in finite time $e(t) \in \Omega_\epsilon$ which implies $\|e_1\| < 2\mu_1\beta/\mu_o + \epsilon$. Therefore from the definition of ρ in (16),

$$\dot{V}_s \leq -2\eta_o\|P_o e_y\| \leq -2\eta_o\eta\sqrt{V_s}$$

where $\eta := \sqrt{\lambda_{min}(P_o)}$. This proves that the output estimation error e_y will reach zero in finite time, and a sliding motion takes place. □

3. ROBUST RECONSTRUCTION OF ACTUATOR FAULTS

In this section the sliding mode observer will be analysed with regard to its ability to robustly reconstruct the fault $f_i(t,u)$ despite the presence of the uncertainty ξ. The analysis will be performed with the condition that $p > q$. Assuming a sliding mode observer has been designed, and that a sliding motion has been achieved, $e_y = \dot{e}_y = 0$ and (14) - (15) become

$$\dot{e}_1(t) = \mathcal{A}_{11}e_1(t) - \mathcal{M}_1\xi \quad (17)$$

$$\nu_{eq} = -\mathcal{A}_{21}e_1(t) + \mathcal{F}_2 f_i(t,u) + \mathcal{M}_2\xi \quad (18)$$

where ν_{eq} is the equivalent output error injection term (the natural analogue of the concept of the equivalent control (Utkin, 1992)) required to maintain a sliding motion. From (Edwards *et al.*, 2000), the signal ν_{eq} can be approximated to any degree of accuracy, and is computable online as

$$\nu_{eq} = -\rho(y,t,u)\frac{P_o e_y}{\|P_o e_y\| + \delta} \quad (19)$$

where δ is a small positive scalar. From (17) and (18), the term ν_{eq} depends on, or equivalently gives information about the fault f_i. The case where there is no uncertainty was investigated by Edwards *et al.* (2000). In the case when $\xi \neq 0$ the attempted reconstruction of f_i will be corrupted by the exogenous signal ξ. Ideally the objective here is to choose L and a scaling of ν_{eq} to minimise the effect of ξ on the fault reconstruction. Define a would-be reconstruction signal $\hat{f}_i = WT^T\nu_{eq}$ where $W := \begin{bmatrix} W_1 & F_o^{-1} \end{bmatrix}$ and $W_1 \in \mathcal{R}^{q\times(p-q)}$ represents design freedom. It follows from (17) - (18) that $\hat{f}_i(t) = f_i(t,u) + \hat{G}(s)\xi$ where the transfer function

$$\hat{G}(s) = WA_{21}(sI - \mathcal{A}_{11})^{-1}\mathcal{M}_1 + WM_2$$

The objective now is to reconstruct the fault $f_i(t,u)$ whilst minimising the effect of $\xi(t,y,u)$. Using the Bounded Real lemma (Chilali and Gahinet, 1996), the $\mathcal{L}_2$ gain from the exogenous signal ξ to $\hat{f}_i$ will not exceed γ if the following holds

$$\begin{bmatrix} \hat{P}\mathcal{A}_{11} + \mathcal{A}_{11}^T\hat{P} & -\hat{P}\mathcal{M}_1 & -(WA_{21})^T \\ -\mathcal{M}_1^T\hat{P} & -\gamma I & (WM_2)^T \\ -WA_{21} & WM_2 & -\gamma I \end{bmatrix} < 0 \quad (20)$$

where the scalar $\gamma > 0$ and $\hat{P} \in \mathcal{R}^{(n-p)\times(n-p)}$ is s.p.d. The objective is to find $\hat{P}, L$ and W to minimise γ subject to (20) and $\hat{P} > 0$. However this must be done in conjunction with satisfying the requirements of obtaining a suitable sliding mode observer as expressed in Proposition 2. Writing P from (7) as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0 \quad (21)$$

where $P_{11} \in \mathcal{R}^{(n-p)\times(n-p)}$ and $P_{12} := \begin{bmatrix} P_{121} & 0 \end{bmatrix}$ with $P_{121} \in \mathcal{R}^{(n-p)\times(p-q)}$, it follows there is a one-to-one correspondence between the variables (P_{11}, P_{12}, P_{22}) and (P_1, L, P_2) since

$$P_1 = P_{11} \quad (22)$$

$$L = P_{11}^{-1}P_{12} \quad (23)$$

$$P_2 = P_{22} - P_{12}^T P_{11}^{-1}P_{12} \quad (24)$$

Choosing $\hat{P} = P_{11}$, it follows

$$\hat{P}\mathcal{A}_{11} = P_{11}A_{11} + P_{12}A_{21} \quad (25)$$

$$\hat{P}\mathcal{M}_1 = P_{11}M_1 + P_{12}M_2 \quad (26)$$

From (25) - (26), it follows that (20) is affine with respect to the variables P_{11}, P_{12}, W_1 and γ and suggests that convex optimisation techniques are appropriate.

4. DESIGNING THE OBSERVER

This section will present a method to design the sliding mode observer gains L^o, G_l and P_o to induce the inequality (20). Specifically in this section it is proposed that the linear gain G_l from (4) be chosen to satisfy

$$\begin{bmatrix} PA_o + A_o^T P & P(G_l D - B_d) & E^T \\ (G_l D - B_d)^T P & -\gamma_o I & H^T \\ E & H & -\gamma_o I \end{bmatrix} < 0 \quad (27)$$

where P is given in (21) and the four matrices $B_d \in \mathcal{R}^{n\times(p+k)}, D \in \mathcal{R}^{p\times(p+k)}, H \in \mathcal{R}^{q\times(p+k)}$ and $E \in \mathcal{R}^{q\times n}$. The scalar $\gamma_o > 0$. Further assume :

$$B_d = \begin{bmatrix} 0 & M \end{bmatrix} \tag{28}$$

$$D = \begin{bmatrix} D_1 & 0 \end{bmatrix} \tag{29}$$

$$H = \begin{bmatrix} 0 & H_2 \end{bmatrix} \tag{30}$$

where $D_1 \in \mathcal{R}^{p\times p}$ is a non-singular user defined parameter and $H_2 \in \mathcal{R}^{q\times k}$ depends on W_1.

If a feasible solution to (21) and (27) exists then the requirements of Proposition 2 will be fulfilled (since (27) implies $PA_o + A_o^T P < 0$). Hence the choice of G_l, the gain matrix L from (23) which follows once P is specified, G_n from (5) and P_o constitute a sliding mode observer design.

Proposition 3. Inequality (27) is feasible if and only if

$$\begin{bmatrix} PA + A^T P - \gamma_o C^T D_x^{-1} C & -PB_d & E^T \\ -B_d^T P & -\gamma_o I & H^T \\ E & H & -\gamma_o I \end{bmatrix} < 0 \tag{31}$$

where $D_x = DD^T$ and an appropriate choice of G_l is given by

$$G_l = \gamma_o P^{-1} C^T D_x^{-1} \tag{32}$$

Proof : Defining $Y := PG_l$, and re-arranging inequality (27) yields

$$\begin{bmatrix} PA + A^T P - YC - (YC)^T & E^T & YD - PB_d \\ E & -\gamma_o I & H \\ (YD - PB_d)^T & H^T & -\gamma_o I \end{bmatrix} < 0 \tag{33}$$

Since $\gamma_o > 0$, using the Schur complement and the fact that $DH^T = 0$ and $DB_d^T = 0$, inequality (33) is equivalent to

$$\begin{bmatrix} PA + A^T P + Q(P,Y) & \frac{1}{\gamma_o}(E - PB_d H^T) \\ \frac{1}{\gamma_o}(E - PB_d H^T)^T & \frac{1}{\gamma_o} HH^T - \gamma_o I \end{bmatrix} < 0 \tag{34}$$

where

$$Q(P,Y) := \triangle - \gamma_o C^T D_x^{-1} C + \frac{1}{\gamma_o} PB_d B_d^T P$$

and

$$\triangle := \frac{1}{\gamma_o}(YD_x - \gamma_o C^T) D_x^{-1} (YD_x - \gamma_o C^T)^T \tag{35}$$

For a choice of $Y = \gamma_o C^T D_x^{-1}$, the term $\triangle = 0$. Thus necessary and sufficient conditions for (34) to hold is that (34) holds when $Y = \gamma_o C^T D_x^{-1}$. From the Schur complement, this is equivalent to

$$\begin{bmatrix} PA + A^T P - \gamma_o C^T D_x^{-1} C & E^T & -PB_d \\ E & -\gamma_o I & H \\ -B_d^T P & H^T & -\gamma_o I \end{bmatrix} < 0 \tag{36}$$

or equivalently,

$$\begin{bmatrix} PA + A^T P - \gamma_o C^T D_x^{-1} C & -PB_d & E^T \\ -B_d^T P & -\gamma_o I & H^T \\ E & H & -\gamma_o I \end{bmatrix} < 0 \tag{37}$$

The choice $Y = \gamma_o C^T D_x^{-1}$ implies G_l has the form given in (32) and the claim is proven. □

The idea is now to relate (31) to (20). By using the partitions for B_d, D and H from (28) - (30), it is straightforward to show that a necessary condition for (31) to hold is that

$$\begin{bmatrix} P_{11}\mathcal{A}_{11} + \mathcal{A}_{11}^T P_{11} & -P_{11}\mathcal{M}_1 & E_1^T \\ -\mathcal{M}_1^T P_{11} & -\gamma_o I & H_2^T \\ E_1 & H_2 & -\gamma_o I \end{bmatrix} < 0 \tag{38}$$

since (38) is 'embedded' in inequality (31). Choosing $E_1 = -WA_{21}$ and $H_2 = WM_2$ will yield the same inequality as (20). The design method can now be summarised to be

> Minimise γ with respect to the variables P and W_1 subject to (31), (20) and (21), where $\gamma_o > 0$ is an a-priori user-defined scalar.

Remarks - Let γ_{min} be the minimum value of γ that satisfies (20), then, since (20) is a 'sub-block' of (31), $\gamma_{min} \leq \gamma_o$ always holds. The optimisation problem above is convex and standard Linear Matrix Inequality software can be used to synthesise numerically γ, P and W_1. Once P has been determined, L can be determined from (23), G_l from (32), G_n from (5), and P_o from Proposition 1.

For a given B_d, D, E and H, inequality (27) can be viewed as resulting from an $\mathcal{H}_\infty$ filtering problem (page 462 of (Zhou *et al.*, 1995)), the idea being to minimise the effect of ξ on $\triangle_z$ (see Figure 1: notation taken from Zhou *et al.* (1995)). However, here, E and H are regarded as design variables, which in particular depend on W.

Once sliding is established, the choice of the linear gain G_l is technically not relevant since the linear output error injection term $G_l e_y \equiv 0$ because $e_y \equiv 0$.

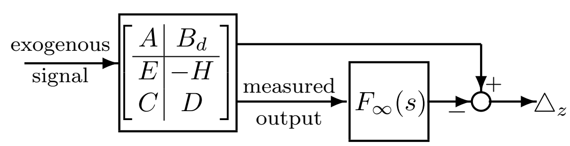

Fig. 1. The $\mathcal{H}_\infty$ filtering problem.

5. ROBUST RECONSTRUCTION OF SENSOR FAULTS

In this section, the actuator fault reconstruction method in §3 will be modified to enable robust sensor fault reconstruction in the presence of uncertainty. In this case, the system under consideration is the following:

$$\dot{x}(t) = Ax(t) + Bu(t) + M\xi(t, y, u) \tag{39}$$

$$y(t) = Cx(t) + Nf_o(t) \tag{40}$$

where $f_o \in \mathcal{R}^r$ is the vector of sensor faults, $N \in \mathcal{R}^{p\times r}$ where $rank(N) = r$ and $r \leq p$. A physical interpretation of this is that some of the sensors are assumed to be perfect. The objective is to transform this problem so that the method described in §3 and §4 can be employed to robustly reconstruct $f_o(t)$. Consider a new state $z_f \in \mathcal{R}^p$ that is a filtered version of y, satisfying

$$\dot{z}_f(t) = -A_f z_f(t) + A_f C x(t) + A_f N f_o(t) \quad (41)$$

where $-A_f \in \mathcal{R}^{p\times p}$ is a stable (filter) matrix. Equations (39) - (41) can be combined to form an augmented state-space system of order $n+p$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}_f(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ A_f C & -A_f \end{bmatrix}}_{A_a} \begin{bmatrix} x(t) \\ z_f(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_a} u(t)$$

$$+ \underbrace{\begin{bmatrix} 0 \\ A_f N \end{bmatrix}}_{F_a} f_o(t) + \underbrace{\begin{bmatrix} M \\ 0 \end{bmatrix}}_{M_a} \xi(t,y,u) \quad (42)$$

$$z_f(t) = \underbrace{\begin{bmatrix} 0 & I_p \end{bmatrix}}_{C_a} \begin{bmatrix} x(t) \\ z_f(t) \end{bmatrix} \quad (43)$$

Equations (42) - (43) are in the form of (1) - (2), and treat the sensor fault as an actuator fault. As described in §3, an observer driven by the signal z_f can be designed, replacing (A, F, C, M) with (A_a, F_a, C_a, M_a) respectively. From the general sliding mode observer theory by Edwards and Spurgeon (1994), an appropriate sliding mode observer exists if

- $rank(C_a F_a) = r$
- any invariant zeros of (A_a, F_a, C_a) lie in the left half plane

Since $rank(N) = r$, the first condition is satisfied since $C_a F_a = A_f N$ and A_f is invertible. An expression pertaining to the invariant zeros of (A_a, F_a, C_a) will now be derived in terms of the system block matrices.

Proposition 4. The invariant zeros of (A_a, F_a, C_a) are a subset of $\lambda(A)$. If $\lambda_i \in \lambda(A)$, then λ_i is an invariant zero if and only if

$$rank \begin{bmatrix} \lambda_i I - \mathcal{A}_{11} & -\mathcal{A}_{12} N \\ -\mathcal{A}_{12} & \lambda_i N - \mathcal{A}_{22} N \end{bmatrix} \neq n - p + r \quad (44)$$

Proof : See Tan and Edwards (2001) $\square$

6. AN EXAMPLE

The robust FDI scheme in this paper will now be demonstrated with an example, which is a VTOL aircraft model by Saif and Guan (1993). Its states, outputs and inputs respectively are

$$x = \begin{bmatrix} \text{horizontal velocity (knots)} \\ \text{vertical velocity (knots)} \\ \text{pitch rate (deg/s)} \\ \text{pitch angle (deg)} \end{bmatrix}$$

$$y = \begin{bmatrix} \text{horizontal velocity (knots)} \\ \text{vertical velocity (knots)} \\ \text{pitch angle (deg)} \end{bmatrix}$$

$$u = \begin{bmatrix} \text{collective pitch control} \\ \text{longitudal cyclic pitch control} \end{bmatrix}$$

In the notation of (1) - (2) and (39) - (40),

$$A = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.4422 \\ 3.5446 \\ -5.5200 \\ 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The (parametric) uncertainty ξ is given by

$$\xi = \begin{bmatrix} \begin{bmatrix} 0 & \triangle a_{32} & \triangle a_{34} \end{bmatrix} y \\ \begin{bmatrix} \triangle b_{21} & 0 \end{bmatrix} u \end{bmatrix}$$

where $\triangle a_{32} = 0.5, \triangle a_{34} = 2$ and $\triangle b_{21} = 2$.

6.1 *Robust reconstruction of actuator faults*

A sliding mode observer was designed using the method presented in §4. The design parameters were chosen as $D_1 = 10 I_3$ (from (29)) and $\gamma_o = 1$ (from (31)). The optimisation routine yielded a value of $\gamma = 5.6728 \times 10^{-4}$. The scalar function ρ from (19) was chosen to be 50, and δ was chosen to be 0.001.

Figure 2 shows the sliding mode observer faithfully reconstructing the actuator fault, rejecting the effect of the uncertainty. The fault condition is taken from Saif and Guan (1993).

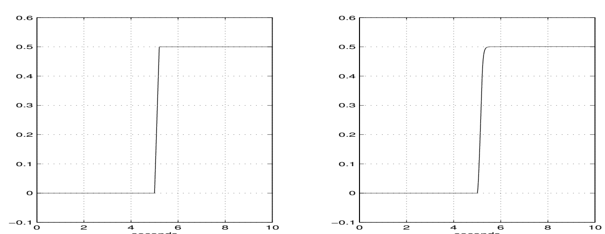

Fig. 2. *The left subfigure is a fault on the first actuator, and the right subfigure is the reconstruction of the fault for the noise free simulation*

In Figure 3 the same scenario is used except that the sensor signals were subject to white noise of standard deviation of 5×10^{-4}. As before the observer replicates the fault, except noise is now overlaid on the reconstruction signal.

6.2 *Robust reconstruction of sensor faults*

The following parameters were chosen for the design of the observer associated with the method described in §5. The filter matrix from (41) was chosen as $A_f = 20 I_3$.

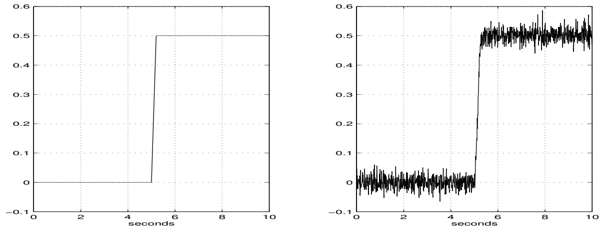

Fig. 3. *The left subfigure is a fault on the first actuator, and the right subfigure is the reconstruction of the fault*

The observer design method proposed in §4 was adopted. The tuning parameter for the linear component of the observer, $D_{1,a}$ from (29), was chosen as $100I_3$ and $\gamma_{o,a}$ was chosen to be unity (where the subscripts 'a' indicate the parameters are for the observer associated with sensor fault reconstruction). The optimisation routine yielded a value of $\gamma_a = 1.6697 \times 10^{-4}$. For this simulation, $\rho_a = 50$ and $\delta_a = 1 \times 10^{-5}$. Once again a fault scenario described in Saif and Guan (1993) is adopted to demonstrate the efficacy of the approach.

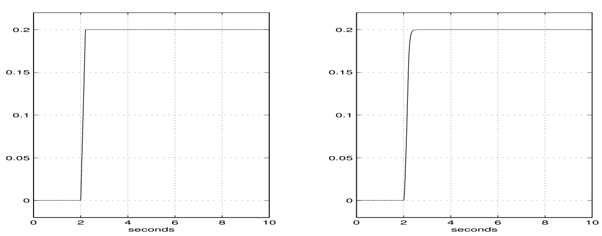

Fig. 4. *The left subfigure is a fault on the third sensor, and the right subfigure is the reconstruction of the fault for the noise free simulation*

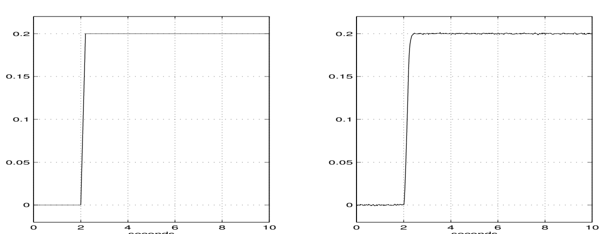

Fig. 5. *The left subfigure is a fault on the third sensor, and the right subfigure is the reconstruction of the fault in the presence of noise*

Figures 4 and 5 show the observer faithfully reconstructing the sensor fault, rejecting the effect of the uncertainty.

7. CONCLUSION

This paper has proposed a method for robust reconstruction of faults using sliding mode observers that minimise the effect of system uncertainty on the reconstruction. The method was developed initially for the reconstruction of actuator faults. This was subsequently extended to the case of sensor faults by the introduction of suitable filters, which enable the sensor faults to be treated as actuator faults. Both methods were demonstrated with an example taken from the FDI literature.

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