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APPLICATION OF H_∞ ROBUST CONTROL TO PARAPLEGIC STANDING

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Abstract: The aim of this work was to design and test robust H_∞ feedback control systems for the control of the upright posture of paraplegic persons standing. While the subject stands in a special apparatus, stabilising torque at the ankle joint is generated by electrical stimulation of the paralyzed calf muscle of both legs using surface electrodes. This allows the subject to stand without the need to hold on to external supports for stability: we call this "unsupported standing". Sensors in the apparatus allow measurement of ankle moments and the inclination angle. A nested loop structure for control of standing is implemented where an inner loop provides control of the total ankle moment while the controller in the outer loop regulates the inclination angle. A difficulty in using electrically-stimulated muscles as actuators in this setup is the very significant degree of nonlinearity in the muscle response. We therefore focus here on robust H_∞-control design for the angle control loop. The design approach is verified in experiments with a paraplegic subject.

Keywords: Functional Electrical Simulation, H_∞ control, Biomedical systems, Rehabilitation engineering.

1. INTRODUCTION

Spinal cord injury results in the interruption of the neurological pathway from the brain to the muscles. For people who have a spinal cord injury at mid to low thoracic level (in the back) the resulting dysfunction includes standing and stepping. After spinal cord injury muscles generally maintain their ability to contract. This offers the possibility of electrical stimulation of the muscles in order to recover useful functionality of the paralysed limbs. This stimulation of muscle (or better: motor nerves) is known as Functional Electrical Stimulation (FES) (Kralj and Bajd, 1989). We have been developing control systems for unsupported standing, using a special apparatus (see figure 1), know as the Wobbler (Hunt *et al.*, 1998*b*; Hunt *et al.*, 1997). While standing in the apparatus the subject wears a body brace which locks the knee and hip joints. Thus we can regard the body in its simplest form as a single link inverted pendulum with movement only around the ankle joints. The ankle is controlled by stimulation of the calf muscles. A major design problem in this setup is the signifi-

Fig. 1. Subject standing in the apparatus.

cant nonlinearity in artificially-stimulated muscle (Gollee and Hunt, 1997; Hunt *et al.*, 1998*a*), and this causes significant limitations in the achievable closed-loop performance.

Our work on unsupported standing has used a nested-loop control structure (see figure 2) (Hunt *et al.*, 1997). In this setup an inner controller regulates the total ankle moment, while the outer loop controller regulates the angle of inclination. In previous work (Hunt *et al.*, 2001) a polynomial

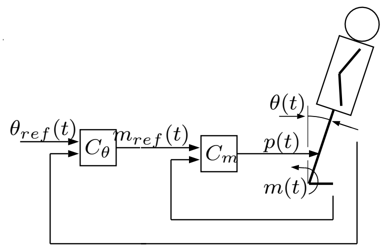

Fig. 2. Nested loop control structure. $\boldsymbol{\theta}$ is the inclination angle, $\boldsymbol{m}$ is the muscle moment and $\boldsymbol{p}$ the pulsewidth of the stimulation. $\boldsymbol{C_m}$ is the moment controller and $\boldsymbol{C_\theta}$ is the angle controller.

pole placement approach has been used for controller design for both angle and ankle moment control. These controllers have been tested with paraplegic subjects. The controllers gave satisfactory performance, with several periods of successful unsupported standing. However, the pole assignment approach does not allow uncertainty in the plant to be dealt with in a direct way during the design process. In this paper we investigate the design of a robust H_∞ controller for the angle control loop. The weighting functions in the design are developed by taking direct account of the uncertainty in the inner loop, which arises principally due to nonlinearity in the stimulated muscle. The design approach is tested in experiments with one paraplegic subject.

The paper is structured as follows: section 2 describes the overall control system and the motivation to apply an H_∞ design approach. In sections 3 and 4 the H_∞-controller design for unsupported subject is summarised. Experimental results are given in section 5 and conclusions in section 6.

2. METHODOLOGY

The nested loop structure for unsupported standing (see figure 3) allows the overall feedback control system to be designed and tested in several steps, starting with the ankle moment control loop and moving then to the body angle controller.

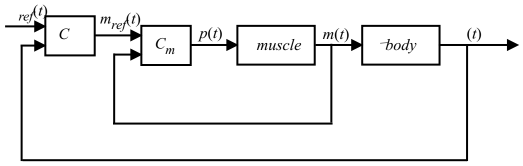

Fig. 3. Nested loop structure for unsupported standing

2.1 Apparatus

The Wobbler apparatus is described in detail in (Donaldson *et al.*, 1997). The Wobbler has been designed to allow investigation of artificial control strategies for unsupported standing without interference from the brain. To this end, a custom-fitted body shell is worn which locks the knee and hip joints; the subject is therefore free to rotate only around the ankle joint. The feet are positioned in footboxes. A load cell between the two boxes allows measurement of total ankle moment. A string attached to the body brace at shoulder level is wound round a pulley attached to a potentiometer placed well behind the subject. This potentiometer is used to measure the inclination angle. Four light ropes are attached from the shoulders of the body brace to a frame attached to the ceiling in order to prevent the subject falling backwards or forwards too far. For angle control experiments the ropes are slackened sufficiently to allow movement back and forth within predefined limits.

For stimulation of the muscles we use the "Stanmore Stimulator" as described in (Phillips *et al.*, 1993), connected to electrodes which are placed on the skin over the calf muscles. The stimulator provides current controlled monophasic rectangular pulses up to a pulse duration of 800μs. In these experiments the stimulator operates at a constant frequency of 20Hz (sample interval 50ms). At the start of each experiment the current is set at a desired constant level . Thus, during the experiments the stimulation pulses have constant frequency and amplitude, and the pulsewidth is varied.

2.2 Subjects

All experiments reported here were carried out with a 44 year old male subject who has a complete spinal cord lesion at level T7/8 and is 4 years post-injury. Muscle training involved alternate stimulation of the ankle plantarflexor and dorsiflexor muscles for initially 30 minutes per day, which increased to 1 hour per day. The subject's muscles had been trained for 6 months prior to the experimental sessions reported here. The plantarflexors are stimulated by pairs of self-adhesive electrodes (75 mm diameter) which are placed over the midline of soleus.

2.3 Identification Test

This is an open-loop test using a stimulation signal where the pulsewidth has a PRBS (Pseudo-Random Binary Sequence) form. The same stimulation pulsewidth is applied to both legs and the total moment (left + right moments) is measured. The PRBS signal can be applied around a range of mean stimulation pulsewidth levels. The amplitude of the PRBS signal at each mean level was set at 35μs. The PRBS signal was set up at a period of 155 samples, i.e 5 samples per digit (Ljung, 1999). The input/output data arising from the PRBS tests are used to identify (Hunt *et al.*, 1998*a*) local linear transfer functions at each operating point. One of the models is chosen as the nominal model for moment control design.

2.4 Design Procedure

The steps involved in control design are:

(1) The closed loop controller for ankle moment is designed; this step establishes a desired closed-loop response between reference moment m_{ref} and measured moment m. This controller is designed using a polynomial pole placement approach and has been verified by testing real time performance (Hunt *et al.*, 2001). The nominal model for this design is chosen as *one* member of the family of identified models.

(2) The closed loop controller for body inclination angle is designed. The plant for angle controller design is taken as the transfer function between the desired moment m_{ref} and the angle θ, i.e. this is a combination of the ankle moment closed loop and the open loop body dynamics. Angle controller design takes account of the uncertainty in the muscle dynamics, as described below.

2.5 Standing Test

Following angle controller design two types of standing test are carried out.

(1) Disturbance rejection: Here, the reference angle is kept constant and disturbances are applied to the body. We applied disturbances by repeatedly pulling the subject forwards or pushing him back.

(2) Angle Tracking : Typically, a square-wave reference angle of a given amplitude and frequency is applied.

3. H_∞ CONTROLLER DESIGN

The H_∞ approach requires a description for both the nominal system and the uncertainties associated with the model. The formulation of the H_∞ based control problem is outlined here.

3.1 Performance and Stability Specification

The controller $K(s)$, which stabilises the nominal plant $G(s)$, is required to ensure stability and meet performance specifications for all possible plants defined by an uncertainty. These concepts are explained for a closed-loop unsupported standing setup whose block diagram is shown in figure 4. In the diagram $D(s)$ denotes the nominal inner closed loop (moment control), $K(s)$ is the feedback controller to be determined and $\theta(s)$, $\theta_{ref}(s)$, $\delta(s)$, and $\eta(s)$ denote the output, reference, disturbance and measurement noise, respectively. Note that $G(s)$ is composed of the open loop body dynamics and the closed moment control loop $D(s)$ (C_m and Muscle, see Figure 3) . $K(0)$ is the static gain controller which ensures a steady state gain of 1 between $\theta_{ref}(s)$ and $\theta(s)$. $W_g(s)$ $W_d(s)$, $W_y(s)$, $W_r(s)$ and $W_n(s)$ are weighting functions which must be chosen by the designer.

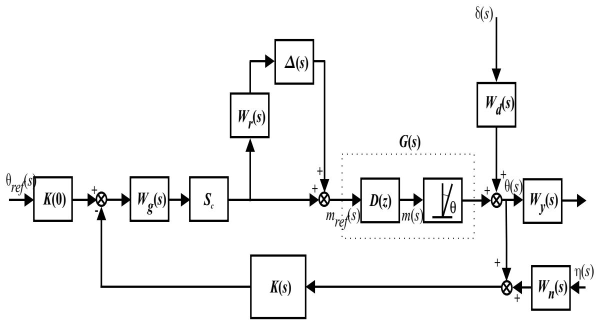

Fig. 4. Generalized and weighted performance block diagram

Now we will look at a typical performance specification, namely tracking reference signals, and see how to quantify it in terms of a weighted H_∞-norm bound. The closed loop performance requirements of the feedback system can be expressed in terms of gain to reduce the influence of disturbances and measurement noises on the output signal. The sensitivity function and the complementary sensitivity function are measured as $|S(j\omega)|$ and $|T(j\omega)|$, and should be as small as possible over the frequency band of the disturbances and measurement noises, respectively. These control objectives can be written using frequency dependent bounds on the sensitivity functions, and norms. Bounds are approximated by gains of transfer function $W_d(s)$, $W_y(s)$ and $W_n(s)$. The sensitivity function performance is represented as :

$$\|W_y S W_d\|_\infty \leq 1 \tag{1}$$

Also, performance of the complementary sensitivity function becomes

$$\|W_y T W_n\|_\infty \leq 1 \tag{2}$$

After the performance problem, we consider the robust stability problem in a weighted norm bound. Suppose the nominal transfer function is G and the plant is perturbed to $\widetilde{G} = (1+\Delta W_r)G$. Here W_r is a fixed real rational weighting function and Δ is a variable real rational transfer function having the following properties:

1. G and $\widetilde{G}$ have the same number of unstable poles,

2. $\|\Delta\|_\infty \leq 1$ (stable), then

$$|(\widetilde{G}(j\omega) - G(j\omega))/G(j\omega)| \leq |W_r\,(j\omega)|\,\forall\omega \tag{3}$$

So $|W_r\,(j\omega)|$ provides the uncertainty profile. The system is robustly stable, for all Δ, with $|\Delta| \leq 1$ ($\forall\omega$), if :

$$\|W_r T\|_\infty \leq 1 \tag{4}$$

Combining these notions acquired previously, we will state that the design specifications are satisfied when the following conditions are satisfied: nominal performance (1)(2) and robust stability (4).Consequently, the final cost function in the H_∞ design is to minimise

$$\left\| \begin{array}{c} W_y S W_d \\ W_y T W_n \\ W_r T \end{array} \right\|_\infty \tag{5}$$

3.2 Standard H_∞ optimisation

Selection of the weighting functions is the most important part of the design process. There are no systematic procedures available for the selection of the weighting functions, other than some guidelines provided in (Skogestad and Postlethwaite, 1997) and (Lundstrom *et al.*, 1991). The following steps are necessary for formulating and solving the weighted mixed sensitivity problem:

(1) The scheme in figure 4 is transformed into the standard H_∞ formulation control problem (Zhou, 1998). It consists of a modified plant $G(s)$, which includes the weighting functions and a controller $K(s)$ which is to be obtained by H_∞.
(2) Choosing the weighting functions based on the knowledge of the plant and design constraints (stability and performance requirements).
(3) Solving the H_∞ optimisation problem and obtaining the controller. More precisely, from the above representation, the H_∞ control problem can be stated as follows; "Find an internally stabilising and feasible controller $K(s)$ for a given plant $P(s)$ such that the H_∞ norm of the linear fractional transformation matrix $F_L(P(s), K(s))$ is below a given level γ", i.e. $\|M\|_\infty < \gamma$;with $\gamma \in \mathbb{R}$ and $\gamma > 0$. The numerical solution of the H_∞ control is solved with MATLAB software which was developed in (Chiang and Safanov, 1988).
(4) There may be situations where a solution may not exist for the H_∞ problem formulated with a particular choice of weighting functions; under such circumstances the design constraints need to be redefined and the above procedures have to be repeated.
(5) The final stage is testing the performance of the designed controller with the real plant and if the performance of the plant with the controller is not satisfactory, adjustments of the weighting functions has to be carried out (i.e in particular if $\|M\|_\infty < 1$ and $\|\Delta\|_\infty \leq 1$, then by the small gain theorem (Zhou, 1998) the perturbed system is robustly stable for all Δ).

4. H_∞ CONTROLLER DESIGN CHOICES FOR UNSUPPORTED STANDING

The equation of motion of the body dynamics (free only to move about the ankle, and maintained upright by a variable moment m about the ankle) is: $-m + \rho g l \sin\theta = J\frac{d^2\theta}{dt^2}$. In this equation ρ is the mass and J is the moment of inertia. The centre of gravity is assumed to be at a distance l from the ankle joint and g is the gravitational acceleration. For small inclination angles we have $\sin\theta \approx \theta$ and the linearised transfer function of the body dynamics becomes :

$$\frac{\theta(s)}{m(s)} = \frac{-1/J}{s^2 - \rho g l} \tag{6}$$

The biomechanical parameters ρ, l and J can be measured for each subject using a simple procedure outlined in (Hunt *et al.*, 1998*a*). The plant considered for angle controller design is composed of the open loop body dynamics and the closed loop muscle moment system.

4.1 Plant perturbation

Three open-loop models of the muscle dynamics are identified at three operating points. One of these was chosen as a nominal model and a moment controller designed using pole assignment, as noted above. This controller together with the family of three open-loop muscle models yields three possible moment loop dynamics; multiplying the open loop body dynamics with the three moment loop transfer functions gives three further transfer functions, and these serve as the possible plants for angle loop design. Thus, the overall system (inverted pendulum with inner loop) is defined by a multiple model representation or family of models. An uncertainty bound will be developed by using the information contained in the models, which is used to carry out robust control design. In order to specify the bound we need to define the uncertainty set by using the family of models. To do that, we have to choose two things:

(1) A nominal plant G.
(2) A multiplicative uncertainty weighting function, W_r.

Given these, the precise definition of the multiplicative uncertainty set is

$$U(G, W_r) := \left\{ \left| \frac{\tilde{G}(j\omega) - G(j\omega)}{G(j\omega)} \right| \leqslant |W_r(j\omega)|, \forall\omega \right\} \tag{7}$$

Choice of the Nominal Model: We select one of the local models as a nominal model. For a larger stability margin we select one of the higher gain models. The nominal model, for angle controller design, then consists of the nominal body dynamics together with the nominal inner loop (closed loop). Table 1 shows the three plant models available for angle control design, together with static gains. These transfer functions were obtained in the following procedure: (i) three open-loop models of the muscle dynamics ($p \to m$) were identified as described above (i.e identification test 2.3); (ii) one of these (Plant 2) was chosen as a nominal model and a moment controller designed using pole assignment - this controller together with the family of three open-loop muscle models yields three possible moment loop dynamics; (iii) the subject's parameters are included in the equation of the body dynamics (6); (iv) multiplying the body dynamics with the three moment loop transfer functions gives the three transfer functions in the table, and these serve as the possible plants for angle loop design. Note that the results reported here are for a subject with physical parameters $J = 90\text{Nms}^2$, $\rho = 90\text{kg}$ and $l = 1\text{m}$.

The proportional plant perturbation is given by $|\Delta(j\omega)| = |(\widetilde{G}(j\omega) - G(j\omega))/G(j\omega)|$. Figure 5 shows plots of the Δ perturbation for various models. The uncertainty bound W_r is chosen as follows :

$$W_r(j\omega) = 0.04\frac{3 + j\omega}{1 + j0.143\omega} \tag{8}$$

Plant	Transfer function	DC-gain
1	$\frac{-0.06694s^3-7.677s^2-285.4s-3471}{s^6+95.65s^5+3347s^4+5.187\times10^4s^3+2.97\times10^5s^2-4.013\times10^5s-2.462\times10^6}$	0.00141
2	$\frac{-0.05542s^3-6.388s^2-238.6s-2913}{s^6+95.9s^5+3316s^4+4.837\times10^4s^3+2.368\times10^5s^2-3.747\times10^5s-2\times10^6}$	0.00145
3	$\frac{-0.04769s^3-5.563s^2-209.7s-2582}{s^6+99.24s^5+3563s^4+5.314\times10^4s^3+2.337\times10^5s^2-4.112\times10^5s-1.991\times10^6}$	0.00129

Table 1. Multiple Model Representation

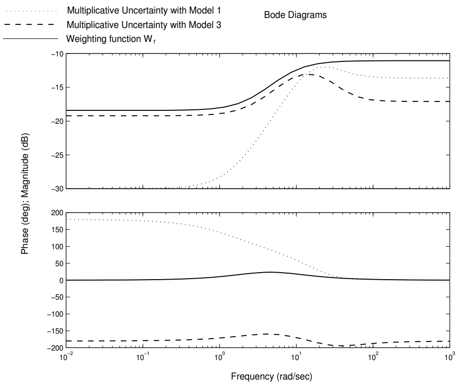

Fig. 5. Plant Perturbation

The numerical values in this transfer function have been chosen such that at each frequency $|W_r(j\omega)|$ represents the maximum potential percentage difference between all of the plants represented by $U(G, W_r)$ and the nominal plant model G. In that sense, $U(G, W_r)$ represents a ball of possible plants, centered at G. On a Nyquist plot, a disk of radius $|W_r(j\omega)G(j\omega)|$, centered at $G(j\omega)$ is the set of possible values that $\widetilde{G}(j\omega)$ can take on, due to the uncertainty description.

4.2 Selection of weighting functions

The closed loop performance objectives are formulated as weighted closed-loop transfer functions which are to be made small through feedback.

Choice of W_g

This post-compensator improves the performance at low frequencies. Therefore W_g is selected by adding integral action in order to achieve low frequency disturbance rejection. We also add a phase-advance term to make the response faster:

$$W_g(s) = \frac{0.9s + 4}{s} \tag{9}$$

Choice of W_d

A disturbance model has been evaluated by investigating the inverted pendulum system, and by consideration of the disturbance forces acting to perturb the body. We choose W_d to give a stable approximation of the disturbances as:

$$W_d(s) = \frac{1}{s + 2.64} \tag{10}$$

Choice of W_y

W_y will be chosen as a constant, for two reasons: first, to keep the order of the augmented system as low possible, and second to achieve the requirement described previously (equations (1) and (2)), thus

$$W_r(s) = 0.5 \tag{11}$$

Choice of W_n

The W_n weight represents frequency domain models of sensor noise. Each sensor measurement feedback to the controller has some noise, which is often higher in one frequency range than another. The W_n weight is derived from laboratory experiments. This weighting function has been evaluated approximately as

$$W_n(s) = \frac{0.3s + 5}{s + 80} \tag{12}$$

In this scheme a scale $S_c = 1000$ has been added because the plant has a low gain and this modification improves conditioning of the H_∞ controller computation. With this choice of weighting functions, the controller design procedure gives the S (sensitivity transfer function from disturbance term δ to output θ) and T (complementary sensitivity transfer function from measurement noise η to output θ) functions shown in figure 6.

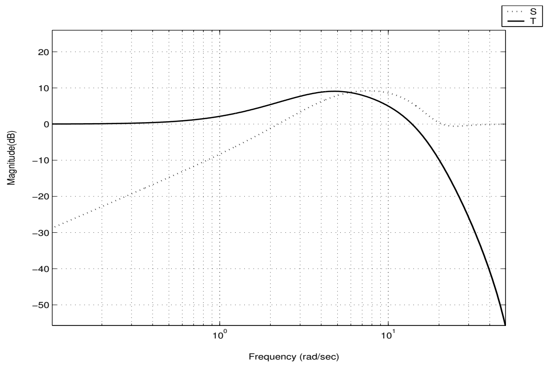

Fig. 6. Sensitivity function $S(j\omega)$ and complementary sensitivity function $T(j\omega)$

5. EXPERIMENTAL RESULTS

The controller design is based on the nominal plant and set of perturbations defined earlier. The following optimal design parameter was found to achieve the best robust stability performance: $\gamma = 1.2$, where γ represents the minimal value of the ∞-norm. Results of the closed loop angle tracking and disturbance rejection tests are shown in figures 7 and 8.

(1) Tracking test: a square wave with a period of 20s (see figure 7) is used as the reference. For the tracking no external disturbance are explicitly applied.
(2) Disturbance test : a constant reference signal is applied (see figure 8), however the standing is disturbed by pulling the subject forward (at t = 9s and t = 20s) and by pushing him backwards (t = 15s and t = 25s).

In all of these plots the upper graph shows the stimulation pulsewidth p, the middle graph shows reference moment m_{ref} and measured moment m, and the lower graph shows reference angle θ_{ref} and measured angle θ. In both cases the controller gives us results satisfying tracking and disturbance rejection, and so achieves robust stability against model uncertainties. Fast response and zero steady state error are achieved.

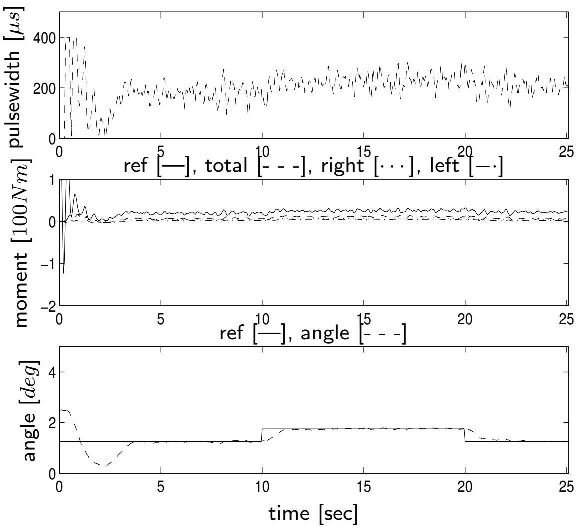

Fig. 7. Test T : Reference tracking (period 20s).

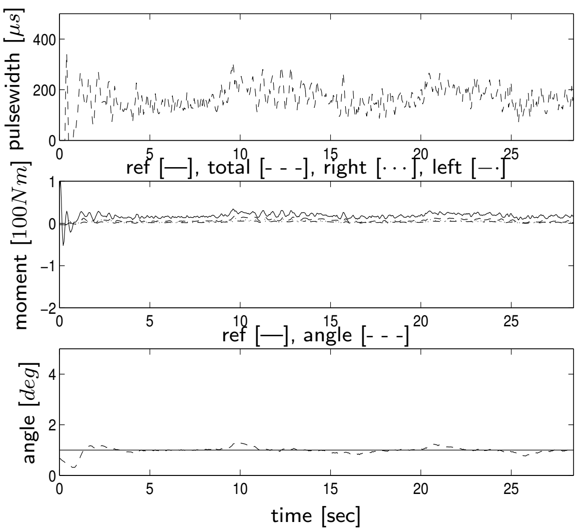

Fig. 8. Test D : Disturbance rejection. The subject is pulled forward at 9s and 20s, and is pushed backwards at 15s and 25s.

6. CONCLUSION

In this paper we have successfully applied robust H_∞ control theory to the control of the upright posture of paraplegic persons standing. Experimental results of reference tracking and disturbance rejection demonstrated good performance and closed loop stability over the whole range of plant operation. The weighted sensitivity approach was found to give a convenient and transparent way to design for performance. Moreover, the perturbation/weight plots, together with the minimal value of the ∞-norm, give a useful measure of the robustness properties of a given design. The resulting minimal ∞-norm value gives a clear way to characterise the controller, and to address the performance and bandwidth trade-offs.

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